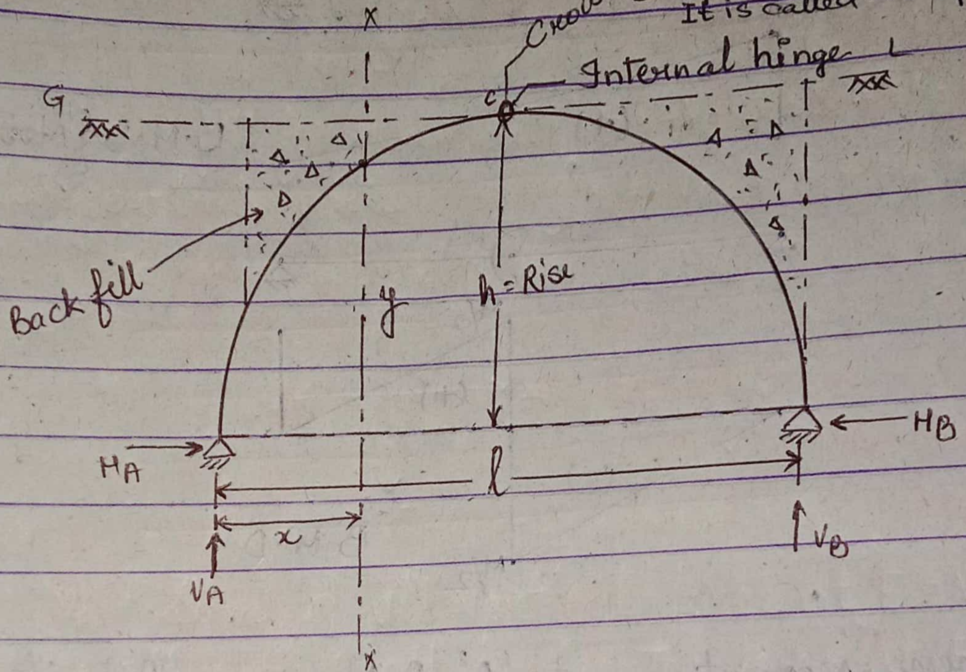
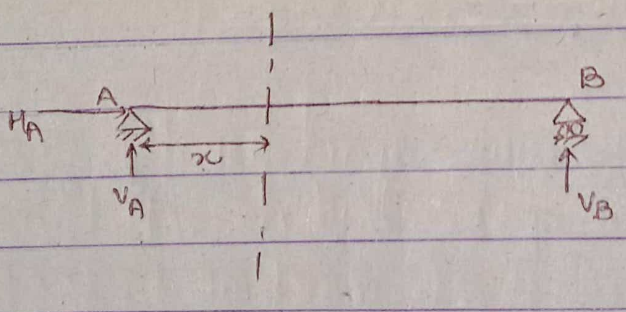
THREE HINGE ARCH



$$M_X = \underbrace{V_A \times x}_{\text{Beam moment}} - \underbrace{H_A \times y}_{\text{Horizontal moment}}$$



$M_x = V_A \times x$

⇒ A beam of same span bending moment at any section in a three hinge arch is less by an amount of $H_A \times y$ (Hy) or horizontal moment.

⇒ Three hinge arch is statically indeterminate structure that is equation of equilibrium alone are sufficient to find all the unknown quantity.

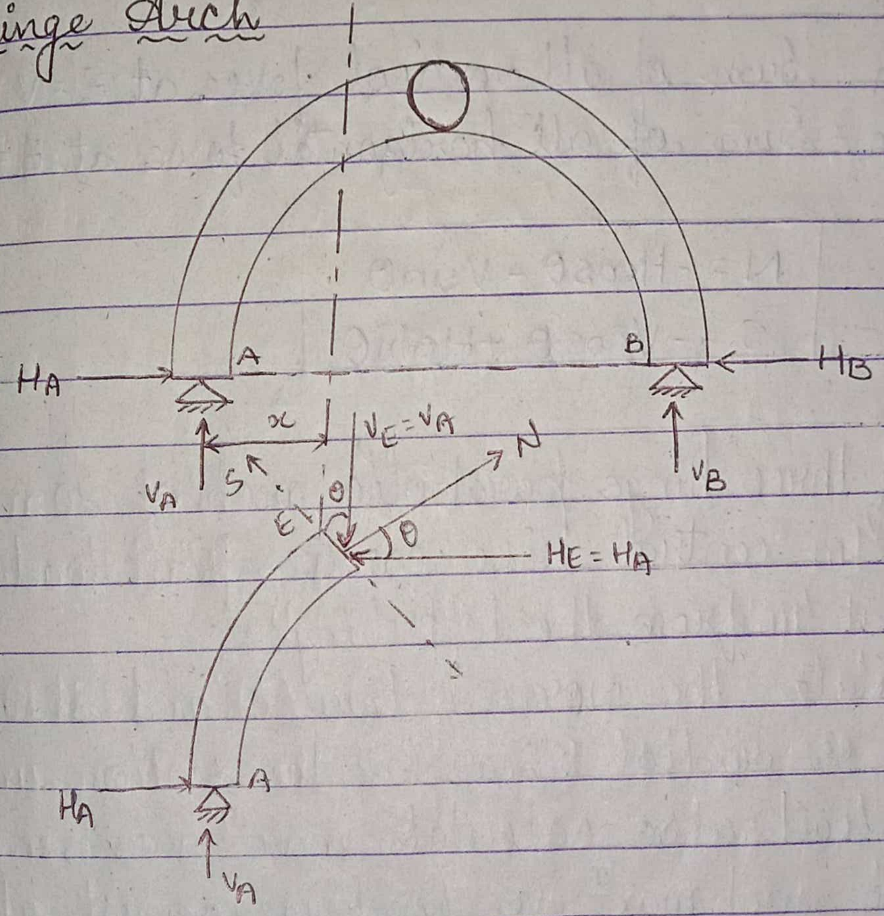
i.e.,

D_s = Total known - Total no. of equilibrium equation available.

$= 4 - (3+1)$ {due to internal hinge}

$[\Sigma H = 0 , \Sigma M = 0 , \Sigma V = 0]$

Normal Thrust and Radial Shear in Three Hinge Arch



N = Normal thrust = Sum of components of all forces (acting at E) Normal to c/s.

$N = -H_E \cos\theta - V_E \sin\theta$ {∴ Resolving all forces about Normal axis}

$N = -H_A \cos\theta - V_A \sin\theta$

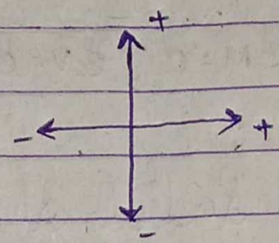
S = Radial shear = Sum of components of all forces (acting at E) parallel to c/s.

$\therefore$ Resolving all forces about tangential axis

$S = -V_E \cos\theta + H_E \sin\theta$

$S = -V_A \cos\theta + H_A \sin\theta$

<u>Note :-</u>



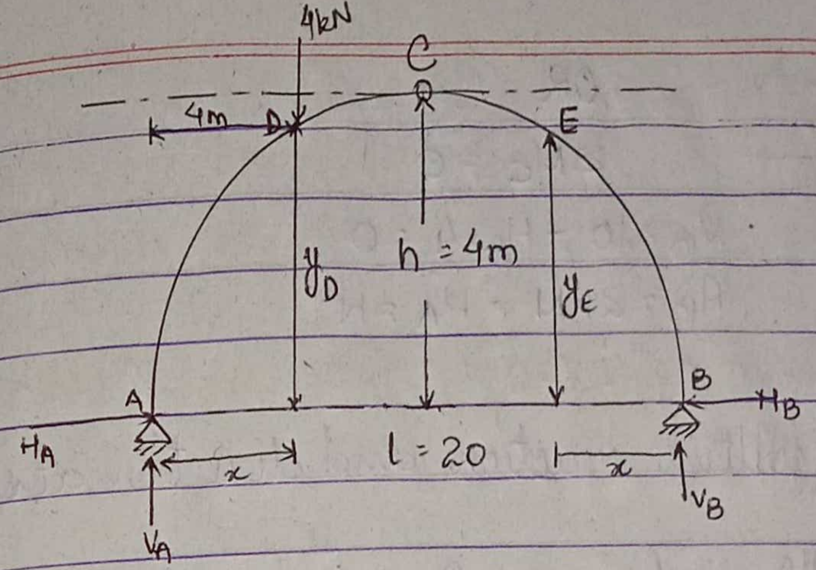
where

V_A = Sum of all vertical force at E = V

H_A = Sum of all horizontal force at E = H

$$\boxed{\begin{aligned} N &= -H\cos\theta - V\sin\theta \\ S &= -V\cos\theta + H\sin\theta \end{aligned}}$$

Ques :- A three hinge parabolic arch of 20 m span and 4 m central rise carries point load of 4 kN at 4 m from the left hinge.
Calculate the normal thrust and shear force or radial shear at the section under the load also calculate max^m +ve bending moment and max^m -ve bending moment also find reaction at support and its direction with horizontal ?



Sol:- Step 1:- Support Reaction

$$\Sigma V = 0$$

$$V_A + V_B = 4 \quad \text{——— (1)}$$

$$\Sigma M_B = 0$$

$$V_A \times 20 - 4 \times 16 = 0$$

$$V_A = \frac{64}{20} = 3.2 \text{ kN}$$

from (1)

$$V_B = 4 - V_A = 4 - 3.2 = 0.8 \text{ kN}$$

$$\Sigma H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = H \text{ (say)}$$

L to R $\rightarrow$ $\Sigma M_C = 0$ (due to internal hinge)

$$V_A \times 10 - 4 \times 6 - H_A \times 4 = 0$$

$$H_A = 2 \text{ kN} = H_B = H$$

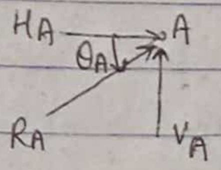
OR

R to L $\rightarrow$ $\Sigma M_C = 0$

$V_B \times 10 - H_B \times 4 = 0$

$H_B = 2 kN = H_A = H$

Step 2: Resultant reaction and direction at support:



$R_A = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{(3.2)^2 + (2)^2}$

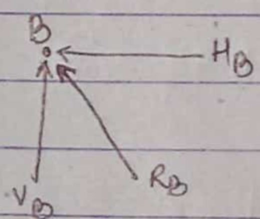
$= 3.77 kN$

$\tan\theta_A = \frac{V_A}{H_A}$ $\left\{ \because \tan\theta = \frac{L}{A} \right\}$

$\theta_A = \tan^{-1} \frac{V_A}{H_A}$

$= \tan^{-1} \frac{3.2}{2}$

$= 58^\circ$



$R_B = \sqrt{V_B^2 + H_B^2}$

$= \sqrt{(0.8)^2 + (2)^2}$ $= 2.15 kN$

$\tan\theta_B = \frac{V_B}{H_B}$

$\theta_B = \tan^{-1} \frac{V_B}{H_B}$

$= \tan^{-1} \frac{0.8}{2}$

$= 21.80^\circ$

Step (3): Normal thrust & Radial shear or Shear force

$$N = -H\cos\theta - V\sin\theta \quad \text{——— (a)}$$
$$S = -V\cos\theta + H\sin\theta \quad \text{——— (b)}$$

$$H = H_A = 2\ kN$$

at D_R
$$V = V_A - 4 = -0.8\ kN$$

at D_L
$$V = V_A = 3.2\ kN$$

We know that from Parabola

$$y = \frac{4hx}{l^2}(l - x)$$

$$y = \frac{4 \times 4 \times x}{(20)^2}(20 - x)$$

$$y = \frac{x}{25}(20 - x)$$

$$= \frac{20}{25}x - \frac{x^2}{25}$$

$$\frac{dy}{dx} = \frac{20}{25} - \frac{2x}{25} = \tan\theta$$

$$(\tan\theta)_{at\ x = 4m} = \frac{20}{25} - \frac{2 \times 4}{25} = 0.48$$

$\theta = \tan^{-1}(0.48) = 25.64^\circ$

from eq^n (a).

$N = -2\cos 25.64^\circ - (-0.8)\sin\theta$ $(v = -0.8)$
$N = -1.46$ kN

$N = -2\cos 25.64^\circ - 3.2\sin\theta$ $(V = 3.2)$
$N = -3.18$ kN

from eq^n (b)

$S = -3.2\cos\theta + 2\sin\theta$ $(V = 3.2)$
$S = -2.01$ kN

$S = -(-0.8)\cos\theta + 2\sin\theta$ $(V = -0.8)$
$S = 1.58$ kN

Step 4: Max^m +ve bending moment & Max^m -ve bending moment :

Max^m +ve bending moment :
We all know that max^m +ve BM just under the point load.

$M_D = M_{max}(+ve) = V_A \times 4 - H_A \times y_D$ — (c)

We know that

$$y = \frac{4hx}{l^2}(l-x)$$

$y_D = +2.56m$

from eqn (C)

M_{max} (+ve) $= 3.2 \times 4 - 2 \times 2.56$
$= +7.68$ kN-m

Max^{m} -ve bending moment :-

Max^{m} -ve bending moment will be somewhere at B and D.

Let,
Max^{m} -ve bending moment at a distance x from B.

$M_E = M_{max}(-ve) = V_B \times x - H_B \times y_E$
we know that

$$y_E = \frac{4hx}{l^2}(l-x)$$

$$= \frac{4 \times 4x}{(20)^2}(20-x)$$

$$= \frac{x}{25}(20-x)$$

$$M_{max}(-ve) = 0.8x - 2 \times \frac{x}{25}(20-x)$$

$$M_{max}(-ve) = 0.8x - \frac{40}{25}x + \frac{2x^2}{25} \quad \text{———} \quad (d)$$

We know that
for M_{max} (-ve)

$$\frac{dM_{max}(-ve)}{dx} = 0$$

$$0.8 - \frac{40}{25} + \frac{2 \times 2x}{25} = 0$$

$$x = 5m$$

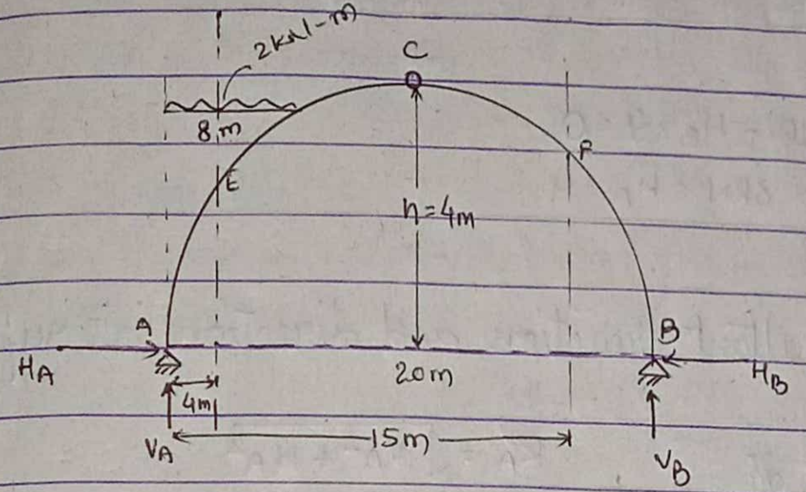
from eq^n (d)

$$M_{max}(-ve) = 0.8 \times 5 - \frac{40}{25} \times 5 + \frac{2(5)^2}{25}$$

$$= -2 \text{ kN-m}$$

Ques:- A parabolic arch hinged at springings at crown has a span of 20 cm. and central rise of the arch is 4m.
If it is loaded with a UDL 2kN/m on the left 8m length from left support. Calculate A direction and magnitude of reaction at the hinge at B. BM, NT & Shear at 4m & 15m

from left end. Calculate Max^{m} +ve and -ve bending moment.



Sol:- Step 1:- Support Reaction

$$\Sigma V = 0$$

$$V_A + V_B = 2 \times 8 = 16 \text{ kN} \quad \text{--- (1)}$$

$$\Sigma M_B = 0$$

$$V_A \times 20 - (2 \times 8) \times \left(\frac{8}{2} + 12\right) = 0$$

$$V_A = 12.8 \text{ kN}$$

from L to R $\longrightarrow$

$$V_B = 16 - 12.8 = 3.2 \text{ kN}$$

$$\Sigma H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = H \text{ (say)}$$

$$\Sigma M_C = 0$$

$$V_A \times 10 - H_A \times 4 - (2 \times 8) \times 6 = 0$$

$$12.8 \times 10 - H_A \times 4 - 16 \times 6 = 0$$

$$128 - H_A \times 4 - 96 = 0$$

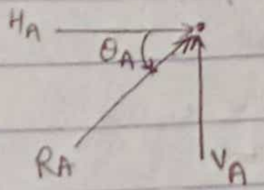
$H_A = 8kN = H_B = H$

OR R to L

$V_B \times 10 - H_B \times 4 = 0$

$H_B = 8kN = H_A = H$

Step 2:- Resultant reaction and direction at support



$R_A = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{(12.8)^2 + (8)^2}$

$= 15.09 kN$

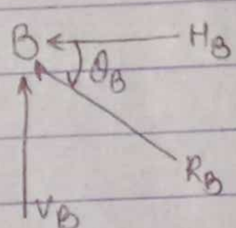
$\tan\theta_A = \frac{V_A}{H_A}$

$\tan\theta_A = \frac{12.8}{8}$

$\theta_A = \tan^{-1} 1.6$

$= 57.99$

$\theta_A = 58'$



$R_B = \sqrt{(V_B)^2 + (H_B)^2}$

$= \sqrt{(3.2)^2 + (8)^2}$

$R_B = 8.61 kN$

$\tan\theta_B = \frac{V_B}{H_B}$

$\theta_B = \tan^{-1} \frac{3.8}{8}$

$\theta_B = 21.80^\circ$

Step 3:- Bending Moment, Normal Thrust & Radial Shear or shear force.

At 4m from left support

$$B.M_{4m} = V_A \times 4 - H_A \times y_4 - 2 \times 4 \times \frac{4}{2} \quad \text{—— (2)}$$

$$B.M_{4m} = 12.8 \times 4 - 8 \times y_4 - 2 \times 4 \times 2$$

We know that from eq^n of parabola

$$y = \frac{4hx}{l^2}(l-x)$$

$$y_4 = \frac{4 \times 4 \times 4}{(20)^2}(20-4)$$

$$y_4 = 2.56\ m$$

from (2)

$$B.M_{4m} = 12.8 \times 4 - 8 \times 2.56 - 2 \times 4 \times 2$$

$$B.M_{4m} = 14.72\ kN\text{-}m$$

We know that

$$N = -H\cos\theta - V\sin\theta \quad \text{—— (a)}$$

$$S = -V\cos\theta + H\sin\theta \quad \text{—— (b)}$$

$H = H_A = 8 kN$

$V = V_A - 2 \times 4 = 12.8 - 8 = 4.8 kN$

also from eq^n of parabola

$$y = \frac{4hx}{l^2}(l-x)$$

$$= \frac{4 \times 4 \times x}{(20)^2}(20-x)$$

$$y = \frac{x}{25}(20-x)$$

$$y = \frac{20x}{25} - \frac{x^2}{25}$$

$$\frac{dy}{dx} = \frac{20x}{25} - \frac{x^2}{25} = \tan\theta$$

$$(\tan\theta)_{at\ 4m} = \frac{20}{25} - \frac{2x}{25}$$

$$= \frac{20}{25} - \frac{2 \times 4}{25}$$

$$= \frac{20}{25} - \frac{2 \times 4}{25} = 0.48$$

$\theta = \tan^{-1} 0.48$

$\theta = 25.64°$

from eq^n (a)

$N = -8\cos 25.64° - 4.8 \sin 25.64°$

$= -9.28 kN$

$S = -4.8 \cos 25.64° + 8 \sin 25.64°$
$= 0.861$ kN

At 15m from left support

$$M_{15m} = V_A \times 15 - H_A \times y_{15} - 2 \times 8 \times \left(\frac{8}{2} + 7\right) \quad \text{——— (3)}$$

We know that from eq^n of parabola

$$y = \frac{4hx}{l^2}(l-x)$$

$$y_{15} = \frac{4 \times 4 \times 15}{(20)^2}(20-15)$$

$$y_{15} = \frac{15}{25}(5)$$

$$y_{15} = \frac{3}{5}(5)$$

$$\boxed{\therefore y_{15} = 3}$$

from eq^n (3)

$$B.M_{15m} = 12.8 \times 15 - 8 \times y_{15} - 2 \times 8 \times \left(\frac{8}{2} + 7\right)$$
$$= 12.8 \times 15 - 8 \times 3 - 2 \times 8 \times (4+7)$$

$$B.M_{15m} = -8 \text{ kN-m}$$

We know that

$N = -H\cos\theta - V\sin\theta$ —— (4)

$S = -V\cos\theta + H\sin\theta$ —— (5)

$H = H_A = 8\,kN$

$V = V_A - 2\times 8$

$V = -3.2$

Also from eqn of parabola

$y = \frac{4hx}{l^2}(l-x)$

$= \frac{4\times 4\times x}{(20)^2}(20-x)$

(tan

$y = \frac{x}{25}(20-x)$

$y = \frac{20x}{25} - \frac{x^2}{25}$

$\frac{dy}{dx} = \frac{20x}{25} - \frac{x^2}{25} = \tan\theta$

$(\tan\theta)_{at\,15m} = \frac{20}{25} - \frac{2x}{25}$

N

$= \frac{20}{25} - \frac{2\times 15}{25} = \frac{20}{25} - \frac{30}{25} = -21.80$

$\theta = -21.80^\circ$

from eq^n (4)

$N = -8\cos(-21.80^\circ) - (-3.2)\sin(-21.80^\circ)$
$N = -8.61$ kN

from eq^n (5)

$S = -(-3.2)\cos(-21.80^\circ) + 8\sin(-21.80^\circ)$
$= 3.2\cos(-21.80^\circ) + 8\sin(-21.80^\circ)$
$= +2.97 - 2.97$
$= 0$

Step 4:- Max^m +ve Bending Moment and Max^m -ve Bending Moment.

Max^m +ve Bending moment

Max^m +ve B.M will occur somewhere between A & D.

Let Max^m +ve B.M is at a distance x from A.

$$M_x = M_{max}(+ve) = V_A \times x - H_A \times y_x - 2 \times x \times \frac{x}{2}$$
$$= 12.8 \times x - 8 \times \frac{x}{25}(20-x) - x^2$$

$$y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 4x}{(20)^2}(20-x) = \frac{x}{25}(20-x)$$

$$M_{max}(+ve) = 12.8x - \frac{160}{25}x + \frac{8x^2}{25} - x^2 \quad \text{——— (c)}$$

For Max^{m} +ve B.M we know that $\frac{dM_{max}(+ve)}{dx} = 0$

$$\frac{dM_{max}(+ve)}{dx} = 0$$

$$\frac{d}{dx}\left(12.8 - \frac{160}{25} + \frac{2 \times 8x}{25} - 2x\right) = 0$$

$$12.8 - 6.4 + \frac{16x}{25} - 2x = 0$$

$$\frac{16x - 50x}{25} = -6.4$$

$$x = 4.70m$$

from eq^{n} (c)

$$M_{max}(+ve) = 12.8(4.70) - \frac{160}{25}(4.70) + \frac{8(4.70)^2}{25} - (4.70)^2$$

$$= 60.16 - 30.08 + 7.06 - 22.09$$

$$M_{max}(+ve) = +15.06 \text{ kN-m}$$

Max^{m} (-ve) BM is at a distance x from B.

$$M_x = M_{max}(-ve) = V_B \times x - H_B \times y_x$$

$$= 3.2 \times x - 8 \times \frac{x}{25}(20-x)$$

$$= 3.2x - \frac{160x}{25} - \frac{8x^2}{25} \quad \text{——— (d)}$$

We know that
for M_{max} (-ve) Bending moment

$$\frac{dM_{max}(-ve)}{dx} = 0$$

$$3.2 - \frac{160}{25} + \frac{2 \times 8 \times x}{25} = 0$$

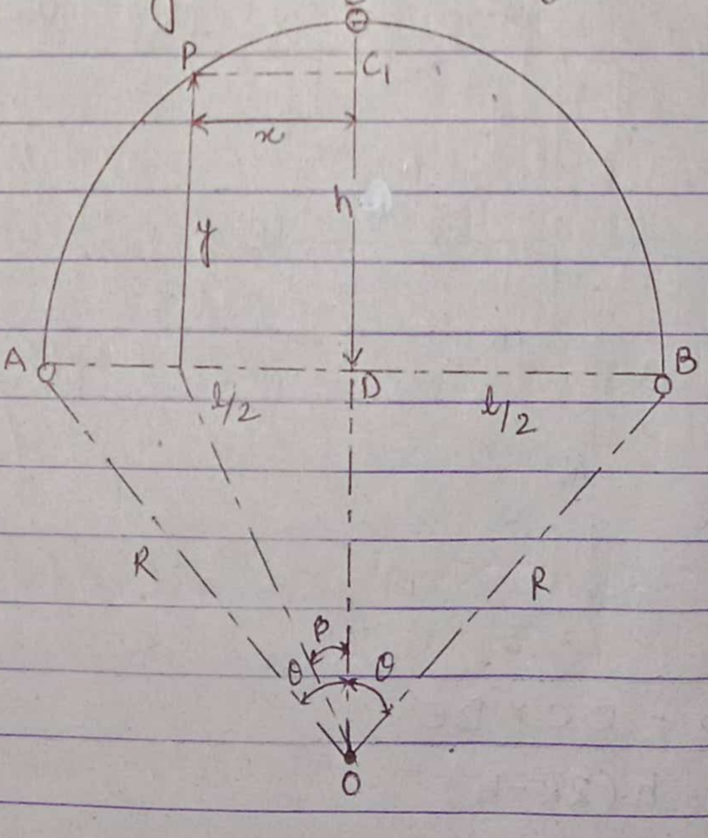
$$x = 5m$$

from eq^n (d)

$$M_{max}(-ve) = 3.2 \times 5 - \frac{160 \times 5}{25} + \frac{8 \times (5)^2}{25}$$

$$M_{Max}(-ve) = -8 \text{ kN-m}.$$

* Three Hinge circular Arch :-



Let's know consider the central line of the arch to be segment of a circle of radius R, subtending an angle of 2θ at the angle its always convental to have the origin D. The middle of the span. Let (x, y) be the coordinates of the point P. Draw the line PC_1 parallel to AB.

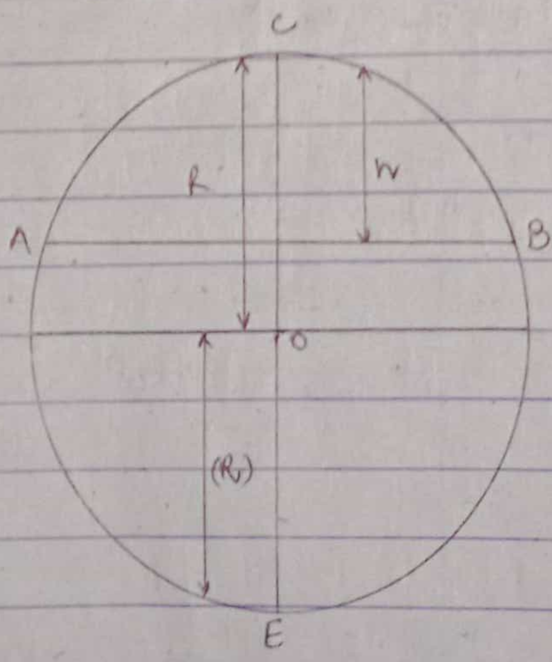
Now,

$$OP^2 = PC_1^2 + OC_1^2 \quad \{\text{Using pythagoras theorem}\}$$

$$R^2 = x^2 + [OD + DC_1]^2$$

$$R^2 = x^2 + [(R-h) + y]^2 \quad \text{--- (1)}$$

From property of circle we can write



$$AD \times DB = DC \times DE$$

$$\frac{l}{2} \times \frac{l}{2} = h(2R-h)$$

$$\frac{l^2}{4} = (2R - h)h \quad \text{—— (2)}$$

From eq^n (2) we can calculate radius R.

⇒ The coordinates of P i.e., (x, y) can also be expressed as trignometric function. Thus if OP makes an angle β with OC then

$$x = OP \sin\beta = R\sin\beta$$

$$y = C_1D = OC_1 - OD = R\cos\beta - R\cos\theta$$

$$y = R(\cos\beta - \cos\theta)$$

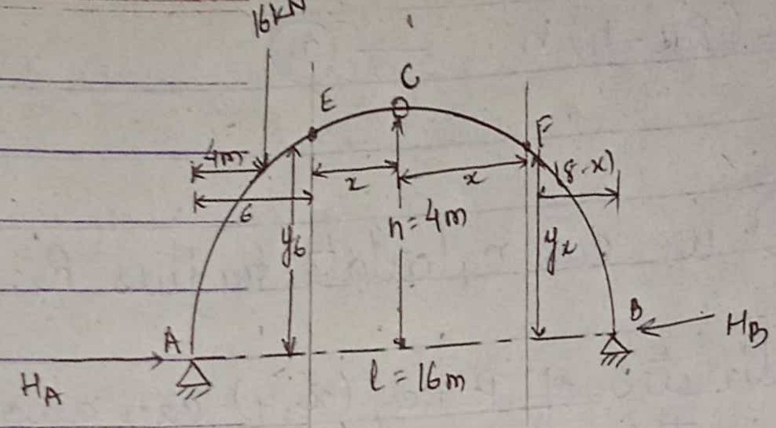
Ques:- A symmetrical three hinge circular arch has a span of 16m and a rise of 4m. it carries a vertical load of 16kN at 4m from the left and end the magnitude of thrust.

Find (a) magnitude of thrust at springing also find direction.

(b) Bending moment at 6m from left end hinge.

(c) Max^m +ve and -ve B.M

(d) Normal thrust, radial shear & shear force under the load.



Sol:- Step (1) : Support Reaction :-

$$\Sigma V = 0$$

$$V_A + V_B = +16\,kN \quad \text{—— (1)}$$

$$\Sigma M_B = 0$$

$$V_A \times 16 - 16 \times 12 = 0$$

$$V_A = 12\,kN$$

from eq^n (1)

$$V_B = 4\,kN$$

$$\Sigma H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = 0$$

$$H_A = H_B = H \text{ (say)}$$

$$\Sigma M_C = 0$$

L to R $\Rightarrow$ $V_A \times 8 - 16 \times 4 - H_A \times 4 = 0$

$$H_A = 8\,kN = H_B = H$$

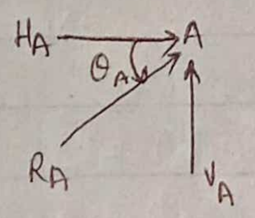
OR

R to L

$$V_B \times 8 - H_B \times 4 = 0$$

$$H_B = 8\,kN = H_A = H$$

Step(2):- Resultant Reaction :-



$$R = \sqrt{(V_A)^2 + (H_A)^2}$$

$$= \sqrt{(12)^2 + (8)^2}$$

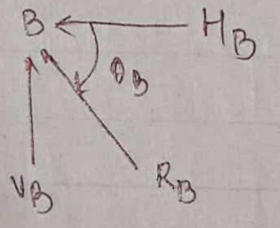
$$\boxed{R = 14.42\,kN}$$

$$\tan\theta_A = \frac{V_A}{H_A}$$

$$= \frac{12}{8}$$

$$\theta_A = \tan^{-1} 1.5$$

$$\boxed{\theta_A = 56.30^\circ}$$



$$R = \sqrt{(4)^2 + (8)^2}$$

$$\boxed{R = 8.94\,kN}$$

$$\tan\theta_A = \frac{V_B}{H_B}$$

$$\tan\theta_A = \frac{4}{8} \Rightarrow \theta_A = \tan^{-1} 0.5$$

$$\boxed{\theta_A = 26.56^\circ}$$

Step (3) :- Relation b/w y & x

from property of circle we know that

$$\frac{l^2}{4} = (2R - h)h$$

$$\frac{(16)^2}{4} = (2R - 4)4$$

$$64 = 8R - 16$$

$$64 + 16 = 8R$$

$$\boxed{R = 10m}$$

And also we know that

$$R^2 = x^2 + [(R-h) + y]^2$$

$$(10)^2 = x^2 + [(10-4) + y]^2$$

$$100 = x^2 + [6+y]^2$$

$$[6+y]^2 = 100 - x^2$$

taking $\frac{1}{2}$ power both side

$$6 + y = \sqrt{100 - x^2}$$

$$y = \sqrt{100 - x^2} - 6$$

$$y = (100 - x^2)^{1/2} - 6 \quad \text{——— (2)}$$

Step (4) :- Bending moment at 6m from left support :

$M_6 = V_A \times 6 - 16 \times 1 - H_A \times y_6$ —— (3)

from eq^n (2)

$y = (100 - x^2)^{1/2} - 6$

y_6 from left support = y_2 from centre

$y_6 = [100 - (2)^2]^{1/2} - 6 = 3.79$ m

from eq^n (3)

$M_6 = 12 \times 6 - 16 \times 2 - 8 \times 3.79$

$M_6 = 9.6$ kN-m.

Step - 5 :- Max^m +ve and Max^m +ve bending moment.

⇒ Max^m +ve bending moment :- We know that Max^m under the point load it means

$M_4 = M_{max}(+ve) = V_A \times 4 - H_A \times y_4$ —— (4)

$y = (100 - x^2)^{1/2} - 6$

y_4 from left support = y_4 from centre.

$y_4 = \left[100 - (4)^2\right]^{1/2} - 6$

$= 3.16$ m.

from eq^n (4)

$M_4 = M_{max}(+ve) = 12\times4 - 8\times3.16$

$M_{max}(+ve) = +22.72$ kN-m.

$\Rightarrow$ Max^m -ve bending moment :- It will occur some where b/w D & B.

Let Max^m -ve B.M is at a distance x from centre towards right.

$$M_x = M_{max}(-ve) = V_B \times (8-x) - H_B \times y_x$$
$$= 4\times(8-x) - 8\times\left[(100-x^2)^{1/2} - 6\right]$$
$$= 32 - 4x - 8(100-x^2)^{1/2} + 48 \longleftarrow (5)$$

For $M_{max}(-ve)$ B.M we know that

$$\frac{dM_{max}(-ve)}{dx} = 0$$

$$0 - 4 - \frac{8}{2}(100-x^2)^{-1/2}(-2x) + 0 = 0$$

$$\frac{8x}{(100-x^2)^{1/2}} = 4$$

$$8x = 4(100-x^2)^{1/2}$$

$$2x = (100-x^2)^{1/2} \Rightarrow (2x)^2 = \left[(100-x^2)^{1/2}\right]^2$$

$$4x^2 = 100 - x^2$$

$$5x^2 = 100$$

$$x^2 = 20$$

$$x = 4.47$$

from eq^n (5)

$= 32 - 4(4.47) - 8\left[100 - (4.47)^2\right]^{1/2} + 48$
$= -9.44 \text{ kN-m}.$

Step 5:- Normal thrust, Radial shear or shear force.

There is no difference in the analysis of three hinge parabolic arch or circular arch by concept of structure point of view. There is only one difference i.e., relation b/w y & x in parabolic arch we use $\boxed{y = \frac{4hx}{l^2}(l-x)}$ and x is consider from either left end or right end.

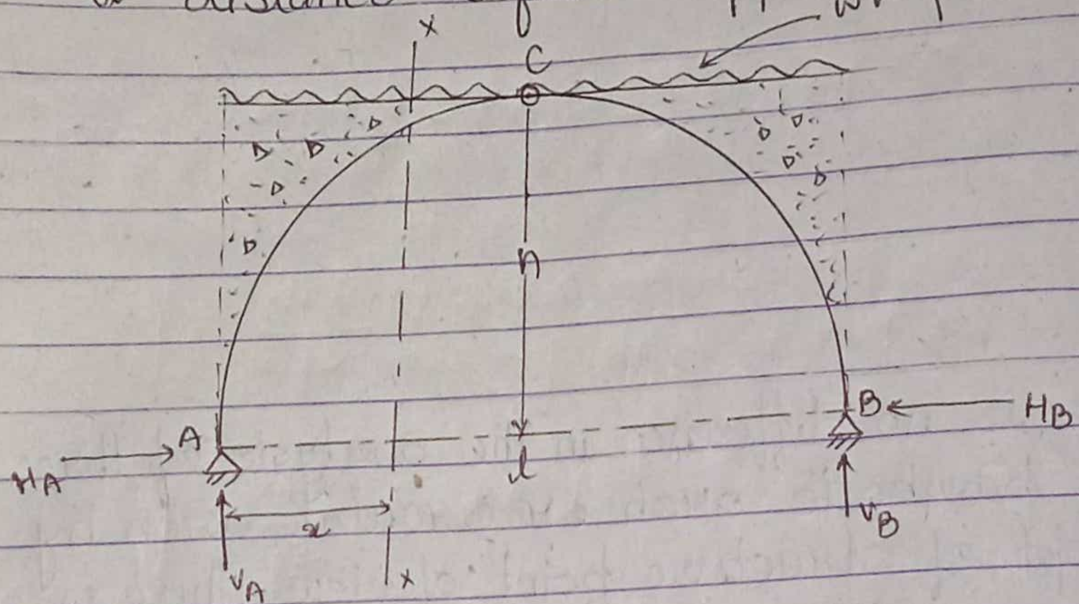
For circular arch to develop relation b/w y & x

* 1^{st}ly define R from the property of circle $\frac{l^2}{4} = (2R-h)h$. and then after use

$$R^2 = x^2 + \left[(R-h) + y\right]^2$$

and x is consider from center of the span always.

Ques:- A three hinge parabolic arch is subjected to UDL of ω kN/m throughout its length findout bending moment, radial shear and normal thrust at a distance x from support.



Step 1:- Support Reaction :-

$$\Sigma V = 0$$

$$V_A + V_B = \omega \times l \quad \text{—— (1)}$$

Since load is symmetric hence

$$V_A = V_B = \frac{\text{Total load}}{2} = \frac{\omega l}{2}$$

$$\Sigma M_C = 0$$

$$V_A \times \frac{l}{2} - H_A \times h - \omega \times \frac{l}{2} \times \frac{1}{2} \times \frac{l}{2} = 0$$

$$V_A \times \frac{l}{2} - H_A \times h - \frac{\omega l^2}{8} = 0$$

$$\frac{wl}{2} \times \frac{l}{2} - H_A \times h - \frac{wl^2}{8} = 0$$

$$-H_A \times h = \frac{wl^2}{8} - \frac{wl^2}{4}$$

$$H_A = \frac{wl^2}{8h} = H_B = H$$

Step(2) :- B.M at x from support

$$M_x = V_A \times x - H_A \times y_x - w_x \times \frac{x}{2}$$

$$M_x = \frac{wl}{2} \times x - \frac{w\cancel{l^2}}{8\cancel{h}} \cdot \frac{4\cancel{h}x}{\cancel{l^2}}(l-x) - \frac{wx^2}{2}$$

$$M_x = \frac{wlx}{2} - \frac{wlx}{2} + \frac{wx^2}{2} - \frac{wx^2}{2}$$

$$\boxed{M_x = 0}$$

NOTE :-

If a three hinge parabolic arch is subjected to UDL throughout its length then B.M is zero at every where.

Step (3) :- Radial shear at $x - x$

$$S = -V\cos\theta + H\sin\theta$$

$$H = H_A = \frac{wl^2}{8h}$$

$$V = V_A - w_x$$

$$\frac{wl}{2} - wx_1$$

$$S = -\left(\frac{wl}{2} - wx\right)\cos\theta + \frac{wl^2}{8h}\sin\theta$$

dividing both side by $\cos\theta$

$$\frac{S}{\cos\theta} = -\left(\frac{wl}{2} - wx\right) + \frac{wl^2}{8h}\tan\theta$$

$$y = \frac{4hx}{l^2}(l-x)$$

$$y = \frac{4h}{l^2}(lx - x^2)$$

$$\frac{dy}{dx} = \frac{4h}{l^2}(lx - x^2) = \tan\theta$$

$$\tan\theta = \frac{4h}{l^2}(l-2x)$$

$$\frac{S}{\cos\theta} = -\left(\frac{wl}{2} - wx\right) + \frac{wl^2}{8h} \times \frac{4h}{l^2}(l-2x)$$

$$S = 0$$

NOTE :- If a three hinge parabolic arch is subjected to UDL throughout its length then radial shear is zero everywhere in the arch.

Step (4):- Normal thrust at x - x

$$N = -H\cos\theta - V\sin\theta$$

$$H = H_A = \frac{wl^2}{8h}$$

$$V = V_A - wx$$

$$= \frac{wl}{2} - wx$$

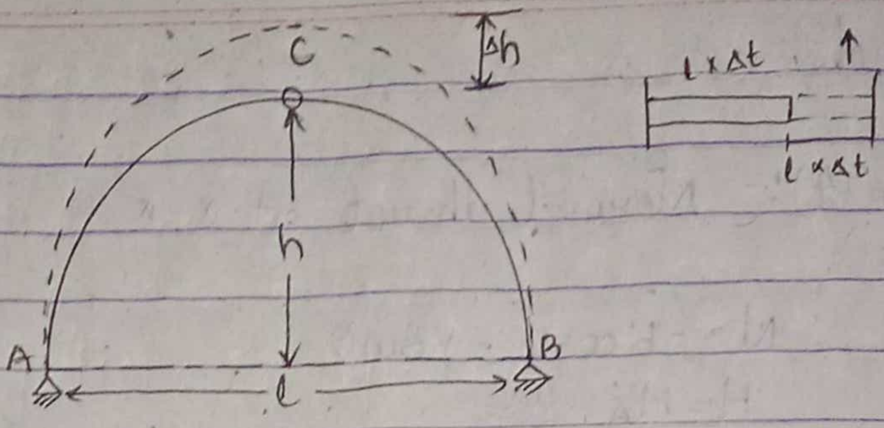
$$N = \frac{-wl^2}{8h}\cos\theta - \left(\frac{wl}{2} - wx\right)\sin\theta$$

$$N \neq 0.$$

NOTE: If a three hinge parabolic arch or two hinge parabolic arch is subjected to UDL throughout its length then BM and radial shear are zero at everywhere in the arch always. The C/s is subjected to normal thrust only.

Temperature Effect on Three Hinge Arch :-

Case 1:- When a three hinge arch is not subjected to any load and subjected to temp. rise only.



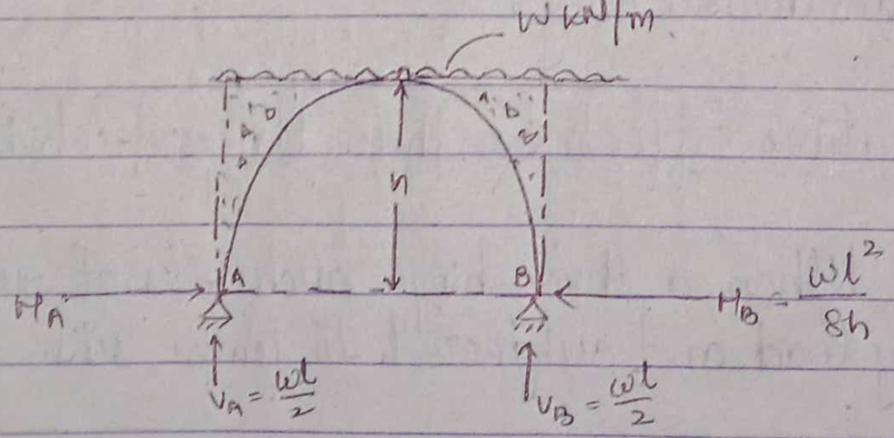
Since three hinge arch is statically determinate structure, temp. stress is not develop anywhere in the arch.

But, due to increase in temp. the crown of the arch will rise by an amount of Δh

$$\Delta h = \left(\frac{l^2 + 4h^2}{4h}\right) \times \Delta t$$

Case (2)

When the arch is subjected to UDL through out its length 1st and then subjected to the temp. rise of t°c.



⇒ Due to applied load

$$H = H_A = H_B = \frac{wl^2}{8h}$$

⇒ Due to increase in temp. h increase by Δh
so H will decrease

$$H = \frac{wl^2}{8h}$$

$$\frac{dH}{dh} = \frac{-wl^2}{8h^2}$$

$$dH = \frac{-wl^2}{8h^2}\,dh$$

% Change in horizontal thrust

$$= \frac{dH}{H} = \frac{\frac{-wl^2}{8h^2}\,dh}{\frac{wl^2}{8h}} = \frac{-dh}{h}$$

$$\boxed{\frac{dH}{H} = \frac{-dh}{h} = \frac{-\Delta h}{h}}$$

∴ where

dH = Change in horizontal thrust
H = actual horizontal thrust
dh = Change in crown height
h = actual crown height

Ques:- A three hinge parabolic arch of rise 4m is subjected to UDL throughout it's length show that horizontal thrust is 10kN. Due to increase in temp. If the crown is rises by 4 cm; change in horizontal thrust is?

Sol:- Given that,

$$H = 10kN$$
$$dh = 4cm$$
$$h = 4m$$

$$\frac{dH}{H} = \frac{-dh}{h}$$

$$dH = -\frac{dh}{h} \times H = \frac{-0.04}{4} \times 10$$

$$dH = -0.1kN$$

-ve sign shows that decrease in horizontal thrust

final reaction

$$dH = 10 - 0.1 = 9.9kN$$