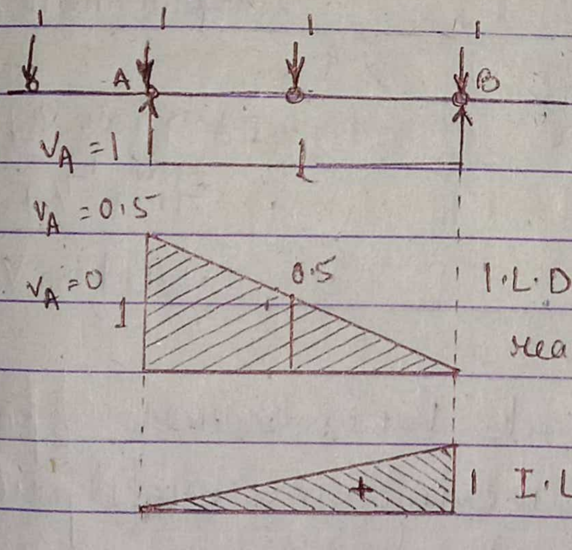
Influence Line Diagram

(A) ILD's for the forces in truss members :-

I.L.D :-

Concepts :- 1. Influence line diagram (I.L.D) :-
It is the graphical representation of variation of reaction, S.F, B.M, etc., as a unit load moves from one end to the other end of a structure.

2. I.L.D for reactions :-



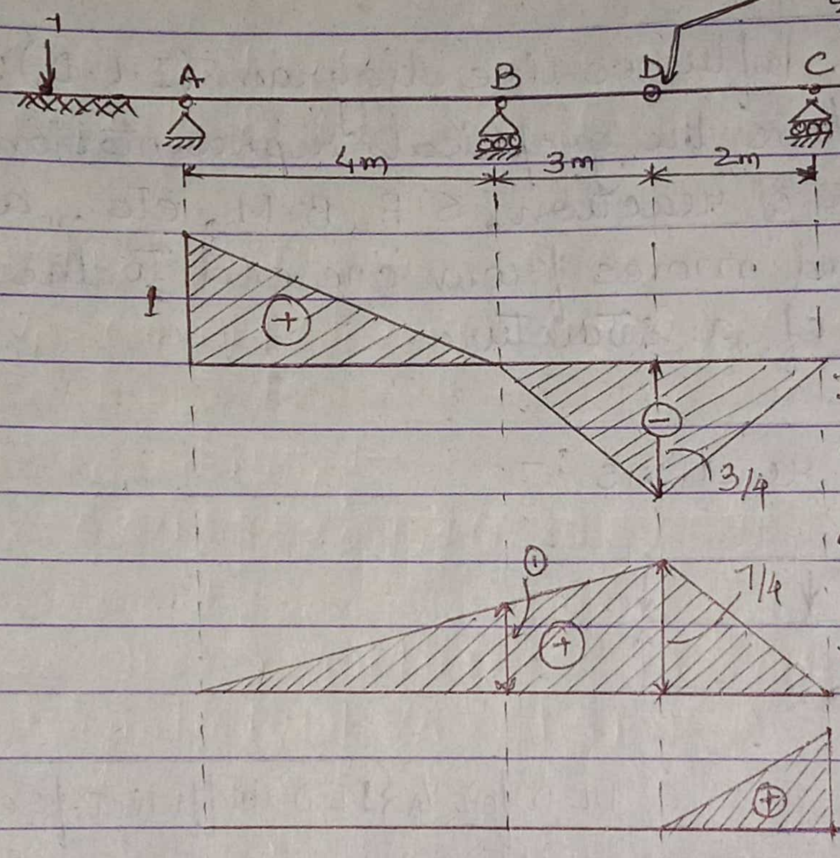
I.L.D for V_A {I.L.D Coefficient for reaction have no units}

1 I.L.D for V_B

To draw I.L.D [illegible] reaction at any support, remove the support & give unit displacement at that support then corresponding deflected shape of the beam itself is the I.L.D for reaction at that support.

Ex - Draw I.L.D for V_A, V_B & V_C

Internal hinge
Floating hinge
{Bolted connection}



$\left[\begin{array}{l} 4m = 1 \\ 3m = \frac{3}{4} \end{array}\right]$

I.L.D for V_A {without removing the supports at 'B' & 'C' give unit displacement at A}

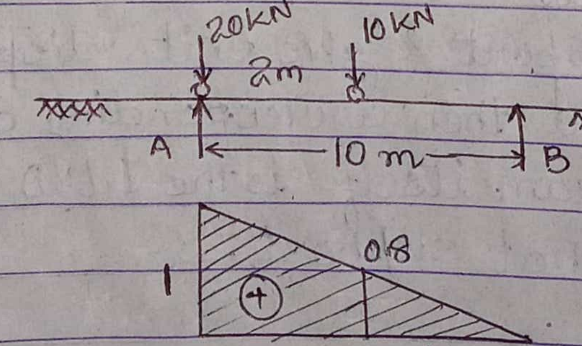
I.L.D for V_B

4m = 1
7m = 7/4

I.L.D for V_C

Ques:- Two wheel loads 10kN, 20kN, spaced 2m apart are moving on a simply supported beam of span 10m. Max^{m} reaction is.

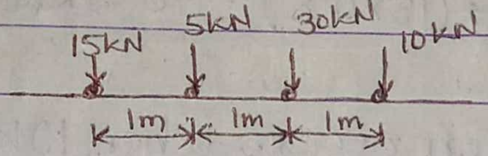
Sol:-



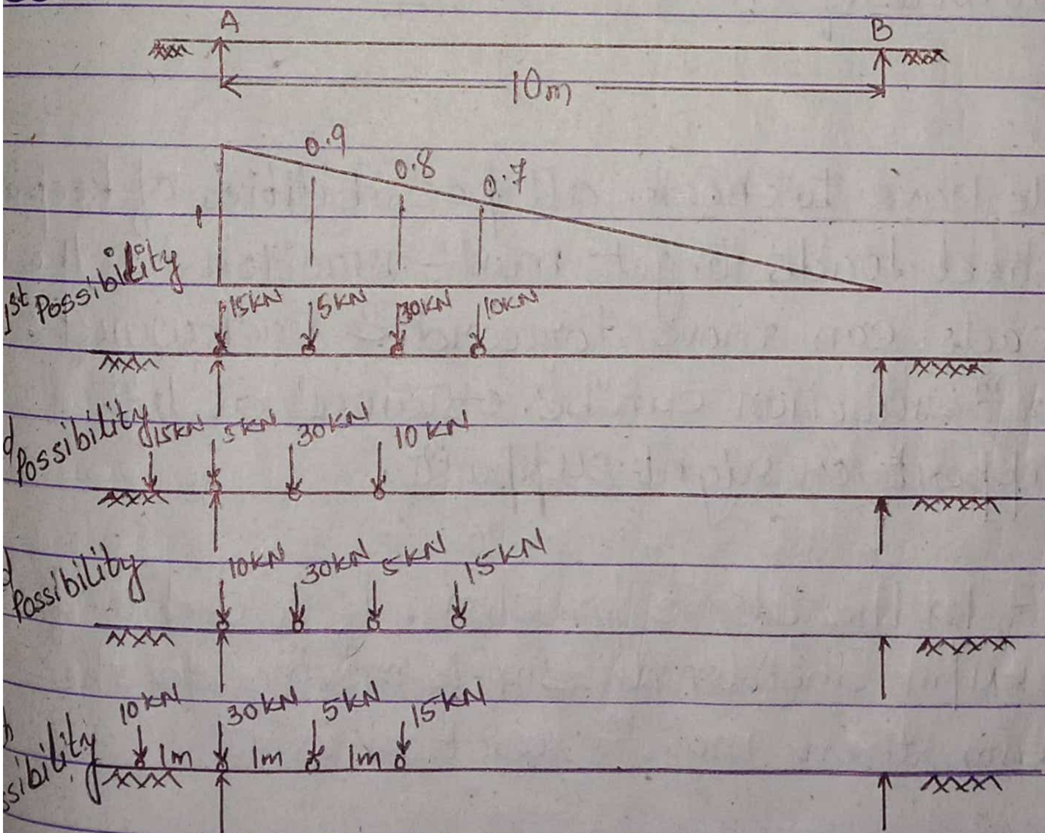
To get max^m reaction, keep 20kN at A & 10kN on the span as shown in fig.

$$V_A = 20 \times 1 + 10 \times 0.8$$
$$= 28 \text{ kN}.$$

Ques:- For the wheel loads 10kN, 30kN, 5kN, 15kN spaced one meter apart are moving on a simply supported beam of span 10m. max^m reaction is?



Sol:-



1^{st} Possibility :-

$V_A = (15\times1) + (5\times0.9) + (30\times0.8) + (10\times0.7) =$

2^{nd} Possibility :-

$V_A = (5\times1) + (30\times0.9) + (10\times0.8) = 40\,kN$

3^{rd} Possibility :-

$V_A = (10\times1) + (30\times0.9) + (5\times0.8) + (15\times0.7) = 51.5\,kN$

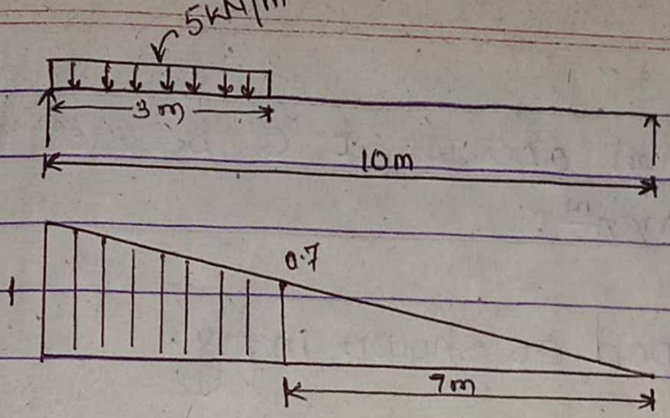
4^{th} Possibility :-

$V_A = (30\times1) + (5\times0.9) + (15\times0.8) = 46.5\,kN$

So, max^m reaction occurs when 10 kN is at A.
Magnitude of $V_A = 51.5\,kN$ Ans:-

Note :-
We have to check all possibilities of keeping of wheel loads to get max^m reaction i.e. the loads can move forward & backward & max^m reaction can be obtained at left support or right support.

Ques:- In the above problem, if a U.d.l of 5 kN/m and length 3m is moving on the beam then max^m reaction is ?



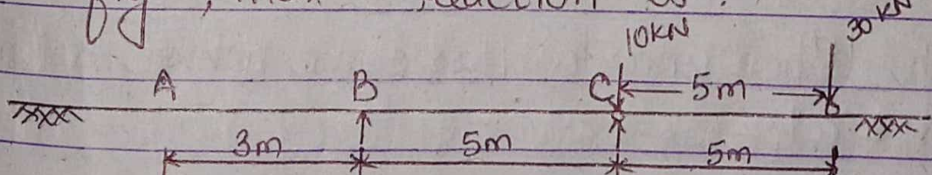
U.d.l is treated as infinite no. of point loads.

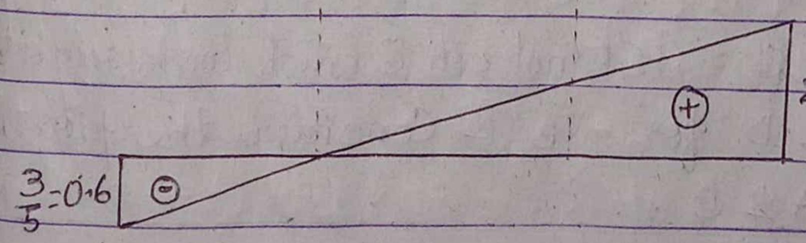
→ To get max^{m} reaction at 'A', keep the U.d.l such that max^{m} area of I.L.D is covered under U.d.l as shown in fig.

V_A = Intensity of U.d.l × area of I.L.D under U.d.l

$$= 5 \times \left[\underbrace{\frac{1+0.7}{2} \times 3}_{\text{Area of trapezoid}} \right] = 12.75\ kN \ \text{Ans:-}$$

Ques:- Two wheel loads 10kN & 30kN spaced 5m apart are moving on a beam as shown in fig., max^{m} reaction is?





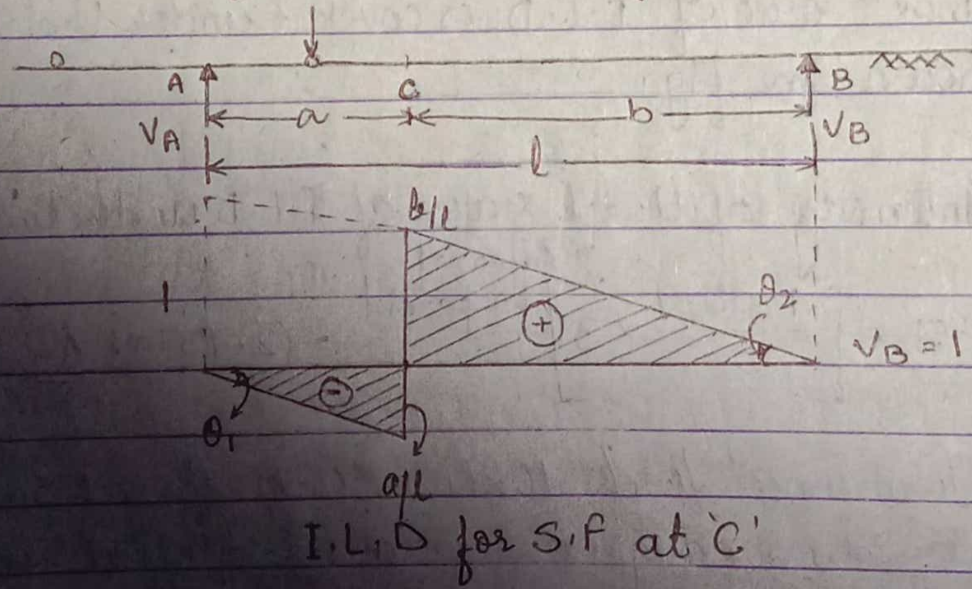
NOTE :-

Max$^{\underline{m}}$ reaction occurs at 'C' because over-hang is max$^{\underline{m}}$.

Keep the load as shown in fig.

$$V_C = (30 \times 2) + (10 \times 1) = 70\ kN$$

(B) I.L.D for S.F at any section 'C' :-



I.L.D for S.F at 'C'

Case I :- When unit load is some where between 'A' & 'C' :-

S.F at C = (Stand at C and look right) = $-V_B$

Draw I.L.D for $-V_B$ & Consider the portion between A & C.

Case II :- When unit load is somewhere betwee

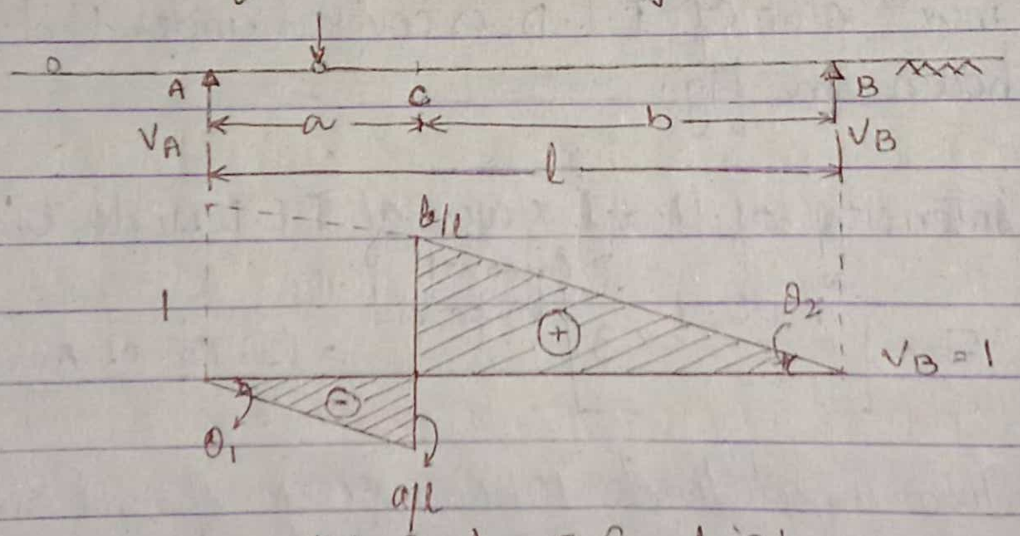
NOTE :-

Max$^{\underline{m}}$ reaction occurs at 'C' because over-hang is max$^{\underline{m}}$.

Keep the load as shown in fig.

$$V_C = (30 \times 2) + (10 \times 1) = 70 \text{ kN}$$

(B) I.L.D for S.F at any section 'C' :-



I.L.D for S.F at 'C'

Case I :- When unit load is some where between 'A' & 'C' :-

S.F at C = (Stand at C and look right) = $-V_B$

Draw I.L.D for $-V_B$ & Consider the portion between A & C.

Case II :- When unit load is somewhere between

C & B :-

S.F at C = (Stand at C and look left) = $+V_A$
Draw I.L.D for V_A & Consider the portion between C & B.

Observations :-

1. $\tan\theta_1 \approx \theta_1 = \dfrac{a/l}{a} = \dfrac{1}{l}$

$\tan\theta_2 \approx \theta_2 = \dfrac{b/l}{b} = \dfrac{1}{l}$

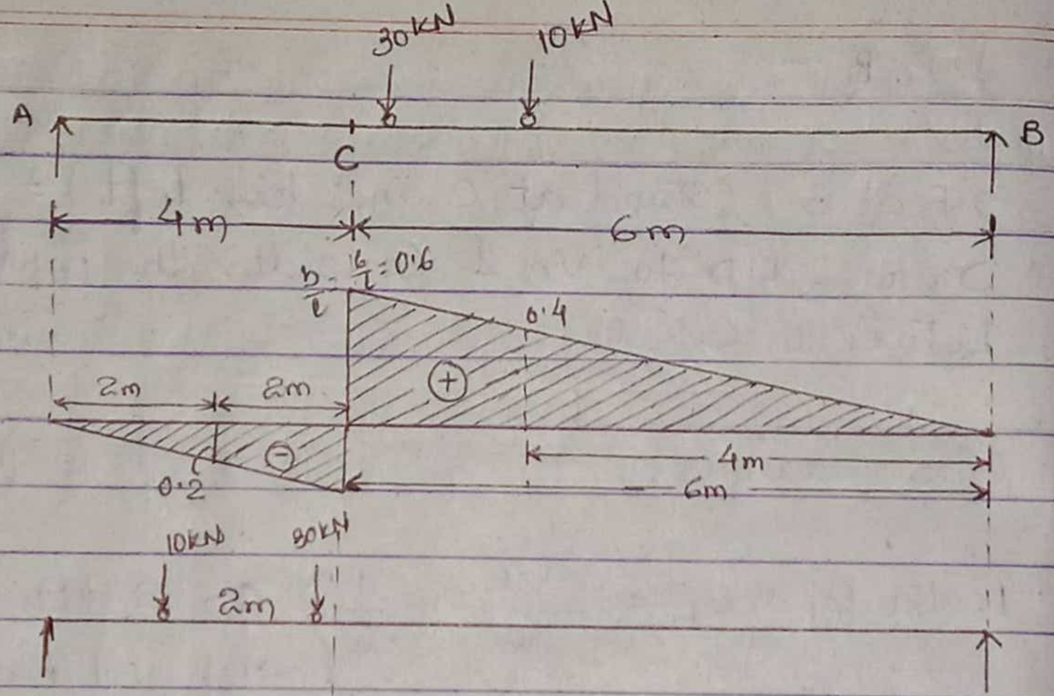
So, $\theta_1 = \theta_2$
Slope at A and B are equal.

2. $\dfrac{a}{l} + \dfrac{b}{l} = \dfrac{a+b}{l} = \dfrac{l}{l} = 1.$

3. From Muller:
To draw I.L.D for S.F at any section 'C', cut the beam at C & lift the beam as shown in fig (without removing the supports). Then the deflected shape of the beam is itself is the I.L.D for the S.F at that section.

Ques:- Two wheel loads 10 kN, 30 kN spaced 2 m apart are moving on a simply supported beam of span 10 m max^{m} S.F at a section 4 m from the left support is ... ?

Sol:-



→ To get max^m S.F at 'C' keep 30 kN at R.H.S of 'C' {not at 'C'} and 10 kN on the span BC as shown in fig.

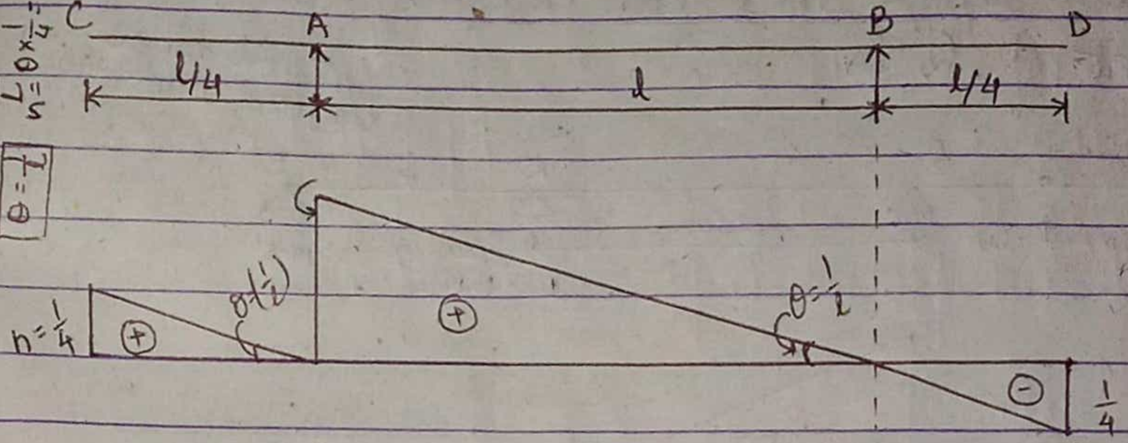
Max^m S.F at C = $(30 \times 0.6) + (10 \times 0.4) = 22$ kN

Ques:- In the above problem max^m −ve S.F at 'C' is ?

Sol:- To get max^m −ve S.F at C, keep 30 kN at L.H.S of 'C' {But not at 'C'} & 10 kN on the span of AC.

Max^m −ve S.F at 'C' = $30 \times (-0.4) + 10 \times (-0.2)$
$= -14$ kN.

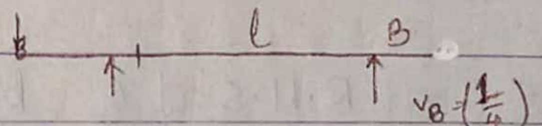
Ques:- I.L.D for S.F at R.H.S of 'A' for the beam shown in fig ?



$$\frac{a}{l} = \frac{0}{l} = 0$$

1. Cut the beam at R.H.S of A and lift the beam by unit amount $\left\{ \frac{a}{l} = 0, \ \frac{b}{l} = \frac{l}{l} = 1 \right\}$

The deflected shape of the beam is the I.L.D for S.F at R.H.S of A. {valid when load is between ABD}

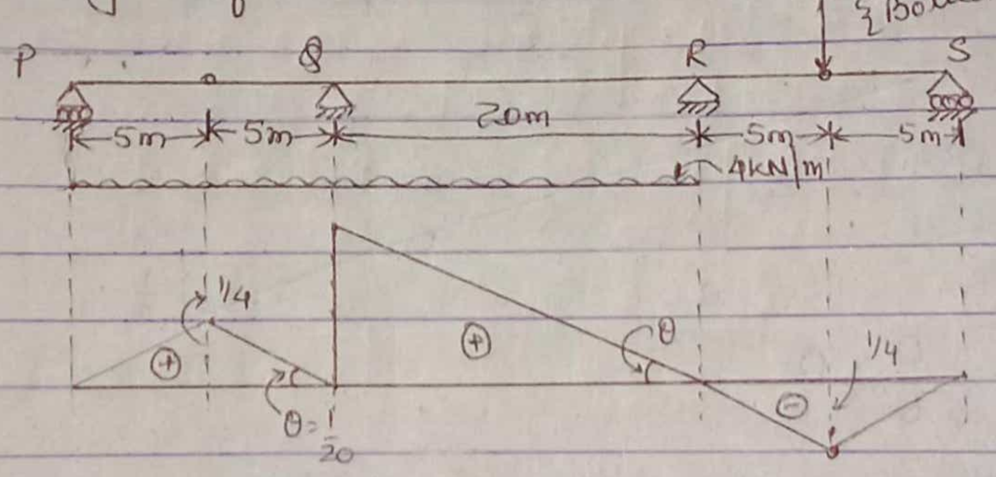


2. When unit load C :-

S.F at R.H.S of A = (Stand at R.H.S of 'A' & look right)

$= -V_B = +\frac{1}{4}$ {So, ILD ordinate at 'C' is $\frac{1}{4}$}.

Ques:- The beam shown in fig. is subjected to a moving distributed load of 4 kN/m. Max^{m} S.F. that can occur just to the right of Q is . . ?



1. Without removing supports at P, Q, R, S. Cut the beam at R.H.S of Q & lift it by unit amount.

2. θ is same for QP also draw I.L.D as shown in fig.

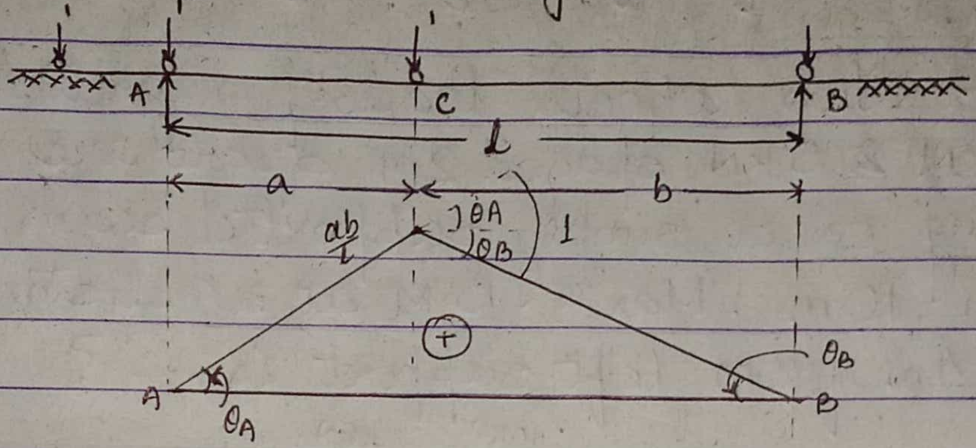
3. To get max^{m} S.F at R.H.S of Q, keep U.d.l between P Q R as shown in fig.

S.F at R.H.S of Q = Intensity of Udl × (area of I.L.D covered under U.d.l)

$$= 4\left[\left(\frac{1}{2} \times 10 \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 20 \times 1\right)\right]$$

$= 45$ kN

(C). ILD for B.M at any section 'C' :-



I.L.D for B.M at 'C'

1. When unit load is at A & B, B.M at C = 0

2. When unit load is at C, B.M at C = $\frac{ab}{l}$

Note :- From Muller :-

To draw I.L.D for B.M at any section C, assume an imaginary hinge at 'C' & lift it by an amount of ab/l {without removing the supports at A & B} then the deflected shape of the beam is the I.L.D for B.M at 'C'.

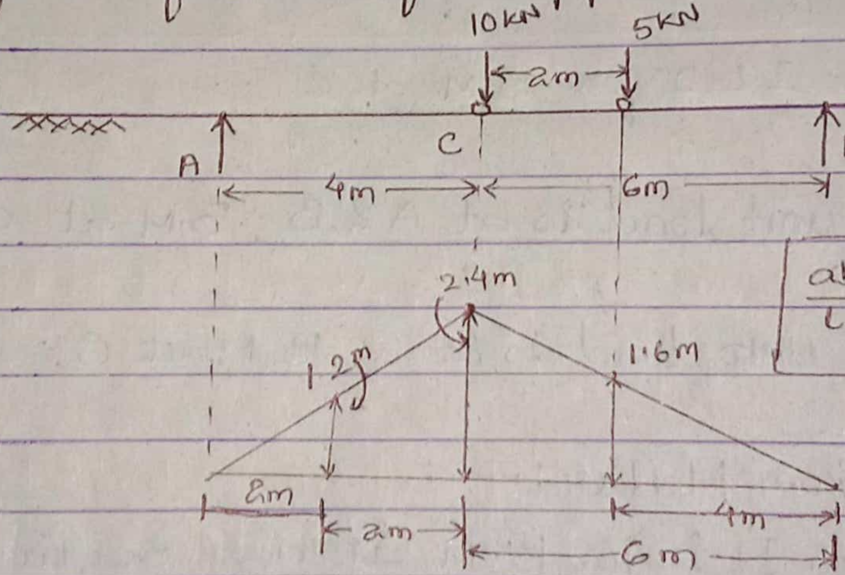
Observations :-

1. $\tan\theta_A = \theta_A = \frac{\frac{ab}{l}}{a} = \frac{b}{l}$

$\tan\theta_B = \theta_B = \frac{\frac{ab}{l}}{b} = \frac{a}{l}$

$$\theta_A + \theta_B = \frac{b}{l} + \frac{a}{l} = \frac{a+b}{l} = \frac{l}{l} = 1$$

Ques:- Two wheel loads of magnitude 10 kN & 5 kN spaced 2m apart are moving on a simply supported beam of span 10 m. Max^m B.M at a distance of 4m from left support is ...?



$$\frac{ab}{l} = \frac{4\times6}{10} = 2.4\,m$$

$6m \rightarrow 2.4$

$4m = \frac{4}{6}\times 2.6$

$= 1.6$

$4m \rightarrow 2.4$

$2m \rightarrow \frac{2.4}{4}\times 2 = 1.2\,m$

B.M at C $= 10\times2.4 + 5\times1.6 = 32$ kN-m Ans:-

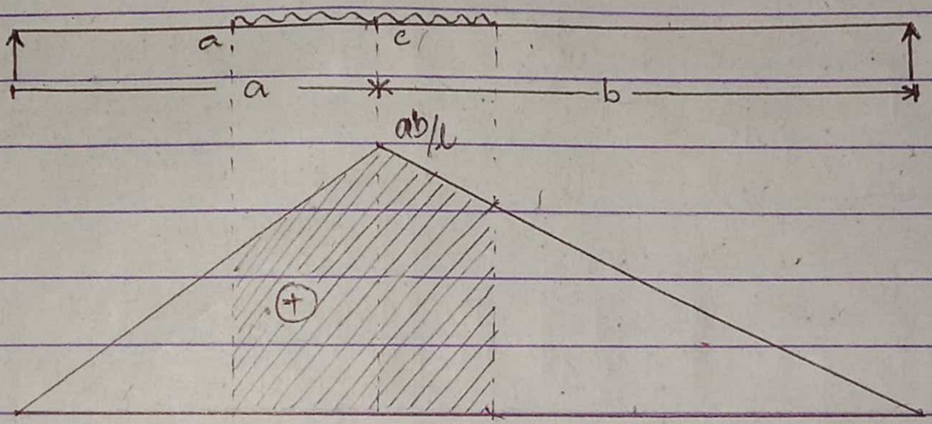
NOTE: {1 marks} → Units for I.L.D coefficient for B.M are 'm'

To get max^m B.M at C, keep greater load at 'C' & less load on span BC

{I.L.D Coefficient is less if load is on AC}

NOTE :-

1. When a U.d.l shorter than the span is moving on the beam, to get max^m B.M at C, keep U.d.l such that [avg. load on AC = avg. load on BC]



I.L.D for B.M at C

Avg. load on AC = Avg. load on BC

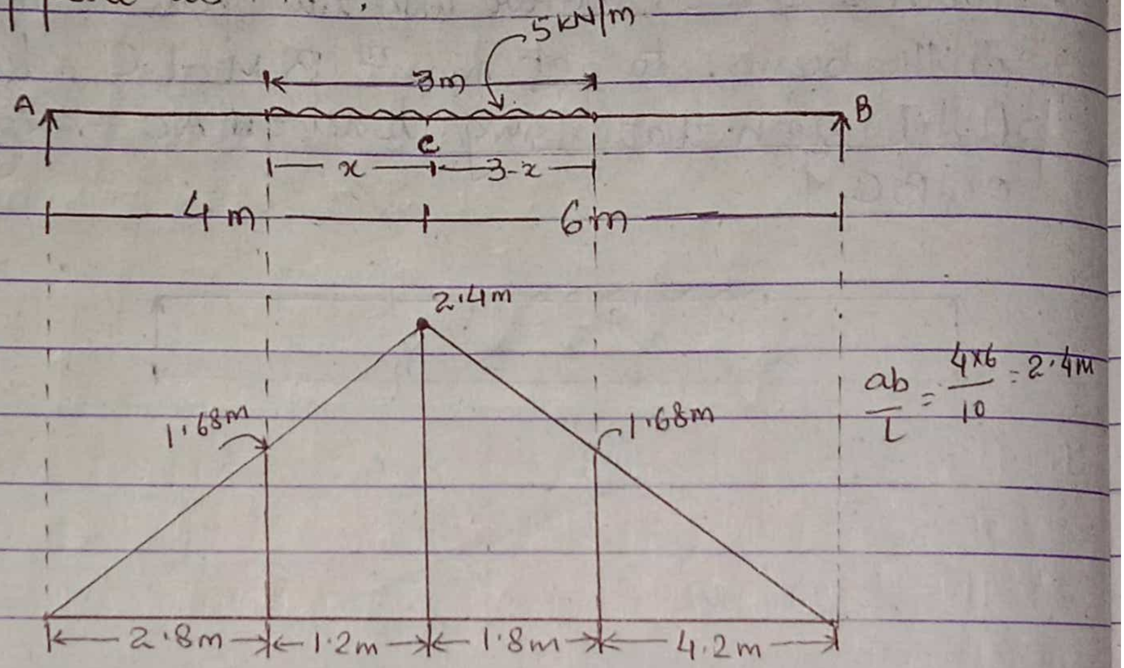
$\frac{\text{load on AC}}{\text{span AC}}$

$$\boxed{\frac{W \times (x)}{a} = \frac{\omega(L_1 - x)}{b}}$$

NOTE : {1 Marks}

If the above condition is satisfied, then I.L.D ordinates at the ends of U.d.l will be equal & max^m area of I.L.D is covered under U.d.l.

Ques :- A U.d.l of 5kN/m & length 3m is moving on a simply supported beam of span 10m. Max^m B.M at a distance of 4m from left support is ...?



Get Max^m B.M at C,

Avg. load on AC = Avg. load on BC

$$\frac{5\times(x)}{4} = \frac{5(3-x)}{6}$$

$$\boxed{x = 1.2m}$$

4m → 2.4m

2.8m → ? $= \frac{2.4}{4} \times 2.4 = 1.68m$

6m → 2.4m

$4.2 = \frac{2.4}{6} \times 4.2 = 1.68m$

Max^m B.M at C = Intensity of U.d.l × area of I.L.D covered under U.d.l

$$= 5 \times \left[(1.68 \times 3) + \frac{1}{2} \times 3 \times (2.4 - 1.68) \right]$$

$$= 30.6 \text{ kN-m}.$$

NOTE 2:-

If more then two point loads are moving on the beam, to get max$^{\underline{m}}$ B.M at any section C. Keep the loads such that avg. load on AC is equal to avg. load on BC.

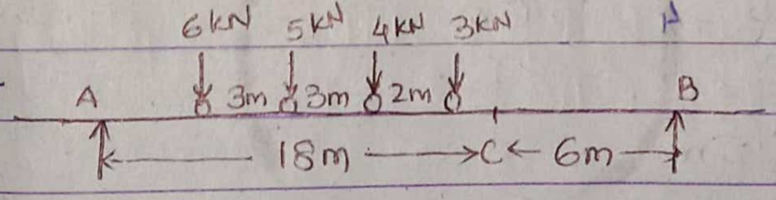
Ques:- {2 Marks}

Four wheel loads 3 kN, 4 kN, 5 kN, 6 kN spaced 2m, 3m, 3m apart move on a simply supported beam of span 24 m with 3 kN load leading from left to right. Find the max$^{\underline{m}}$ B.M at 18 m from left support, the load that must be placed at the section is ...?

6kN ↓ 3m ↓ 5kN 3m ↓ 4kN 2m ↓ 3kN → given load cond^n

(A) 3 kN (B) 4 kN (C) 5 kN (D) 6 kN

Sol:-

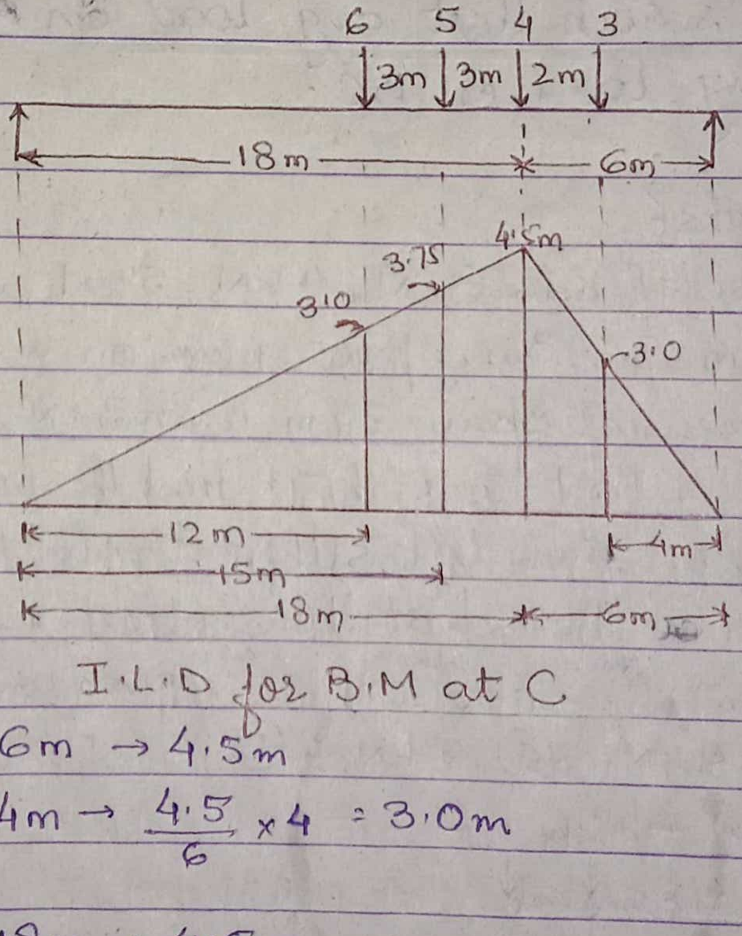


	Avg. load on AC	Avg. load on BC
1. When all loads are on span AC	$\frac{6+5+4+3}{18} = 1 >$	0
2. When 3kN crosses 'C'	$\frac{6+5+4}{18} = \frac{15}{18} >$	$\frac{3}{6}$
3. When 4kN crosses 'C'.	$\frac{6+5}{18} = \frac{11}{18} <$	$\frac{3+4}{6} = \frac{7}{6}$

Conclusion :-

When 4kN load crosses 'C', avg. load on AC becomes less than avg. load on BC. So, kept 4kN at 'C' to get max^m B.M at 'C'.

Ques :- In the above problem the magnitude of max^m B.M at C is ?



$$\frac{ab}{l} = \frac{18 \times 6}{24} = 4.5m$$

I.L.D for B.M at C

6m → 4.5m

$4m \rightarrow \frac{4.5}{6} \times 4 = 3.0m$

18m → 4.5m

$15m \rightarrow \frac{15}{18} \times 4.5 = 3.75m$

$12m \rightarrow \frac{12}{18} \times 4.5 = 3.0m$

Max^m B.M at 'C' $= (6 \times 3) + (5 \times 3.75) + (4 \times 4.5) + (3 \times 3)$

$= 63.75$ kN-m

NOTE 3:- {1 Marks}

When a series of wheel loads are moving on a beam, then to get max^{m} B.M under a choosen wheel load, keep the load such that the choosen wheel load and the resultant of wheel load system are at equal distance from the centre of the span.

Then max^{m} B.M will occur under the choosen wheel load. {But not at centre}

Ques: {20 Marks}

5 points loads 10 kN, 12 kN, 12 kN, 16 kN & 20 kN, spaced 5m apart roll over a simply supported beam of span 80 m from left to right with 20 kN leading. Calculate the position & magnitude of max^{m} B.M which may occur anywhere in the beam.

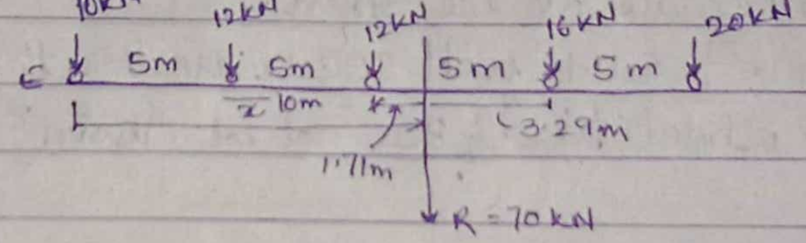
NOTE :-

(i) In this problem the section of the beam is not specified. We have to choose the location of B.M, max^{m} B.M occurs near centre.

(ii) To get max^{m} B.M under a choosen wheel load, the resultant of the load system & the chosen load must be at equal distance from the centre then max^{m} B.M occurs under the chosen wheel load.

(iii) The location of the resultant is found from Varignon's theorem { moment of a force about any point is equal to moment summation of its components about that point }.

Sol:-



$R \times \bar{x} = 10 \times 0 + (12 \times 5) + (12 \times 10) + (16 \times 15) + (20 \times 20)$

$\bar{x} = 11.71$ m From E

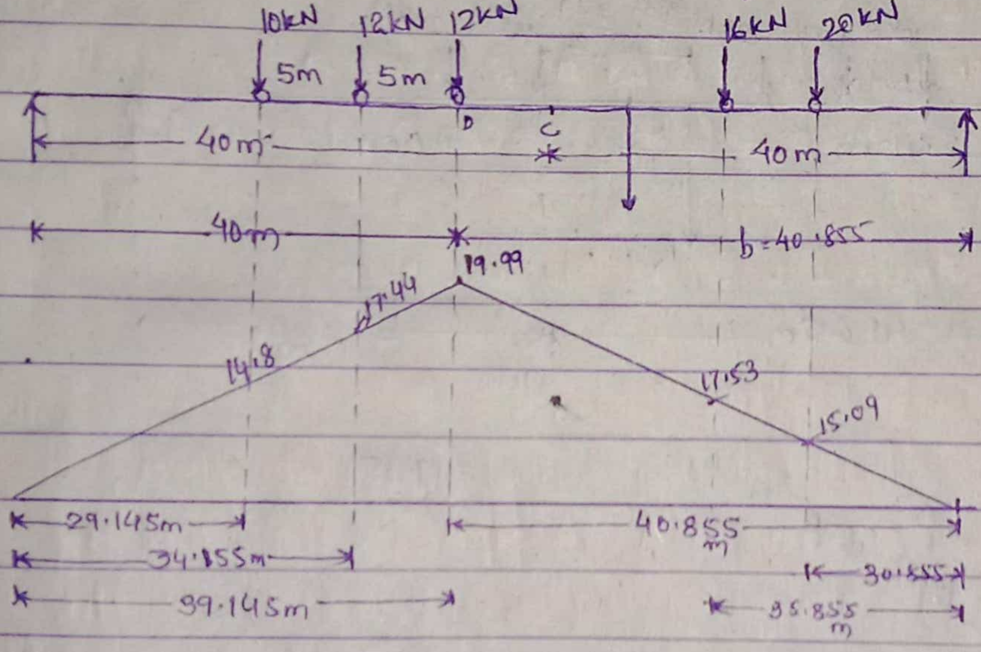
(V). In this problem wheel load is not chosen. We have to choose the wheel load under which max^m B.M may happen in this case 12 kN is nearer to the resultant but greater load 16kN is slightly away from the resultant. So, we have to check the two possibility of getting max^m B.M under 12 kN load & 16 kN.

(v) If it all, 16 kN load is nearer to the resultant, then max^m B.M will occur under 16 kN load only, the other possibility need not to be check.

Sol:-

1^{st} possibility :-

B.M under 12 kN load { Keep 12 kN load & resultant at equal distance from the centre }



I.L.D for B.M at D

$$\frac{ab}{l} = \frac{39.145 \times 40.855}{80} = 19.99m$$

39.145m $\longrightarrow$ 19.99m

34.145 m $\longrightarrow$ $\frac{34.145}{39.145} \times 19.99 = 17.44$

29.145 $\longrightarrow$ $\frac{29.145}{34.145} \times 19.99 = 14.89$

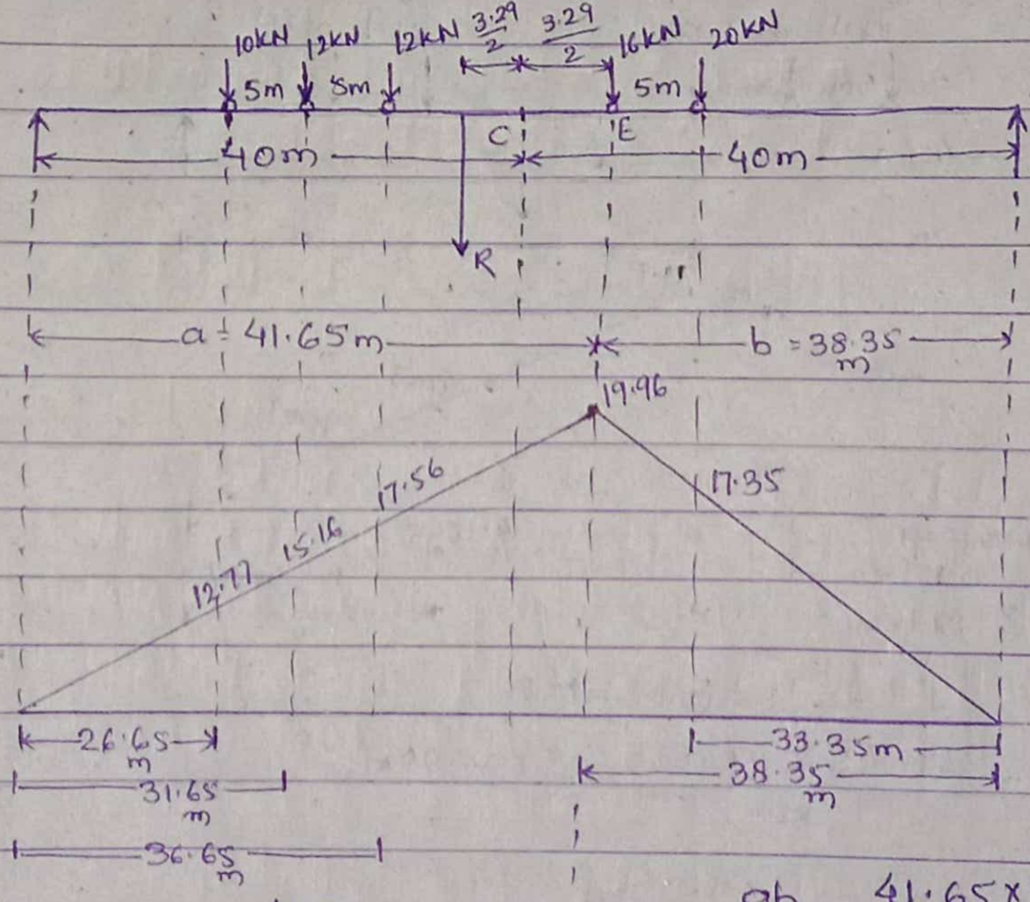
B.M under 12 kN Load $= (10 \times 14.89) + (12 \times 17.44)$

$+ (12 \times 19.99) + (16 \times 17.53) + (20 \times 15.09)$

$= 1180.34$ kN-m.

2nd possibility :-

B.M under 16 kN load :- { Keep 16 kN load & resultant at equal distance from centre }.



$$\frac{ab}{l} = \frac{41.65 \times 38.35}{80}$$

$$= 19.96 \text{ m}$$

B.M under 16 kN load at E

$= (10 \times 12.77) + (12 \times 15.16) + (12 \times 17.56) + (16 \times 19.96) + (20 \times 17.35)$

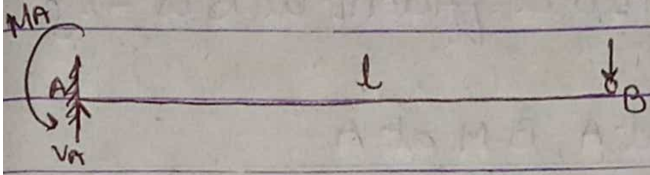
$= 11869$ kN-m

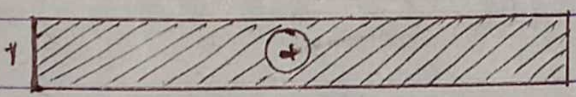
Conclusion :-

Max^{m} B.M occurs under 16 kN, when it is at a distance of 41.65m from left support.

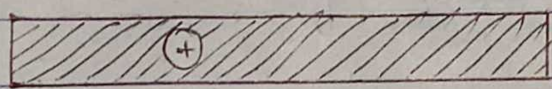
Magnitude of max^{m} B.M = 1186.9 kN-m Ans:-

II Cantilever Beam:-

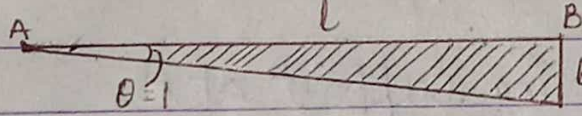




1 I.L.D for V_A



I.L.D for S.F at RHS of 'A'



l I.L.D for B.M at A.

$$\tan\theta \approx \theta = \frac{l}{l} = 1$$

(i) I.L.D for reaction at 'A' :-

As the unit loads moves from one end to the other end, V_A value does not change, So, $\boxed{V_A = 1}$, For all position of unit load. So draw I.L.D as a rectangle as shown in fig.

(ii) I.L.D for S.F at R.H.S of 'A' :-

S.F at R.H.S of A = (Stand at R.H.S of A & look left) = $+V_A$

Draw I.L.D for V_A. It is the I.L.D for S.F at R.H.S of 'A' also.

(iii) I.L.D for B.M at R.H.S of A {at A} :-

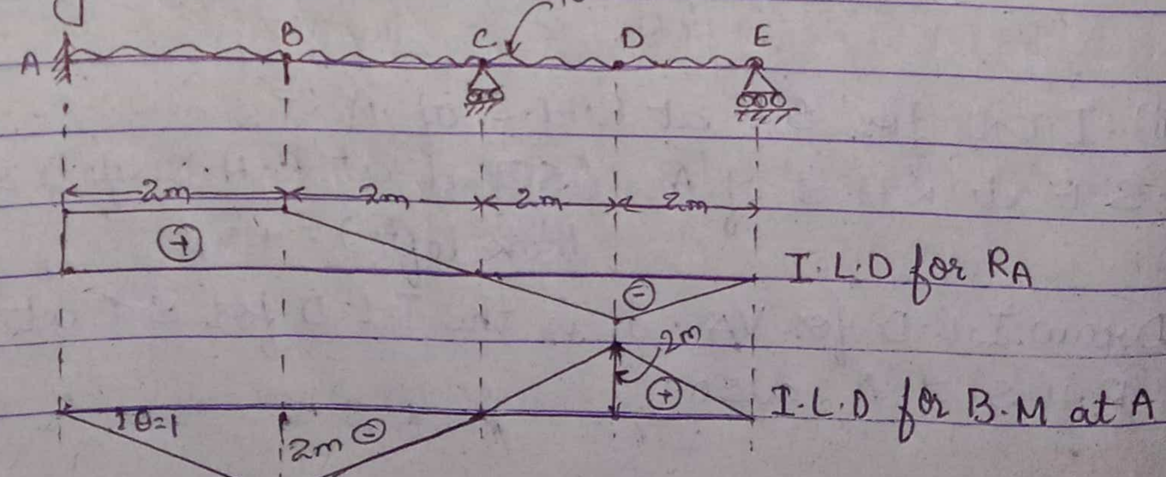
a) When unit load is at B, B.M at A
$= -1 \times l$ {So, I.L.D coefficient at B is $-l$}

b) When unit load is at A, B.M at A
$= 0$ {So, I.L.D coefficient at A is 0}

So, Draw I.L.D as shown in fig.

NOTE :- From Muller :-
Without removing support at A, release the moment A {Assume a hinge at A} rotate the beam by unit amount then the deflected shape of the beam itself is the I.L.D of the B.M at 'A'.

Ques:- Find the B.M at A for the beam as shown in fig. when it is subjected to a U.d.l of 10 kN/m throughout its length.



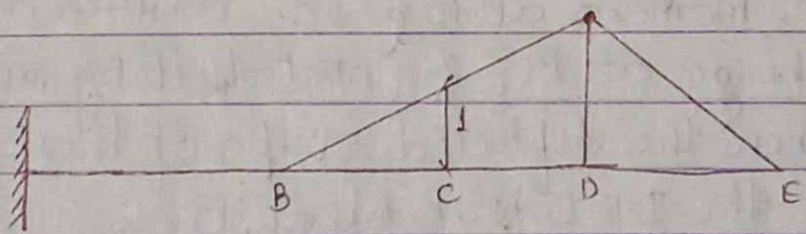
$\boxed{\text{Net Area of I.L.D} = 0}$

B.M at A = Intensity of U.d.l × Area of I.L.D covered under U.d.l

= 10 × 0 = 0

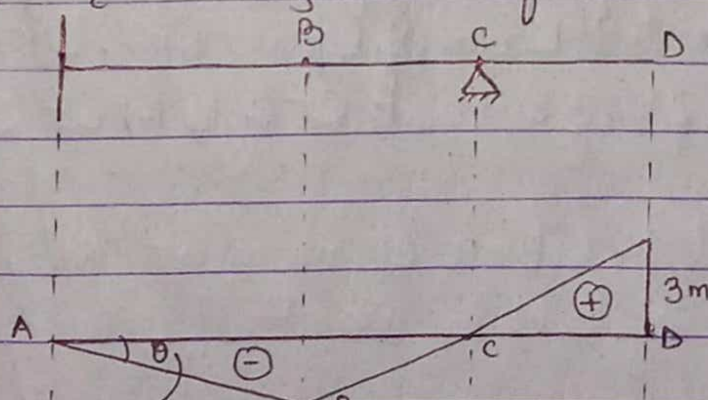
{without removing the support at A, C, E release the moment at A} {assume an imaginary hinge at A and rotate by unit amound}

I.L.D for reaction at A {without releasing moment at A, release the support and lift it by unit amount deflected shape is the I.L.D for reaction at A}



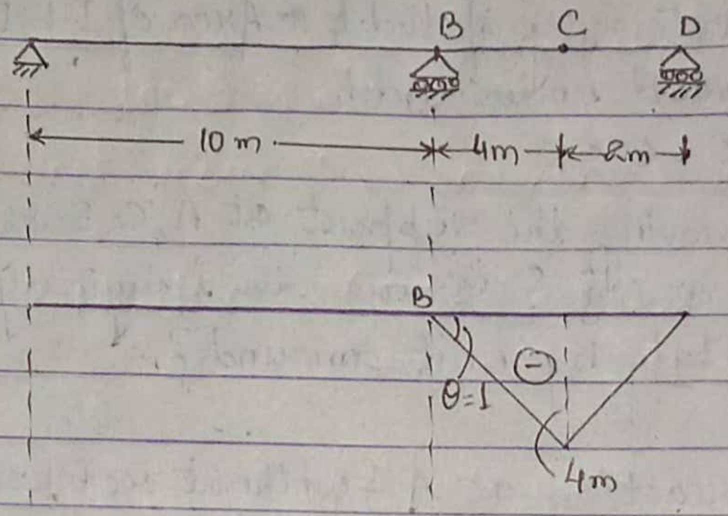
I.L.D for reaction at 'C'

Ques:- {6 Marks} I.L.D for B.M at A?



I.L.D for B.M at A.

Ques:- I.L.D for B.M at B is

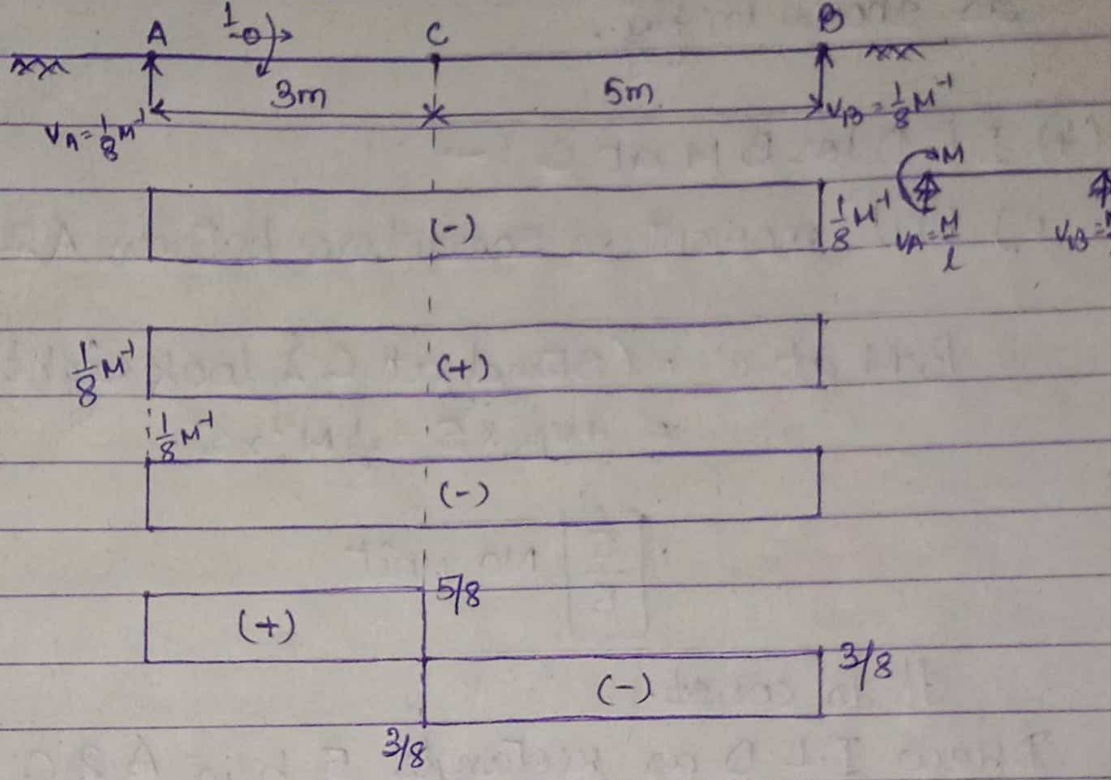


I.L.D for B.M at 'B'

Without removing the support at A, B & D, replace the moment at B {i.e., assume an imaginary hinge at B} & rotated it by unit moment, then the deflected shape of the beam itself is the I.L.D for B.M at B.

Ques:- Unit moment {magnitude is 1 & has no units} is moving from left to right of a simply supported beam of span 8m. Draw I.L.D for V_A, V_B, S.F at C & B.M at C.

Sol:-



1. I.L.D for reaction at A :-
→ For all position of unit moment V_A value does not change. So, draw I.L.D for V_A as a rectangle as shown in fig.

2. I.L.D for V_B :-
→ For all position of unit moment reaction at B is const. So draw I.L.D for V_B as a rectangle as shown in fig.

3. I.L.D for S.F at C :-
→ S.F at 'C' = (stand at 'C' & look right) = $-V_B$
for all position of unit moment S.F at C = $-V_B$,

So draw I.L.D for S.F at 'C' as a rectangle as shown in fig.

(4) I.L.D for B.M at C :→

a) Unit moment is somewhere between A & C :→

B.M at 'C' = (Stand at C & look right)

$= +V_B \times 5 = \frac{1}{8} M^{-1} \times 5$

$= \boxed{\frac{5}{8}}$ No unit

It is const.

Draw I.L.D as rectangle $\frac{5}{8}$ b/w A & C.

(b) Unit moment in somewhere b/w C & B :→

B.M at C = (Stand at 'C' & look left)

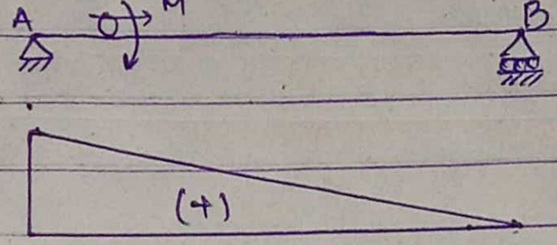
$= -\frac{1}{8} M^{-1} \times 3$

$= -\frac{3}{8}$

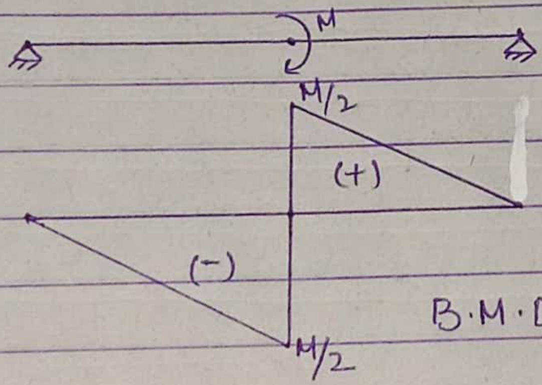
Draw I.L.D for $\frac{3}{8}$ as a rectangle.

Ques:- For what position of moment 'M', magnitude of B.M is max^{m}.

Sol:-



B·M·D {Not I·L·D}



B·M·D

When moment is at 'A' B·M is max^{m} & its value is M.