CE6PC02	STRUCTURAL ANALYSIS II	PC-II	3-1-0	3 Credits

P

uisites: Structural Analysis I

Detailed Syllabus:

MODULE	CONTENTS	Hrs
1.	Analysis of fixed beams, continuous beam, simple frames and redundant frames with and without translation of points. Method of consistent deformation, Strain energy method, Slope deflection method, Moment distribution method.	12
2.	Analysis of two hinged arches. Suspension bridges with two hinged stiffening girder.	10
3.	Structural theorems:-Linearity principle of superposition,virtualwork,energy theorems, reciprocal theorems, Muller's Breslau's principles.	6
4.	Basics of force and displacement matrix methods for beams,plane frame (rigid and pin-pointed)	10
5.	Influence lines:-Influence lines for propped cantilevers,continuous beams and two hinged arches	10

PROFESSIONAL CORE – III

Module - 1

1. MTD to analyse statically Indeterminate str.

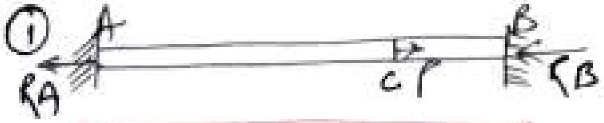
→ All the MTD of analysis can be classified into two types, depending on the types of unknown chosen for the analysis {i.e., Force MTD & Displacement MTD)

FFC
Force/Flexibility/Compatibility

i) Forces are taken as unknowns {i.e., Reactions S.F & B.M} & equations are expressed in terms of these forces.

ii) Additional eq^n. called Compatibility conditions are developed to find all the unknown forces.

Ex:- In strength of Materials

①


$\boxed{DS = r - s = 2 - 1 = 1}$

So, 1 compatibility condition are reqd. to find all forces.

$\Sigma x = 0 \Rightarrow \boxed{-R_A - R_B + P = 0}$

Compatibility Condition ⇒
elongation in AC = Contraction BC.
(A relationship b/w deformation)

DSE ✓
Displacement/Stiffness/Equilibrium

i) Displacement {i.e., slope & deflections} are taken as unknown and eq^n. are expressed in terms of these unknown displacement {slope-deflection eq^n.}.

ii) Additional joint equilibrium eq^n. are developed to find unknown displacement.

Ex:- ⓐ Slope deflection MTD.
ⓑ Moment distribution MTD {Sucessive approch MTD}
ⓒ Kani MTD {Iterative Version of SdM}
ⓓ Stiffeneus Matrix MTD {Matrix version of S.d.M}.

2. Flitched beams:-

Equi. eq^n $\Rightarrow$ $(M.R) = (M.R)_{steel} + (M.R)_{load}$ — ①

Compatibility condition $\rightarrow$

$(R)_{steel} = (R)_{wood}$ — ②

$(e)_{steel} = (e)_{wood}$

3. In Structural Analysis:-

① Theorem of 3-Moments {Compatibility condition. $\theta_{BA} = \theta_{BC}$}

② Redundant trusses {by strain energy or unit load MTD}

> Compatibility condition

$\Rightarrow \left(\delta_{AC} = \frac{-XL}{AE}\right)$

③ 2-hinges Arch > Compatibility condition $\Rightarrow \frac{\partial U}{\partial x} = 0$

④ Flexibility Matrix Mtd.

Note:- If Ds is less than this MTD is used.

Note:-

If Dk is less then these MTD are suitable.

1. Moment Distribution MTD :-

Concepts :-

① Carry Over Moment (C.O.M) :-

→ It is the moment developed at one end due to applied moment at the other end.
{C.O.M are defined only when Moments are applied at one end}.

Case 1.

When far end is fixed, $\boxed{C.O.M = \frac{M}{2}}$

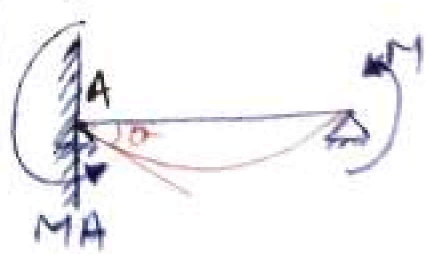
$MA = \frac{M}{2}$ (COM) {It is developed to make slope zero}

OR

Reaching Moment

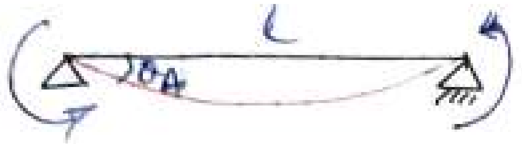
Induced Moment

Developed Moment



Case ii :-

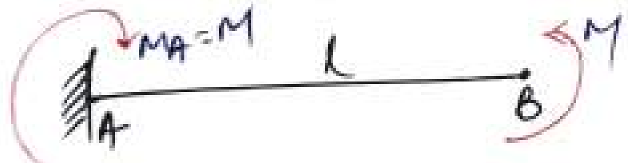
When far end is hinged, $\boxed{C.O.M = 0}$

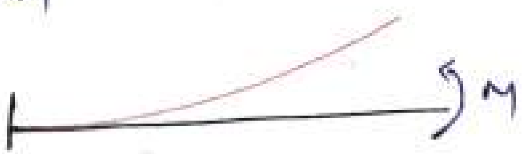


M clapping Moment

$MA = (C.O.M) = 0$ { 'O' b.c.c, slope need not to be make to zero }.

Case iii) :- For Cantilever Beam, $\boxed{C.O.M = -M}$
{ -M because, C.O.M and applied moment are opposite to each other. }





Sag { Due to Sagging B.M }

CONCEPTS :-

Carry Over Factor :- (C.O.F)

⇒ It is the ratio of carry over Moment and applied Moment.

Case i :-

When far end is fixed, $C.O.F = \frac{C.O.M}{\text{Applied Moment}} = \frac{M/2}{M}$

$= \boxed{\frac{1}{2}}$

Case ii :-

When far end is hinged, $C.O.F = \frac{0}{M} = \boxed{0}$

Case iii :-

For cantilever beam, $C.O.F = \frac{-M}{M} = \boxed{-1}$

Concepts :-

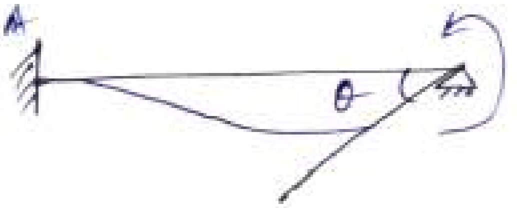
Stiffness Factor (S) :-

It is the moment req.d to produce unit ratation.

Case (i) :-

When far end is fixed, stiffeness factor =

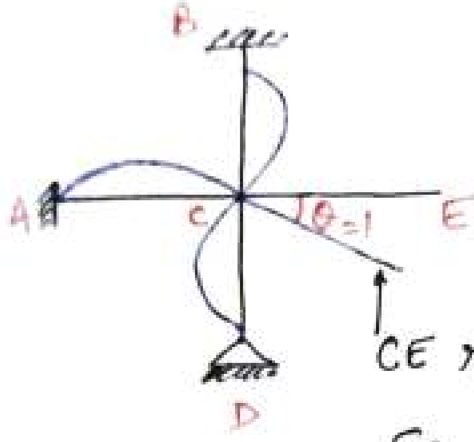
$\boxed{S = \frac{4EI}{L}}$



$M = \frac{4EI}{L} = S$

Case III :-

When far end is free, stiffeness factor



CE rotates freely.

So, stiffeness factor = $s = 0$

CONCEPTS :-

Relative Stiffeness (k) :-

It is the ratio of Moment of inertia and length of the member.

Case (i)

when far end is fixed, relative stiffness

$$\boxed{K = \frac{I}{L}}$$

Case ii

when far end is hinged, relative stiffness

$$\boxed{K = \frac{3}{4} \cdot \frac{I}{L}}$$

$$\therefore \boxed{\begin{aligned} &\frac{4EI}{l} \rightarrow \frac{I}{L} \\ &\frac{3EI}{l} = \frac{3}{4} \cdot \frac{I}{L} \end{aligned}}$$

Case iii

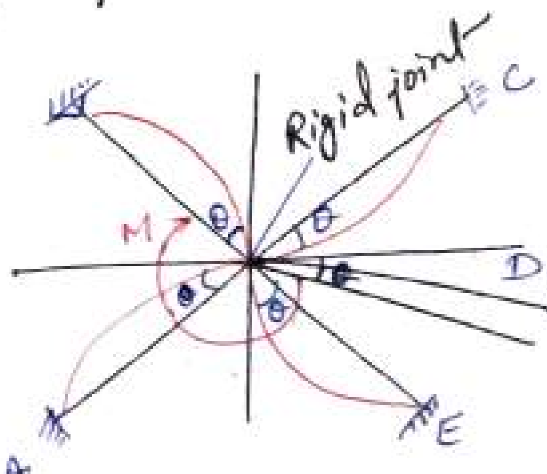
When far end is free, relative stiffness

$K = 0$ {Bcz stiffeness factor, $S = 0$}

Distribution factor (D.F) :-

{It is defind for member meeting at a "rigid joint only}.

⇨ It is the ratio in which the applied moment is distributed to varies members meeting at a rigid joints.



OD member, DF = 0
bcz, K = 0
S = 0

$$\boxed{D.F = \frac{K}{\Sigma K}}$$

Compatibility Conditions ⇒

At a rigid joint, all members rotates same amount. so, D.F can be defined.

NOTE :-

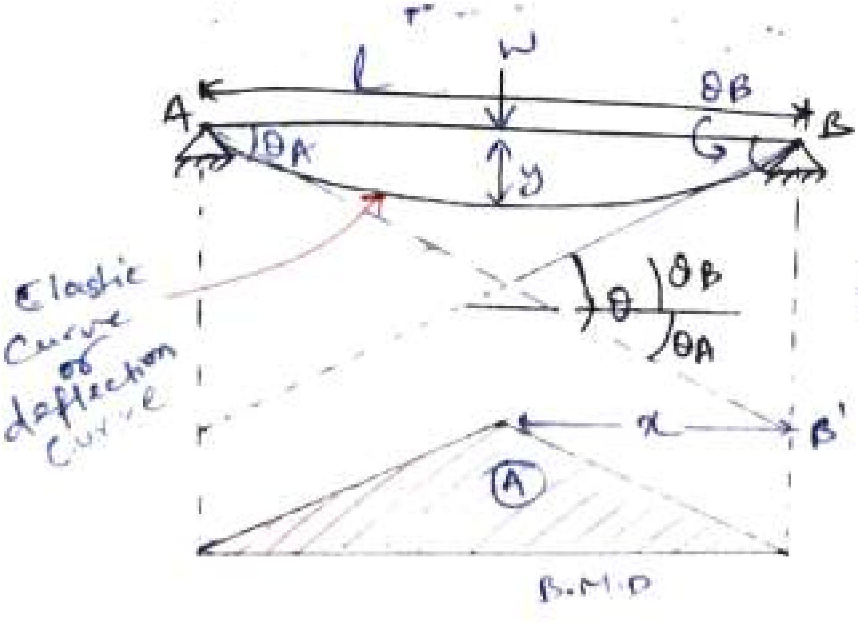
1. At a rigid joint, the sum of the distribution factor of all members meeting at that joint is equal to one.

CONCEPT ⑥

ⓐ It is a Semi analysis MTD, used to find slopes and deflection in Beams.

ⓑ If finding Area of B.M.D finding C.G of Area of B.M.D are simple, the use Moment-Area MTD to find slopes and deflection quickly.

ⓒ Mohr's theoram I :-



$\theta = \theta_A + \theta_B$

→ It states "the angle b/w the two tangents drawn on the elastic line is given by the area of the B.M.D b/w those two tangents devided by EI".

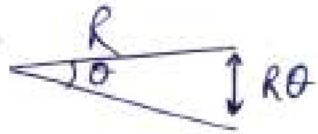
i.e., $\boxed{\theta = \frac{A}{EI}}$

(d) Mohr's theoram II :- $\left\{ \delta = BB' = \frac{A\bar{x}}{EI} \right\}$.

It states

" The deviation of point 'B', away from the tangent drawn at A {i.e., BB'} is given by are moment of area of B.M.D about B. divided by EI.

i.e., $\boxed{BB' = \frac{A\bar{x}}{EI}}$



NOTE :-

To find θ_A :-

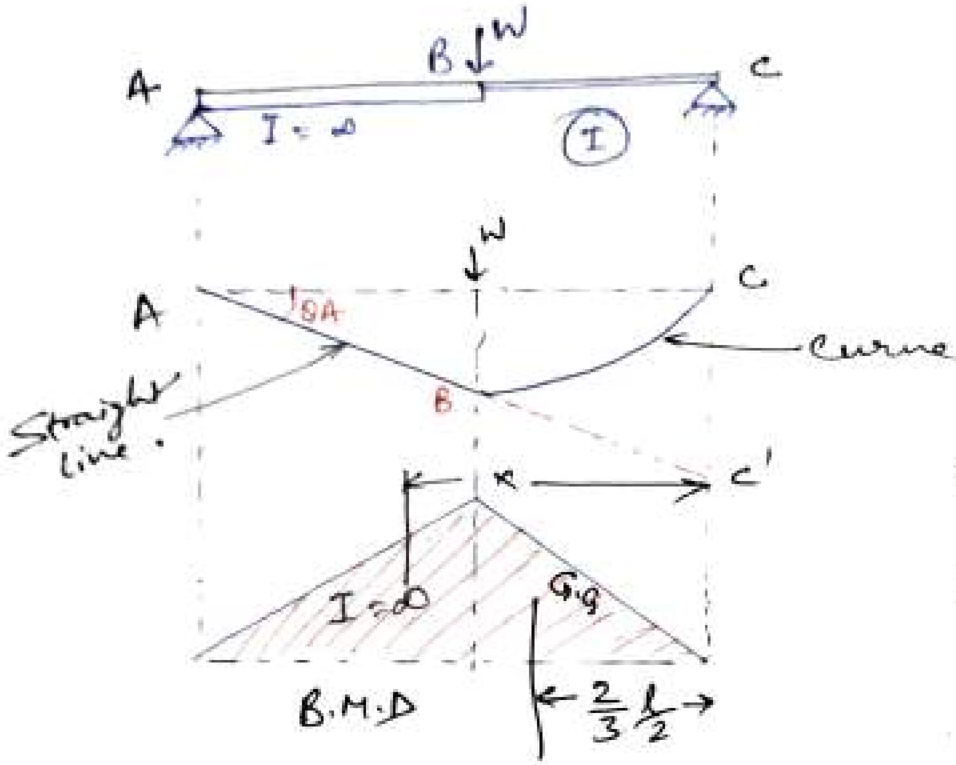
→ first find BB' using Mohr's theoram II

then, $l\theta_A = BB'$

$\Rightarrow \boxed{\theta_A = \frac{BB'}{l}}$

Case 1:-

Find 'θ_A' for the beam shown in fig. [2 Marks]



I→∞ → means flexure rigid ie, not bend.

$I \to \infty$ implies flexurally rigid i.e., beam doesnot bend. It remains as a st. line

→ But axially not rigid.

$\boxed{AE \neq \infty}$, $\boxed{\text{only } EI = \infty}$

$l \times \theta_A = CC'$

$\theta_A = \frac{CC'}{l}$

$CC' = \frac{A\bar{x}}{EI} = \frac{\left(\frac{1}{2} \times \frac{l}{2} \times \frac{Wl}{4}\right)\bar{x}}{E \times \infty}$

$= \frac{\left(\frac{1}{2} \times \frac{l}{2} \times \frac{Wl}{4}\right) \times \left(\frac{2}{3} \times \frac{l}{2}\right)}{EI}$

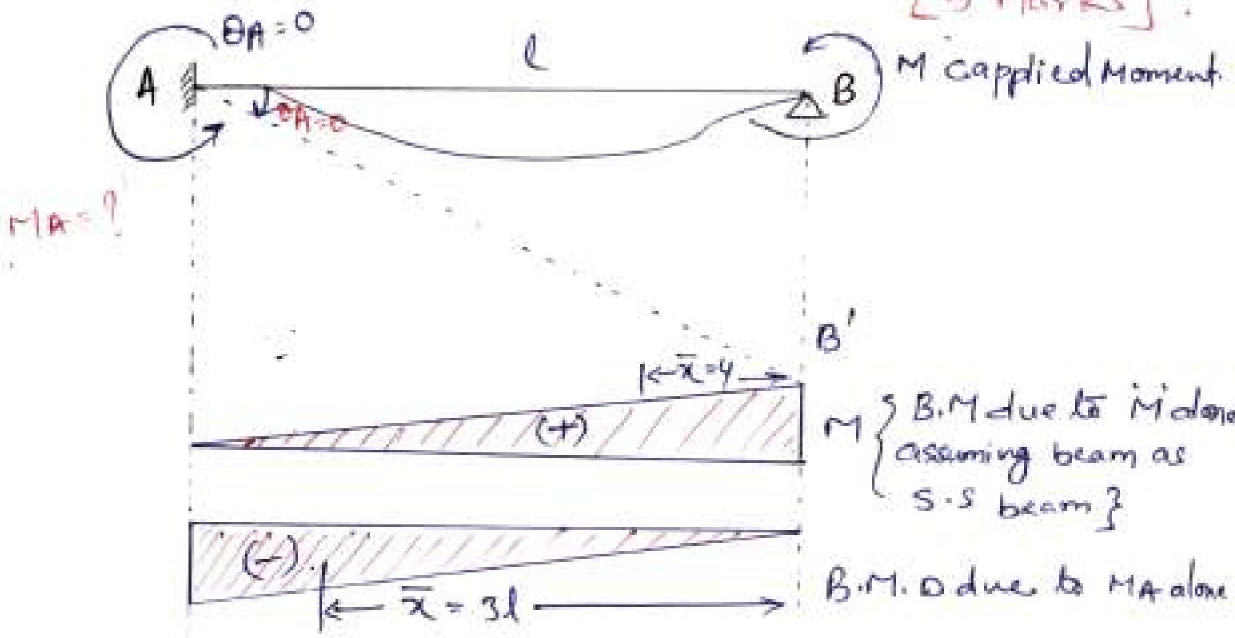
$= \frac{Wl^3}{48EI}$

$$\boxed{\theta_A = \frac{CC'}{l} = \frac{wl^2}{48EI}}$$

$\Rightarrow$ Deflection at centre $= \frac{l}{2} \times \theta_A = \frac{l}{2} \times \frac{wl^2}{48EI}$

$$\boxed{y_B = \frac{wl^3}{96EI}}$$

Q. Prove that C.O.M is $\frac{M}{2}$ when far end is fixed.
[5 Marks].



$\theta_A = 0$

$\Rightarrow l \times \theta_A = BB' \Rightarrow \theta_A = \frac{BB'}{l} = 0$

$\Rightarrow BB' = \frac{A\bar{x}}{EI} = 0$

$$\boxed{A\bar{x} = 0}$$

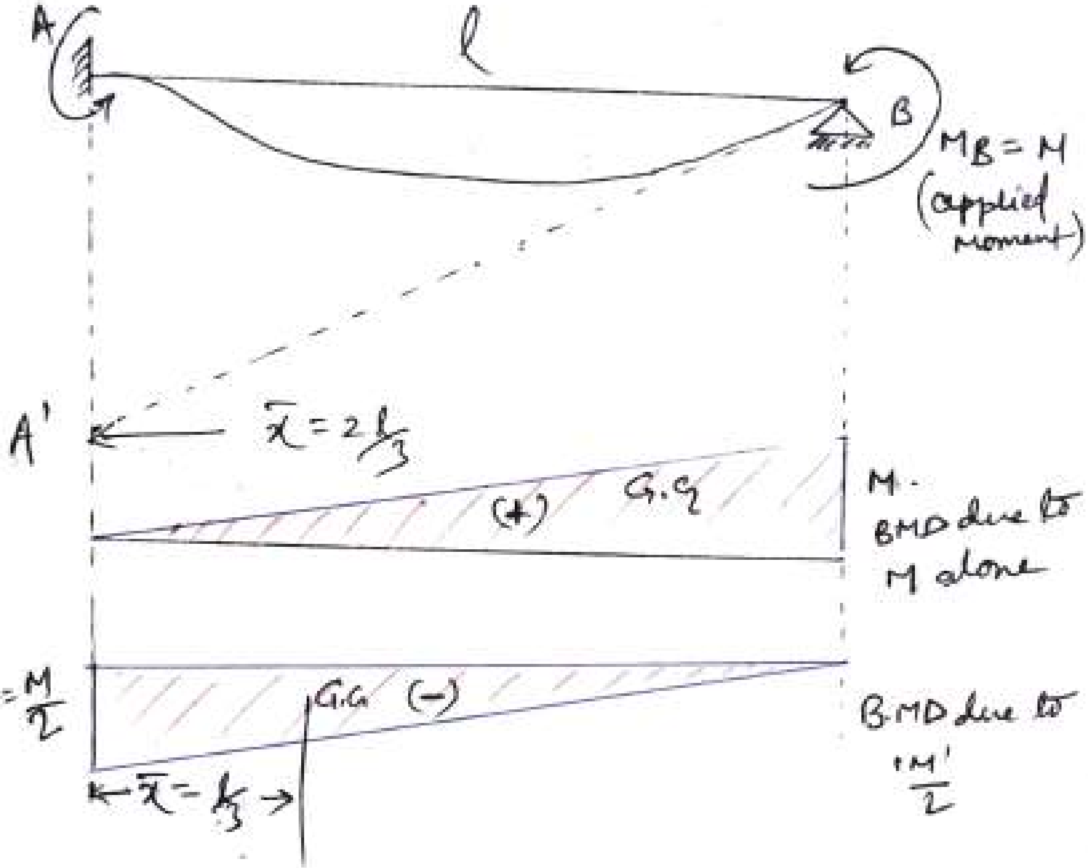
$\Rightarrow \left[\left(\frac{1}{2} \times l \times M\right) \times \frac{l}{3}\right] + \left[-\left(\frac{1}{2} \times l \times M_A\right) \times \frac{2l}{3}\right] = 0$

+ve area — -ve area.

$\Rightarrow \boxed{M_A = \frac{M}{2}}$

Q. Prove that stiffeness factor 's' = $\frac{4EI}{l}$ when far end is fixed.

proof:-



Stiffness factor = S = Moment reqd. to produce unit rotation = $\frac{M_B}{\theta_B}$

$$l \times \theta_B = AA' \Rightarrow \theta_B = \frac{AA'}{l}$$

$$AA' = \frac{A\bar{x}}{EI} = \left[\frac{(\frac{1}{2} \times l \times M)(\frac{2l}{3})}{EI}\right] + \left[\frac{-(\frac{1}{2} \times l \times M/2) \times l/3}{EI}\right]$$

$$= \frac{Ml^2}{3EI} - \frac{Ml^2}{12EI} = \frac{Ml^2}{4EI}$$

$$\boxed{\theta_B = \frac{AA'}{l} = \frac{Ml}{4EI}}$$

So, $S = \frac{M}{\theta_B} = \frac{M}{\left(\frac{Ml}{4EI}\right)} = \boxed{\frac{4EI}{l}}$ Imp.

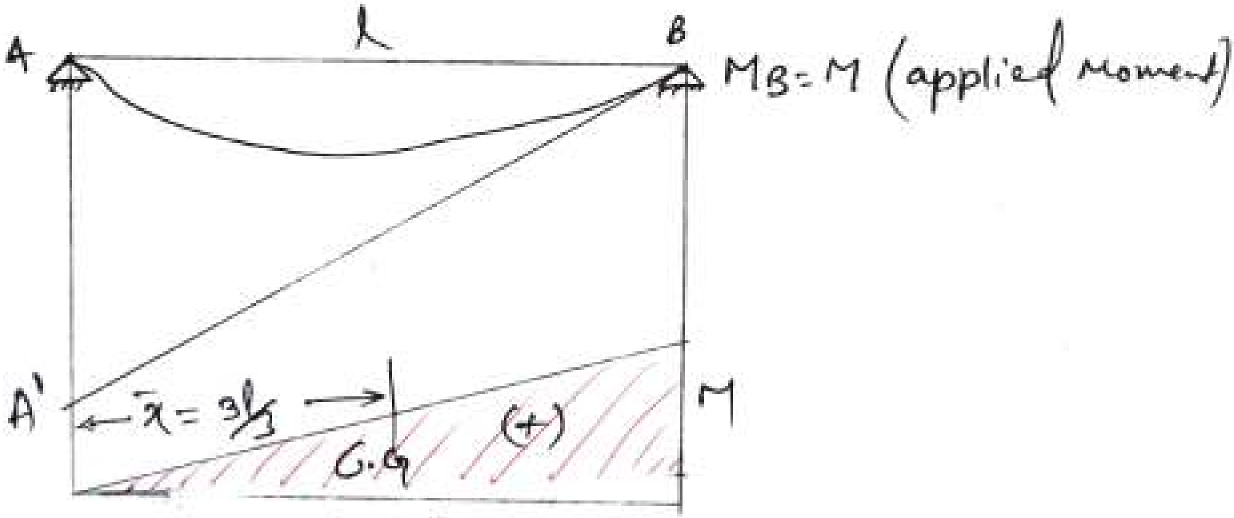
2nd Method

From Mohr's theorem

$\cancel{\theta_A}^{\,0} + \theta_B = \theta \Rightarrow \theta_B = \frac{A}{EI} = \frac{\frac{1}{2} \times l \times M}{EI} - \frac{\frac{1}{2} l \times M/2}{EI}$

$\theta_B - \theta_A = \frac{Ml}{4EI}$

Q. Prove that stiffness factor $S = \frac{3EI}{l}$, when far end is hinged?



B.M.D

Stiffness factor $= S = \frac{M_B}{\theta_B}$

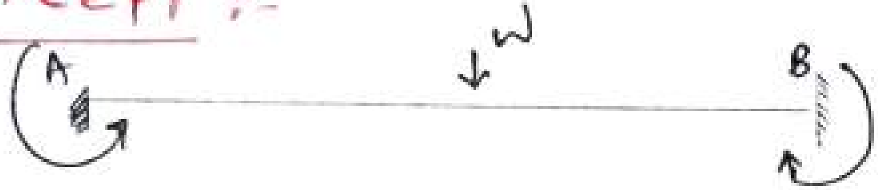
$\theta_B = \frac{AA'}{l}$

$AA' = \frac{A\bar{x}}{EI} = \frac{\left(\frac{1}{2} \times l \times M\right) \times \left(2l/3\right)}{EI} = \frac{Ml^2}{3EI}$

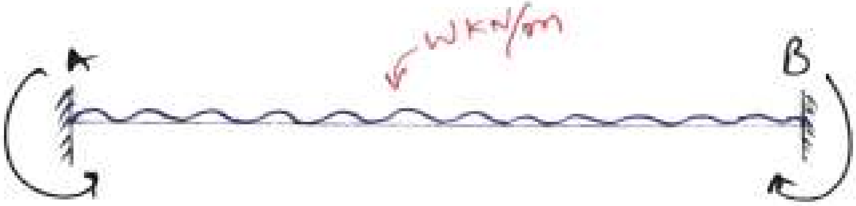
$\theta_B = \frac{AA'}{l} = \boxed{\frac{Ml}{3EI}}$

$S = \frac{M_B}{\theta_B} = \frac{M}{\frac{Ml}{3EI}} = \boxed{\frac{3EI}{l}}$ imp

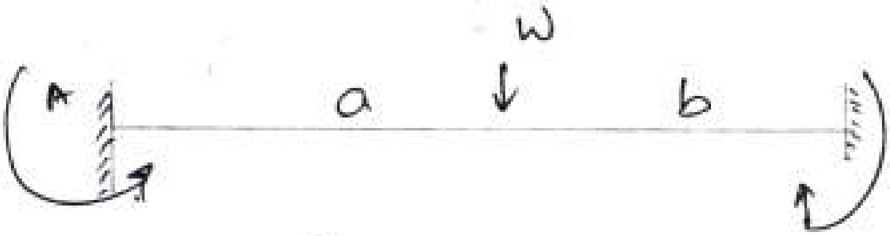
CONCEPT :-



$M_{FBA} = +\frac{wl}{8}$



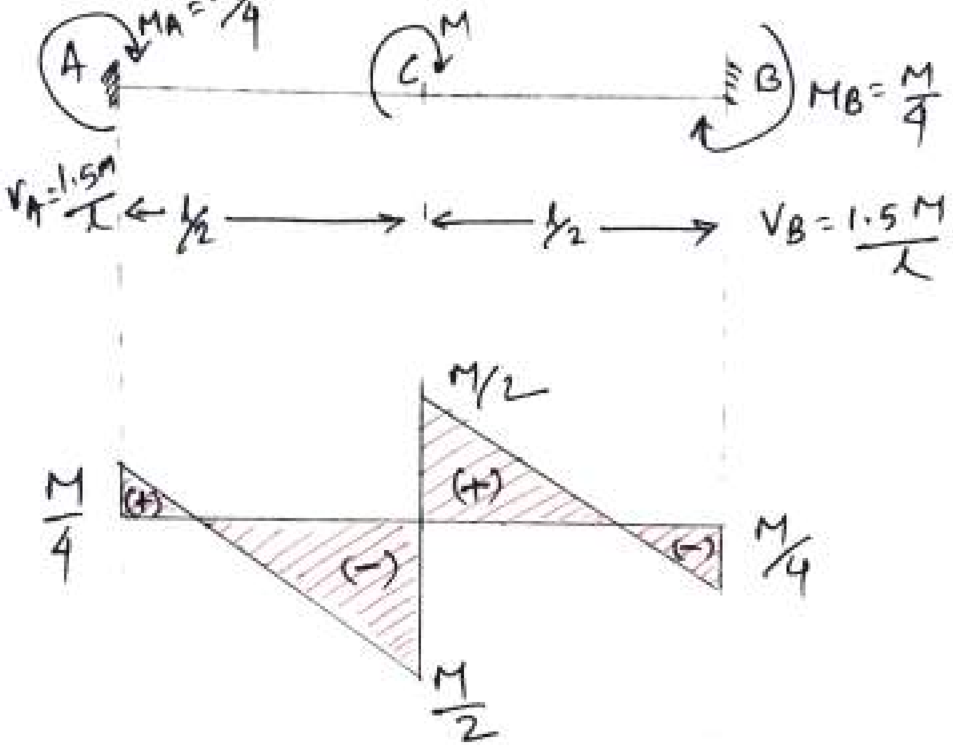
$M_{FBA} = \frac{wl^2}{12}$

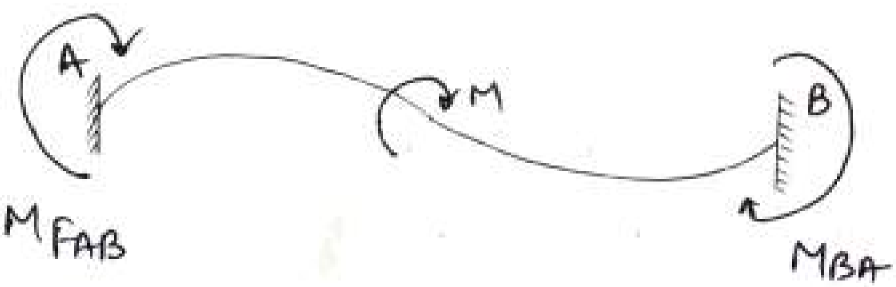


$M_{FAB} = -\frac{wab^2}{l^2}$

$M_{ABA} = +\frac{wa^2b}{l^2}$

* Clockwise end moment taken as +ve
* Anticlockwise end moment taken as -ve.





The purpose of F.E.M is only to make slope Zero

$\boxed{\text{No. of point of contra flexure} = 3}$

CONCEPT 8

Sign Convention in M.D MTD :-

(a) ↻+ ↺− → for endimoments

→ This sign convention is used while distributing end Moment only.

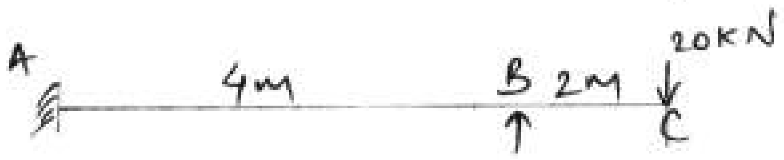
(b) Sagging B.M is taken '+ve' and Hogging B.M is taken as '−ve'

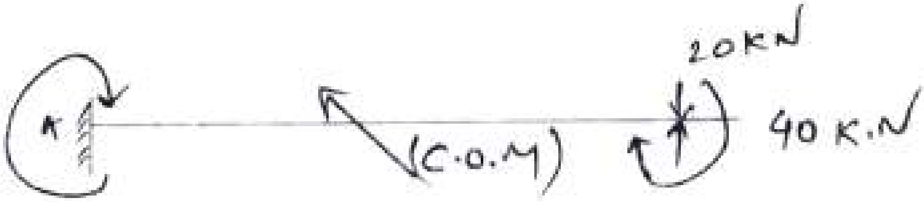
This sign convention is used while drawn B.M.D only.

Note :- Axial load & Axial deformation are neglected in MD. MTD.

Q. [2 Marks]

Moment at A is ?

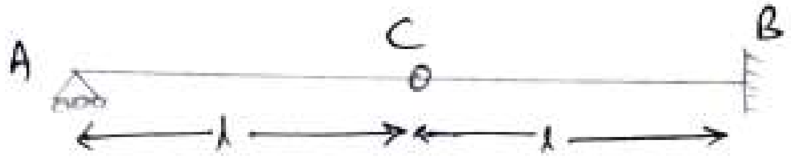




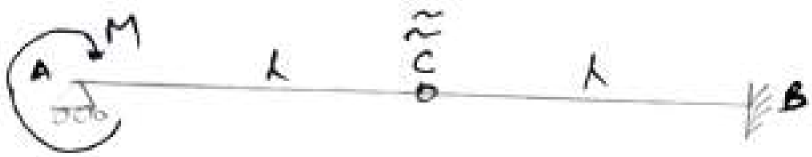
$$M_A = \frac{M}{2}$$

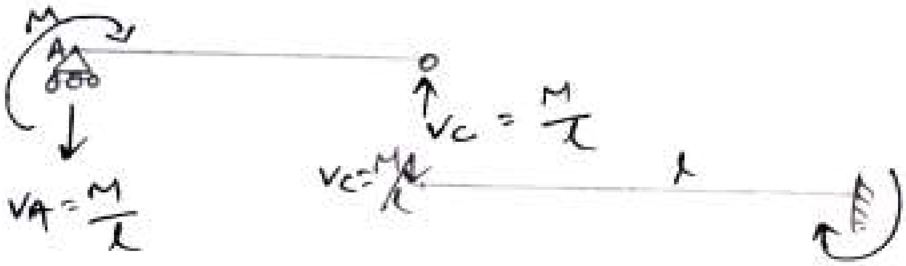
$$= \frac{40}{2} = 20\,KN.M \text{ (clockwise)}.$$

Q. C.O.F CAB is ? [2 Marks]



$$C.O.F = C_{AB} = \frac{(C.O.M) \text{ is at } B}{\text{Applied Moment at } A}$$

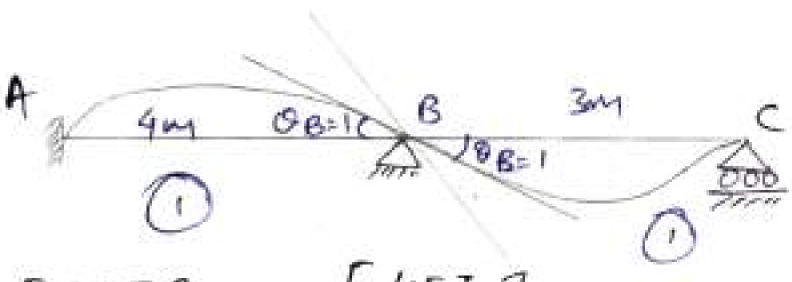




$M_B = M$ {C.O.M}

$$C_{AB} = \frac{C.O.M \text{ at } B}{\text{Applied Moment at } A} = \frac{M}{M} = 1$$

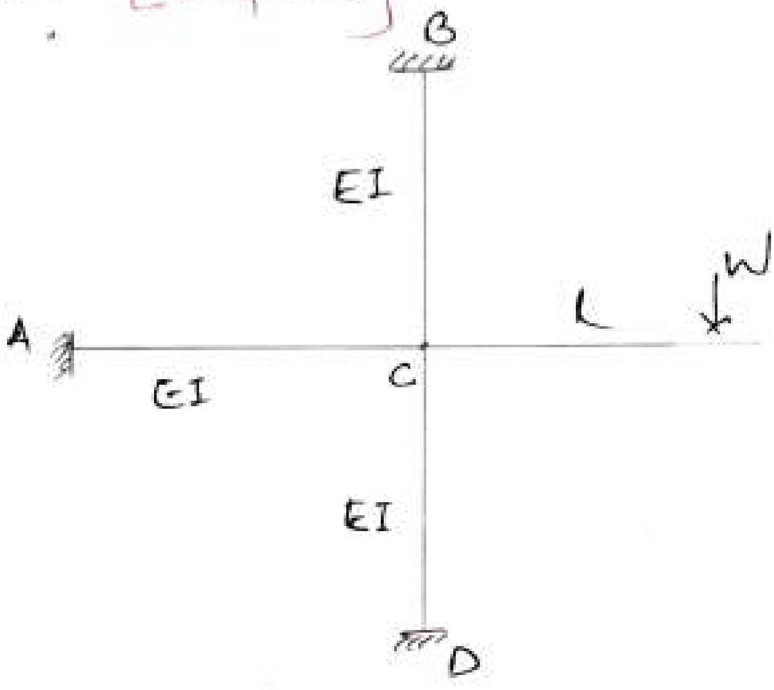
Q. For the continous beam shown in fig. Moment reqd. to produce units slope at B is ?



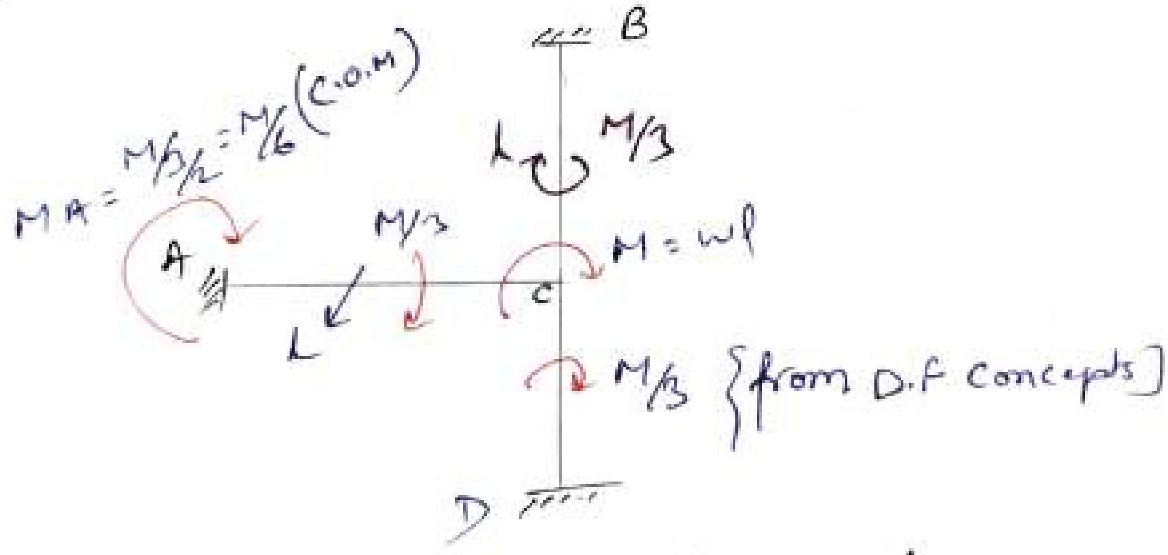
$$M = \left[\frac{3EI}{l}\right]_{BC} + \left[\frac{4EI}{l}\right]_{BA}$$

$$= \left[\frac{3EI}{3}\right]_{BC} + \left[\frac{4EI}{4}\right]_{BA} = 2EI \text{ KN-m.}$$

Q For the frame shown in fig. Moment at A is ...? [2 Marks]

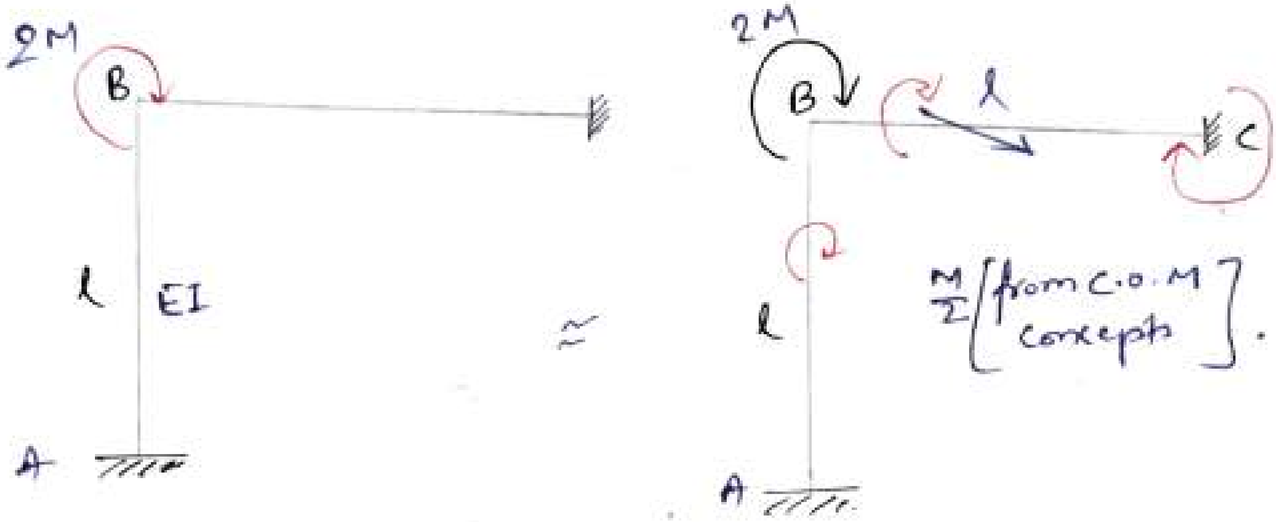


Solⁿ:

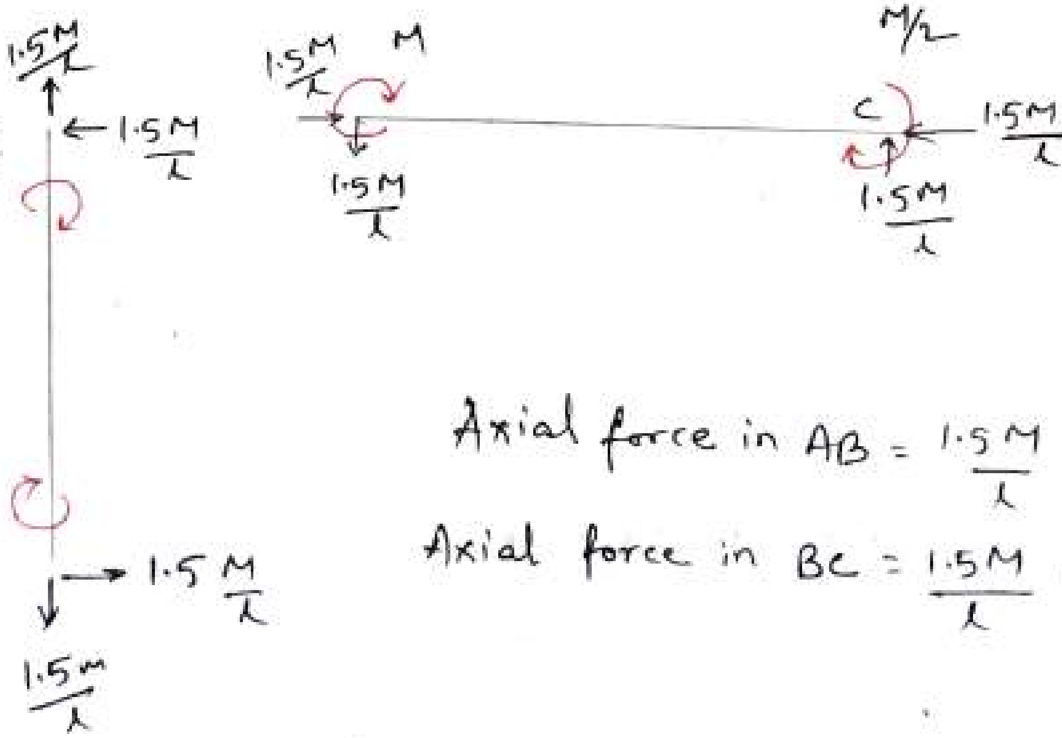


So, Moment at A = $M_A = \frac{M}{6} = \frac{wl}{6}$

Q. Axial forces in member AB is ?



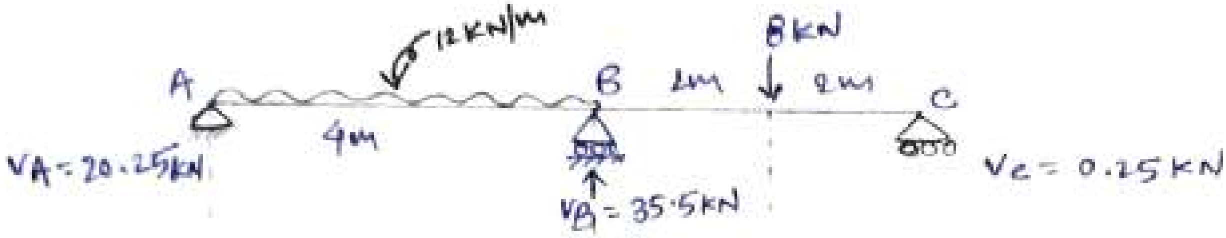
F.B.D

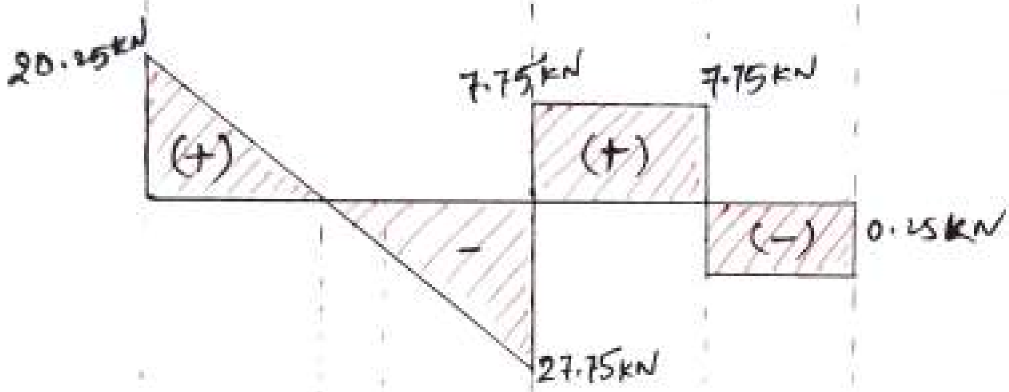


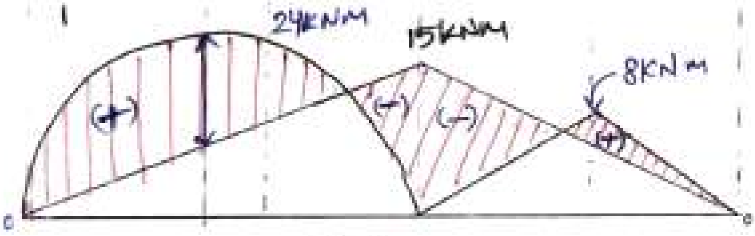
Axial force in AB = $\frac{1.5M}{l}$ (T)

Axial force in BC = $\frac{1.5M}{l}$ (C)

Q. Analyse the continous Beam shown in fig. using M.D M.T.D & Draw S.F.D & B.M.D.







BMD

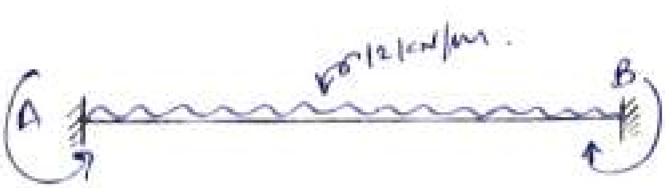
$\frac{wl^2}{8} = \frac{12 \times 4^2}{8}$

$= 24 \text{ kN}$

$\frac{wl}{4} = \frac{8 \times 4}{4}$

$= 8 \text{ kN}$

Step 1:- Find fixed end Moments { (+) ↻ }

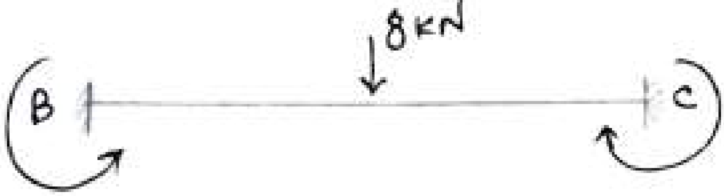


$M_{fab} = \frac{-wl^2}{12}$

$= -16 \text{ KNm}$

$M_{Fba} = +\frac{wl^2}{12}$

$= +16 \text{ KNm}$



$M_{Fbc} = -\frac{wl}{8}$ $\qquad$ $M_{Fcb} = +\frac{wl}{8}$

$= -4$ kN-m $\qquad$ $= +4$ kN-m.

Step 2:-

D.F's

Note:-

If there is only one Member meeting at a joint it's D.f is always (1) {whether, the joint is fixed or hinged}.

Joint	Member	K	ΣK	$D.f = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\frac{I}{4}$	$\frac{3}{2}\frac{I}{4}$	0.5
	BC	$\frac{3}{4}\frac{I}{4}$		0.5

Step 3:- End Moment Distribution:-

Joint	A [1]	B [0.5]	[0.5]	C [1]
F.E.M	-16	+16	-4	+4
'A' & 'C'	+16	-8	-2	
C.O.M				
Net F.E.M	0	+24	-6	0
Balance		-9	-9	
C.O.M	0			0
Final end Moments	0	+15	-15	0

NOTE :-

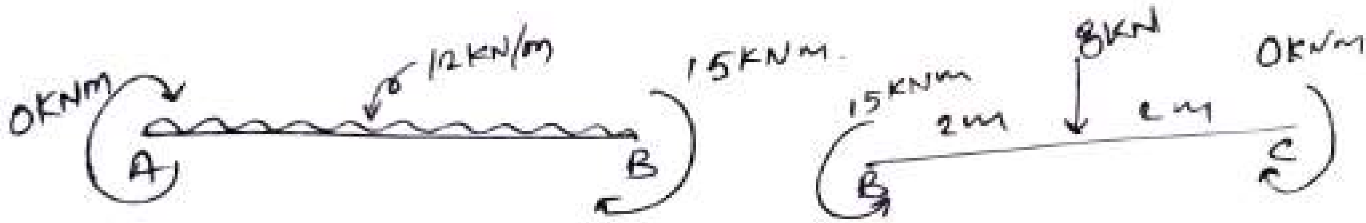
1. Releasing joints A & C means realesing the fixities at the joints at 'A' & 'C'.

2. In end Moment distribution, we fix all the joints & fixed end Moments curve Calculated if ends joints are hinged, then realese those fixities by applying moment opposite to fixed end Moments. & Net fixed end Moments are calculated. Realease the fixities of all the intermediate joints also, this process is called realeasing or balancing.

⇒ Repeat the some procedure till no fixity is developed at any intermidiate joint.

4. Step :- Support reaction :-

F.B.D of span AB & BC :-



1. Reaction due to applied loads	24 KN (↑)	24 KN (↑)	4 KN (↑)	4 KN (↑)
2) Reaction due to end Moments	$\frac{15}{4} = 3.75$ (↓)	3.75 KN (↑)	$\frac{15}{4} = 3.75$ (↑)	3.75 (↓)
final reaction	20.25 KN (↑) (VA)	35.5 (↑) VB		0.25 KN (↑) VC

NOTE :-

(i) Exactly under a point load shear force is indeterminate that is why we calculate at L.H.S of 'D' & R.H.S of 'D' but Note at 'D'.

→ Similarly exactly under a couple B.M is indeterminate.

(ii) If shear force changes it's sign from (+ve) to (-ve) then (+ve) B.M is Maxumum at that Section. If shear force changes it's sign from (-ve) to (+ve) the (-ve) B.M is Max^m. at that section.

(iii) If shear force is zero at @ Section B.M is Max^m. or min.

∵ for 'y' to be Max. or Min, $\frac{dy}{dx} = 0$

→ Similarly,

for 'M' to Max^m. or Min^m. $\frac{dM}{dx} = V = 0$.

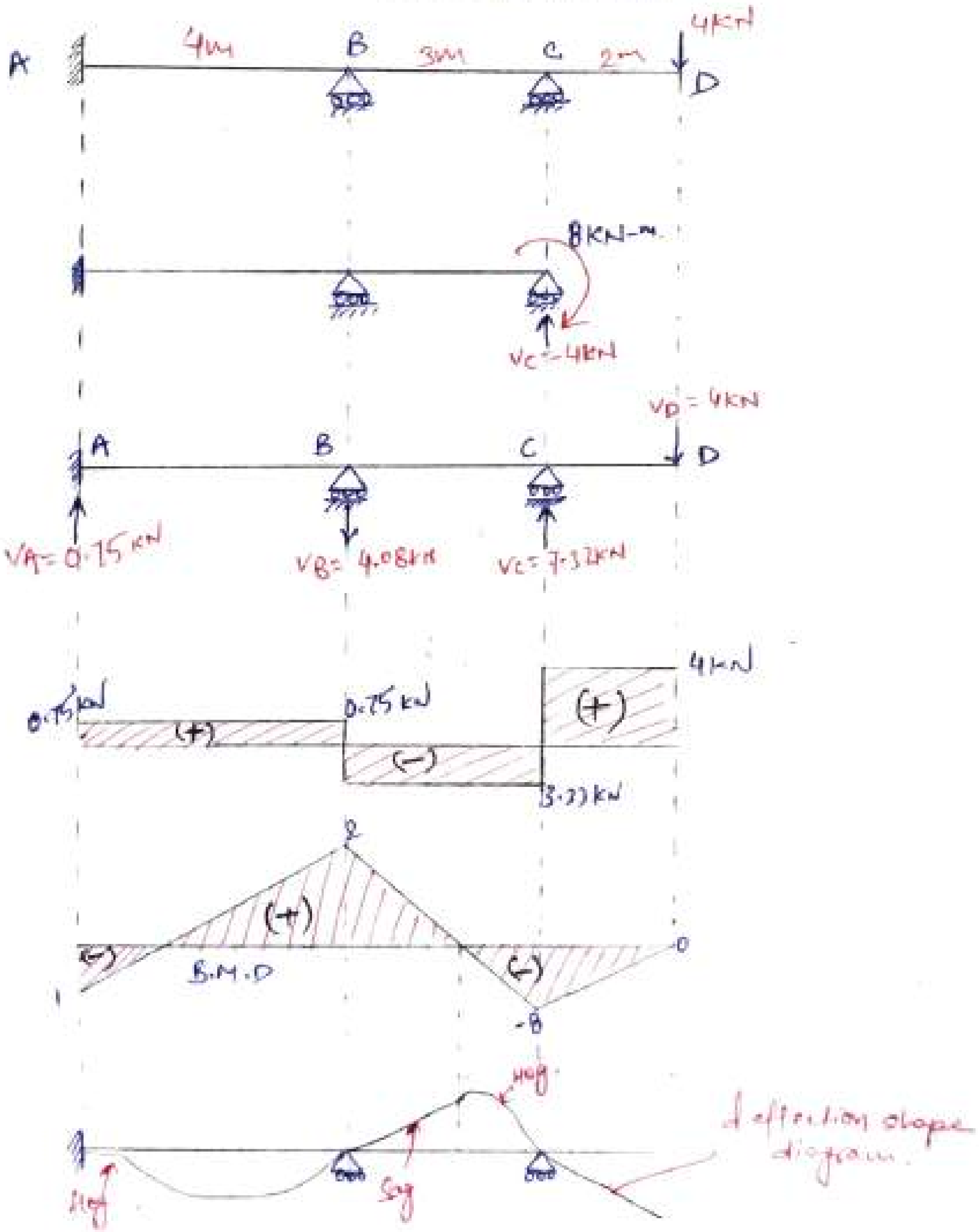
→

(iv) If B.M is zero at a section there Nothing is Known about S.F {It can any value}.

because where,

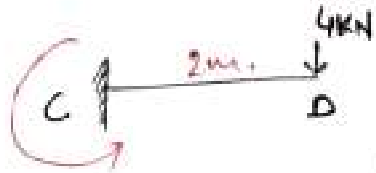
No, relation like $\left(\frac{dv}{dx} = M\right)$

Q. Analyse the continous beam shown in fig. using Moment distribution MTD & Draw it's S.F.D & B.M.D.



Step 1. F.E.M.S :-

$M_{Fab} = M_{Fba} = M_{Fbc} = M_{Fcb} = 0.$



{ free end 'D' should not be fixed Bcz, it get, deflects. }

$M_{FCD} = -4 \times 2 = -8 \text{ kN-m.}$ { -ve because Anticlockwise }

Step 2 :-

D.F's { Idealised 'C' as hinge support }

joint	Member	K	ΣK	$D.f = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4}$	$\frac{I}{2}$	0.5
	BC	$\frac{3}{4} \cdot \frac{I}{3} = \frac{I}{4}$		0.5

'C' is hinged.

Step 3 :- End Moment Distribution :-

Joint	A	B		C		D
D.f	1	0.5	0.5	1	0	-
F.E.M	0	0	0	0	-8	
Realease 'C'				+8		
C.O.M			+4			
Balanced		-2	-2			
C.O.M	-1			0		
final end Moment	-1	-2	+2	+8	-8	

'0' because 'C' is idealise as hinged.

Step 4 :- Support Reaction :-

	A	B	B	C	C	D
1) Reactions due to applied loads	0	0	0	0	4KN↑	-
2) Due to end Moment	$\frac{3}{4}$: 0.75 ↑	0.75	$\frac{10}{3}$: 3.33 ↓	3.33 ↑	-	-
	0.75 KN (↑)	4.08 KN (↓)		7.33 KN (↑)		-
	VA	VB		VC		-

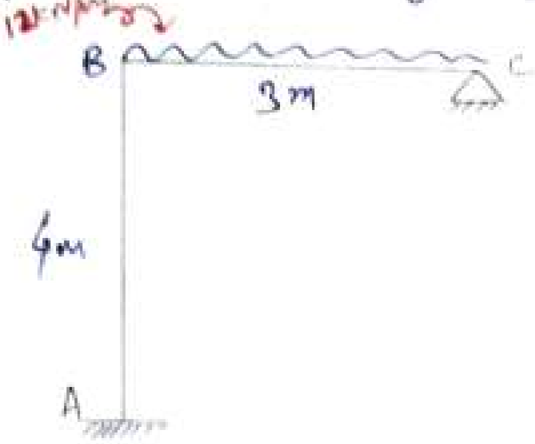
No support

at (B)
So, no reaction

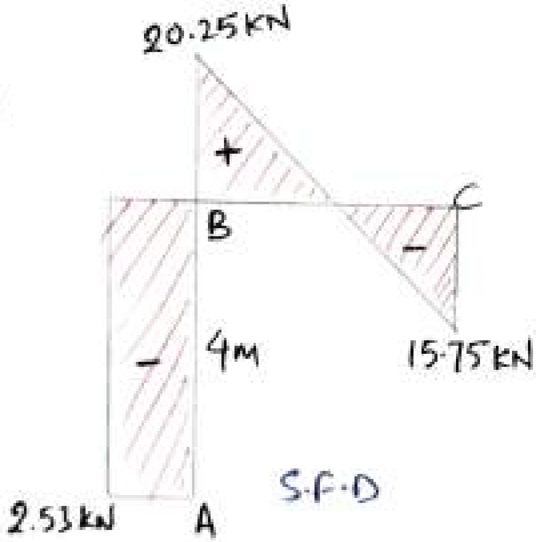
It is in balanced condition that is why no need to give reaction due to Moment.

Step V :- Deflected shape diagram is as shown in fig.

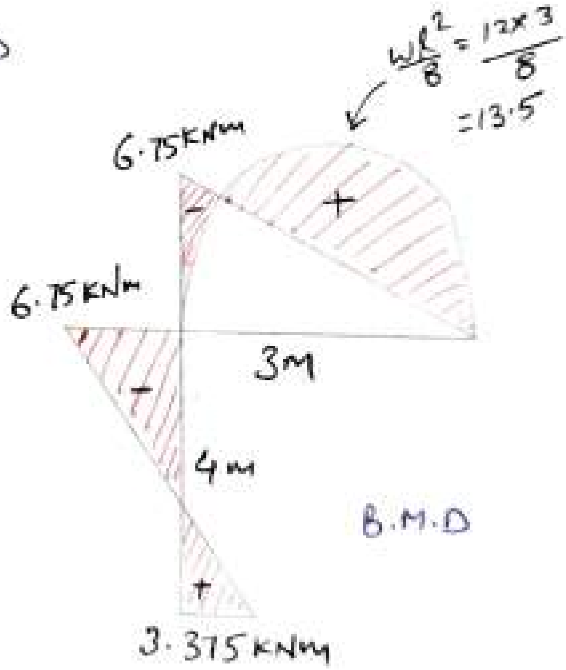
Q. Analyse the frame shown in fig. using Moment distribution MTD & Draw S.F.D & B.M.D, ATD {Axial thrust diagram} & deflection profile.



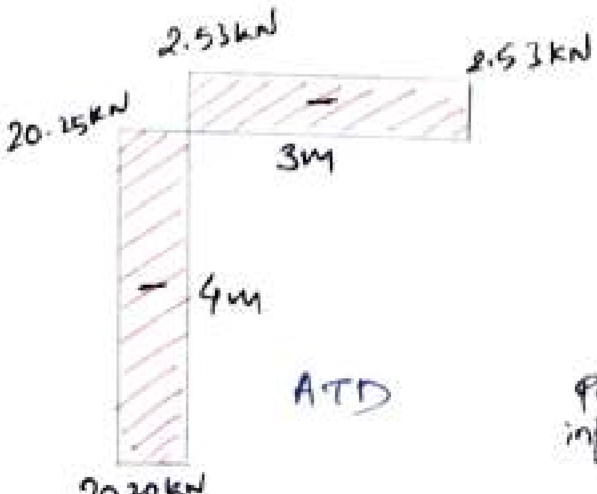
Step (V) :- S.F.D, B.M.D, A.T.D



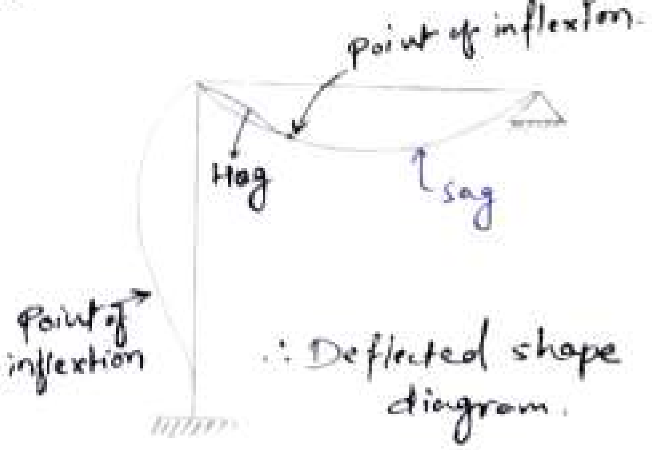
S.F.D



B.M.D



ATD



∴ Deflected shape diagram.

NOTE :-

Due to Horizontal reaction at 'A', 'C', the frame will not sway in Any direction.

Step I:- F.E.M :-

$M_{Fab} = M_{Fba} = 0$ {BCZ there is no. loads on the span AB}

$M_{Fbc} = -\frac{wl^2}{12} = \frac{-12 \times 3^2}{12} = -9$ KN-m.

$M_{Fcb} = +\frac{wl^2}{12} = +9$ KN-m.

Step II :- D.F.S :-

Joint	Member	K	ΣK	$D.f = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4}$	$I/2$	0.5
	BC	$\frac{3}{4} \times \frac{I}{3} = \frac{I}{4}$		0.5

Step III. End Moment Distribution :-

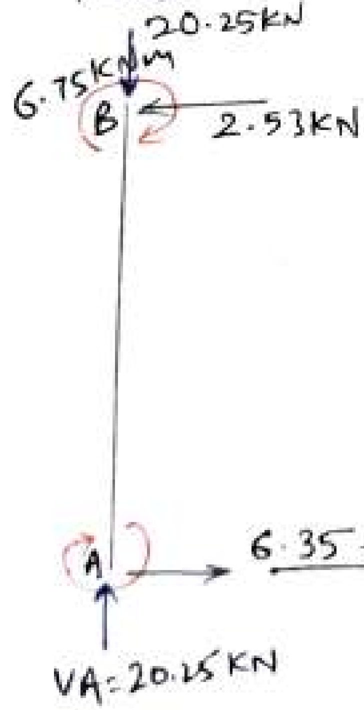
Joint	A	B		C
D.F	1	0.5	0.5	1
F.E.M	0	0	-9	+9
Release 'c' C.O.M			-4.5	-9
Net. F.E.M	0	0	-13.5	0
Balance		+6.75	+6.75	
C.O.M	+3.375			0
Final end Moments.	+3.375	+6.75	-6.75	0

Hinged support.

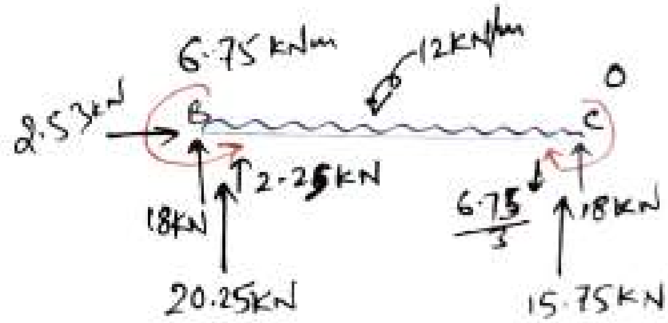
Step iv :- Support Reaction:-

* first Beam
* then column.

F.B.D



$$\frac{6.35+3.75-2.53}{4}\text{ kN} = 2.53\text{ kN}$$



VA = 20.25 KN (↑)

VC = 15.75 KN (↑)

HA = 2.53 KN (→)

HC = 2.53 KN (←)

MA = 3.75 KNm (↻)

$V_A = V_B = 12 \times \frac{3}{2} = 18$ KN(↑)

$V_B = \frac{6.75}{3} = 2.25$ KN(↑).

$V_B = 18 + 2.25 = 20.25$ KN(↑) (same dir.)

$V_C = 18 - 2.25 = 15.75$ KN(↑) (opp. dir.)

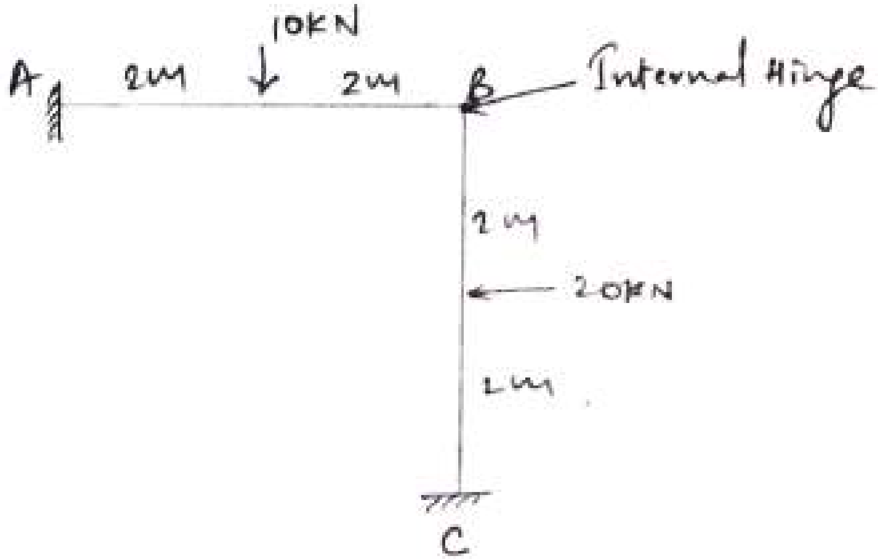
$H_B = \frac{3.375 + 6.75}{4}$

$= \frac{10.125}{4} = 2.53$ (←)

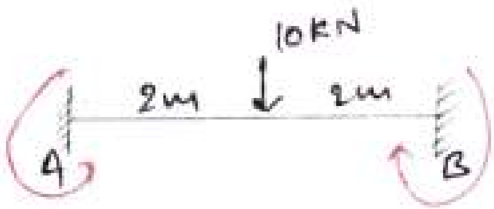
$H_A = \frac{10.125}{4} = 2.53$ (→)

$H_C = 2.25$ (←) (equill. cond.)

Q: Analysis the frame shown in fig. using Moment Distribution MTD (MDM). & Draw the B.M.D.



Step 1:- F.E.M's



$$M_{Fab} = \frac{-wl}{8} = \frac{-10 \times 4}{8} = -5\ KNm.$$

$$M_{Fba} = +5\ KNm$$

$$F_{FBC} = \frac{-20 \times 4}{8} = -10\ KNm.$$

$$M_{FCB} = +10\ KN\text{-}m.$$

Step 2 :- D.f's

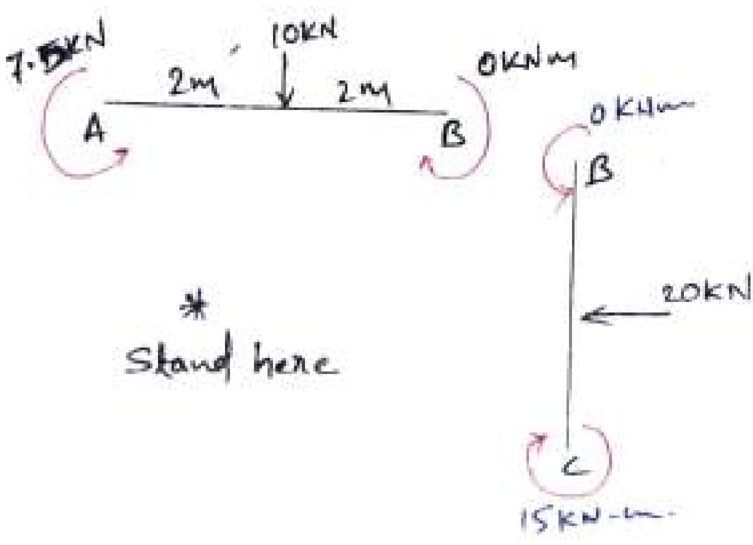
NOTE :- Since, joint 'B' is not a rigid joint, D.f can not be exist or forced.

Step 3 :- End Moment Distribution :-

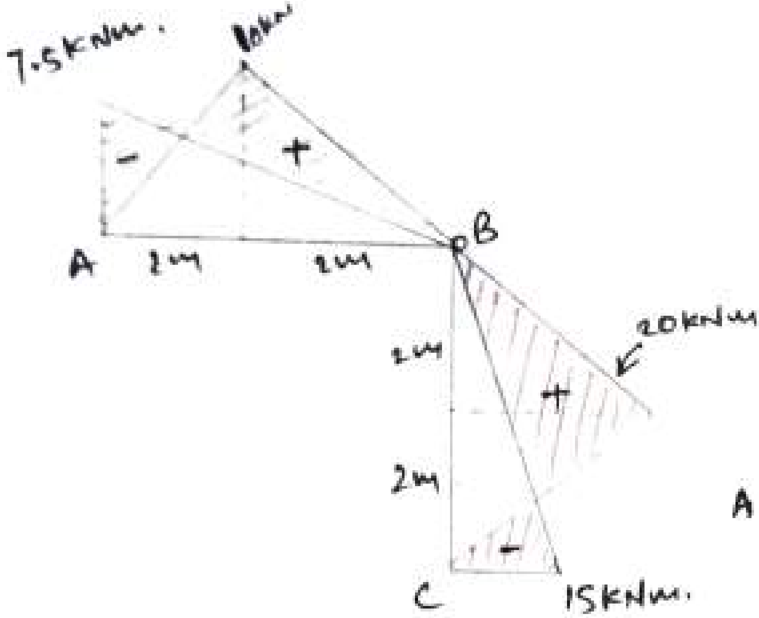
Joint	A		B		C
D.f	$\boxed{1}$		$\boxed{-}$	$\boxed{-}$	$\boxed{1}$
F.E.M	-5		+5	-10	+10
Balanced			-5	10	
C.O.M	-2.5				+5
Final end Moment	-7.5		0	0	+15

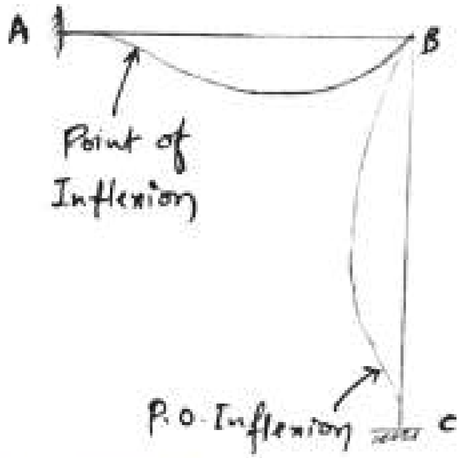
Step IV :- B.M.D

F.B.D :-



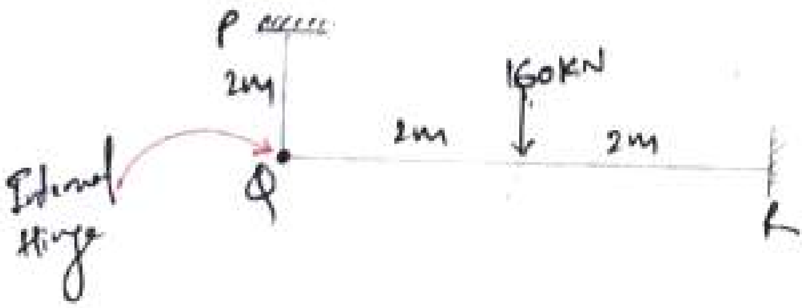
*
Stand here





Deflection shape diagram.

Q. The axial load in a member 'PQ' for the arrangement shown in fig. [$\frac{5.26}{27}$ Gate 2014][2 marks]



F.O.M.

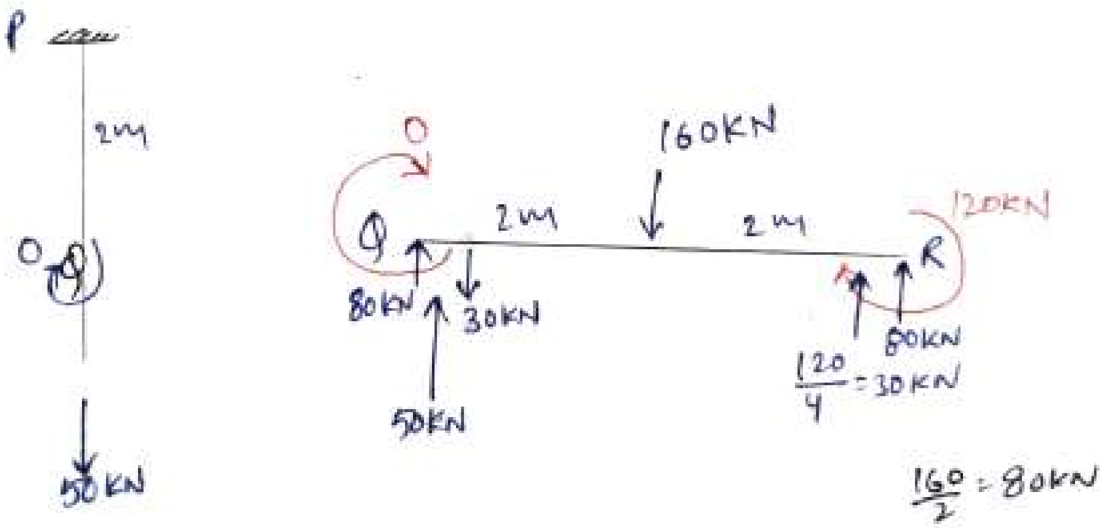
MQR =

MRQ =

Sol^n:- Step III :- End Moment Distribution :-

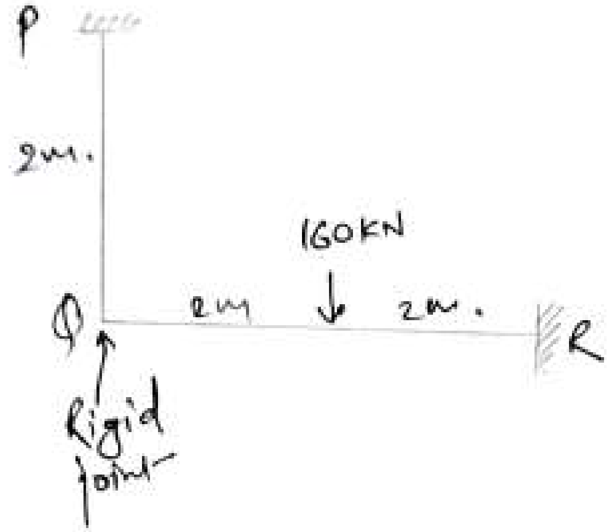
Joint	P	Q		R
D.F	1	-	-	1
F.E.M	0	0	-80	+80
Balance Q			+80	
C.O.M	0			+40
Final end Moments	0	0	0	+120

Step IV :- E.R.D



Axial force in PQ = 50KN(↑) (T)

Q. For the frame shown in fig. the axial force in the member 'PK' is ?



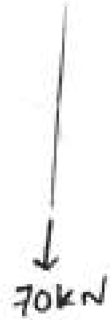
Sol^n:- Step 1 :- D.F

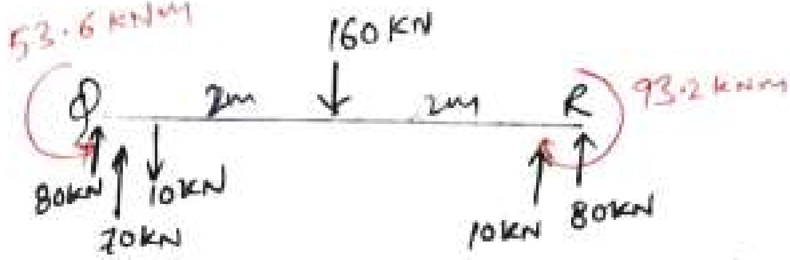
Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
Q	QP	$1/2$	$\frac{3I}{4}$	0.67
	QR	$1/4$		0.33

Step 2 :- End Moment distribution :-

Joint	P	Q		R
D.F	1	0.67	0.33	1
F.E.M	0	0	-80	+80
Balance		+53.6	+26.4	
C.O.M	26.8			13.2
Final end Moment	+26.8	+53.6	-53.6	+93.6

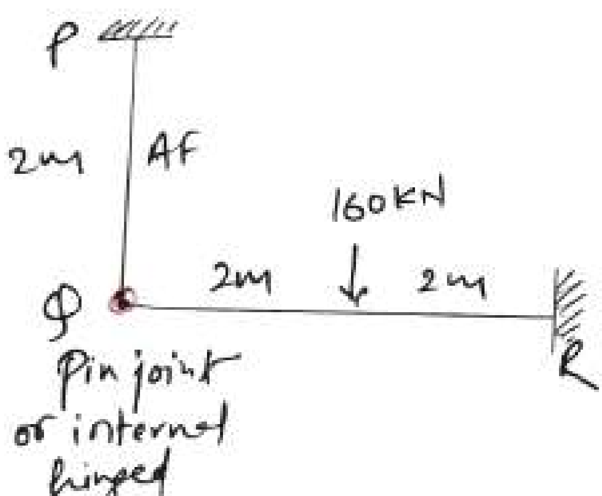
Step 3 :- F.B.D





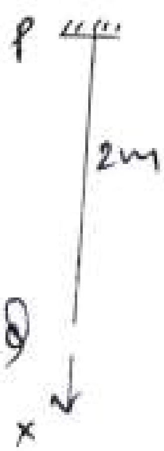
$= 93.2 - 53.6 = 4.$

Q For the arangement shown in fig. if axial rigidity AE & flexural rigidity is EI for all Member. find axial force in the Member PQ.

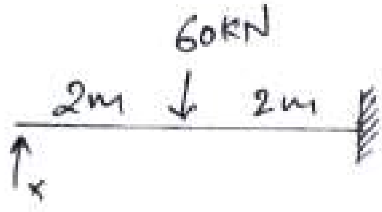


Solⁿ :-

Step 1. F.B.D :-



F.B.D of PQ

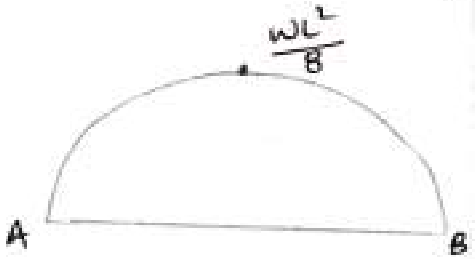


F.B.D of QR

Fixed end Moments

$$M_{AB}^{F} = -\frac{wl}{8}$$

$$M_{BA}^{F} = \frac{wl}{8}$$

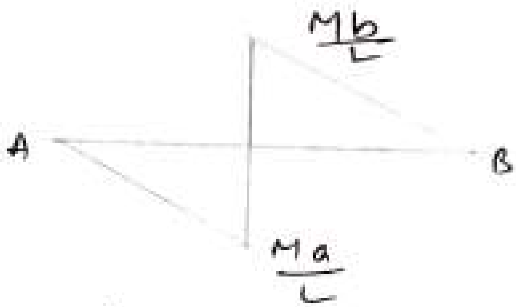


$$M_{AB}^{F} = -\frac{WL^2}{12}$$

$$M_{BA}^{F} = \frac{WL^2}{12}$$

$$M_{AB}^{F} = -\frac{wab^2}{L^2}$$

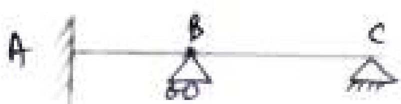
$$M_{BA}^{F} = \frac{Wa^2b}{L^2}$$



$$M_{AB}^{F} = \frac{Mb(2a-b)}{L^2}$$

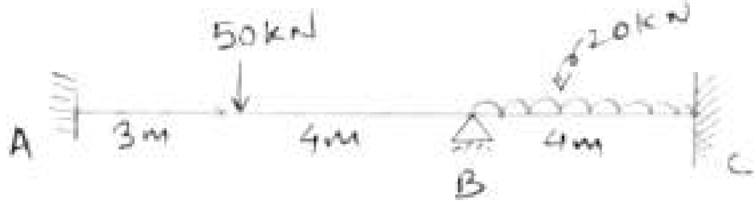
$$M_{BA}^{F} = \frac{Ma(2b-a)}{L^2}$$

* NOTE :- when calculating fixed End Moment consider all the points are fixed.



→ Consider B and C is also fixed only while calculating fixed end moments.

Q. Analysis the beam by Moment-Distribution Method and Draw SFD and BMD.



Sol^n:- FEM :-

$$M_{AB}^F = -\frac{wab^2}{l^2} = \frac{-50\times3\times4^2}{7^2} = -48.98\,kNm$$

$$\bar{M}_{BA} = \frac{wa^2b}{l^2} = \frac{-50\times3^2\times4}{7^2} = 36.73\,kNm$$

$$\bar{M}_{BC} = -\frac{wl^2}{12} = \frac{-20\times4^2}{12} = -26.67\,kNm$$

$$\bar{M}_{CB} = +\frac{wl^2}{12} = \frac{-20\times4^2}{12} = 26.67\,kN.m$$

Step 2:- Find D.F :-

Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{L} = \frac{1}{7} = 0.14I$	$0.14I + 0.25I = 0.39I$	$\frac{0.14}{0.39} = 0.36$
	BC	$\frac{I}{L} = \frac{1}{4} = 0.25I$		$\frac{0.25}{0.39} = 0.64$

Step 3 :- Moment Distribution :-

Joint	A		B	C
Member	AB	BA	BC	CB
D.F	0	0.36	0.64	0
FEM	-48.98	$\boxed{36.73}$	$\boxed{-26.67}$	26.67
ΣM		-10.06		
ΣM×DF	0	-3.62	-6.43	0
C.O $\frac{\Sigma M \times DF}{2}$	-1.81	0	0	-3.21
Final Moments	-50.79	33.11	-33.1	23.46

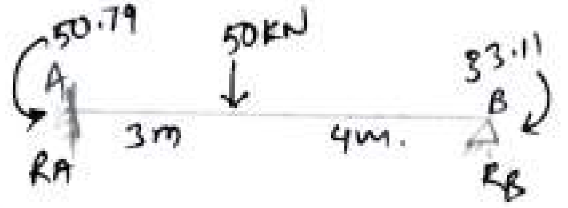
* Don't include D.F & ΣM while final Moment

$M_{AB} = -50.79$ kNm.

$M_{BA} = 33.11$ kNm

$M_{BC} = -33.1$ kNm.

$M_{CB} = 23.46$ kNm.



$\Sigma M_A = 0$

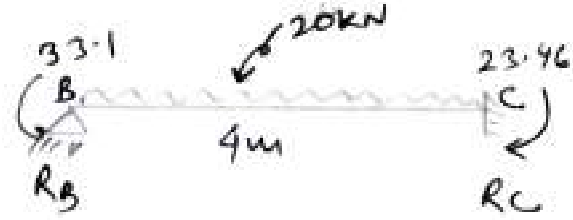
$-50.79 + 50 \times 3 - RB \times 7 + 33.11 = 0$

$RB = 18.9 \uparrow kN$

$\Sigma F_y = 0 \uparrow +$

$-50 + 18.9 + RA = 0$

$RA = 31.1 \uparrow kN$



$\Sigma M_B = 0$ ↓+

$-33.1 + 20 \times 4 \times 2 - RC \times 4 + 23.46 = 0$

$RC = 37.59 \uparrow kN$

$\Sigma F_y = 0 \uparrow +$

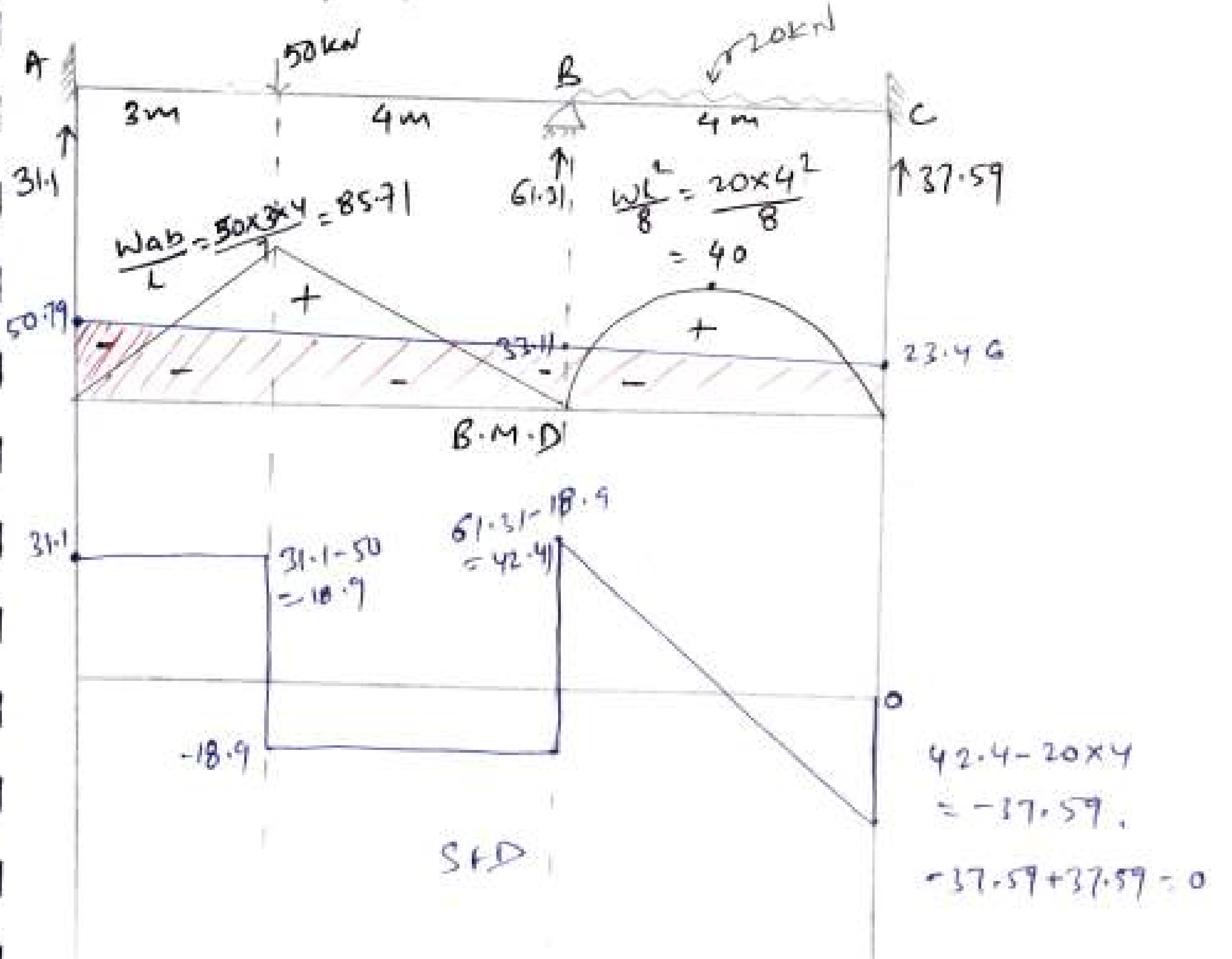
$RB + 37.59 - 20 \times 4 = 0$

$RB = 42.41 \uparrow kN.$

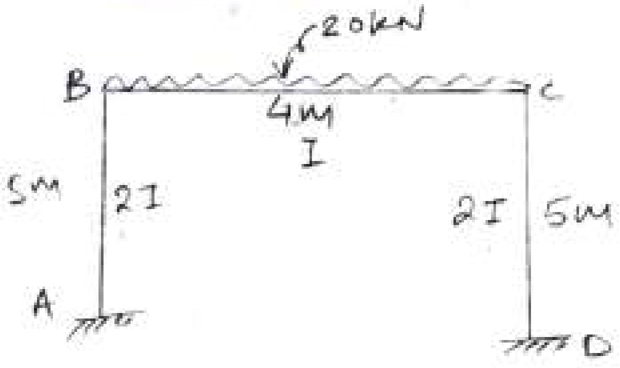
Final Reaction :-

$RA = 31.1 \uparrow kN$; $RC = 37.59 \uparrow kN$

$RB = 18.9 + 42.41$
$= 61.31 \uparrow kN$



Q. Analyse the Frame by Moment Distribution Method and Draw BMD.



Step 1:- FEM :-

$\bar{M}_{AB} = 0$
$\bar{M}_{BA} = 0$ } No loading.

$\bar{M}_{BC} = -\frac{wl^2}{12} = -\frac{20\times4^2}{12} = -26.67\,kNm.$

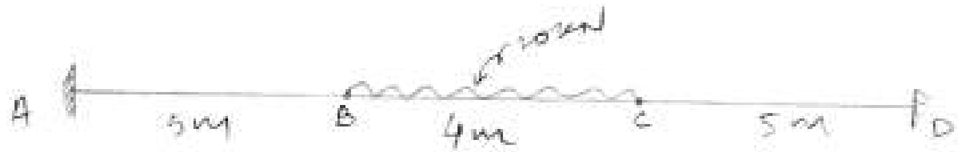
$\bar{M}_{CB} = \frac{wl^2}{12} = \frac{20\times4^2}{12} = 26.67\,kNm.$

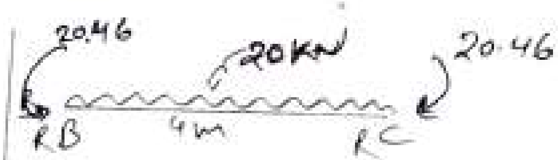
$\bar{M}_{CD} = 0$
$\bar{M}_{DC} = 0$ } No loading.

Step 2 D.F

Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{L} = \frac{2I}{5} = 0.4I$	0.65	0.62
	BC	$\frac{I}{L} = \frac{1}{4} = 0.25I$		0.38
C	CB	$\frac{I}{L} = \frac{1}{4} = 0.25I$	0.65	0.38
	CD	$\frac{I}{L} = \frac{2}{5} = 0.4I$		0.62

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.62	0.38	0.38	0.62	0
FEM	0	0	-26.67	26.67	0	0
ΣM×DF	0	16.53	10.13	-10.13	-16.53	0
C.O	8.265	0	-5.06	5.06	0	-8.26
ΣM		5.06		5.06		
ΣM×DF	0	3.13	1.92	-1.92	-3.13	0
C.O	1.36	0	-0.96	0.96		-1.56
ΣM		0.96		-0.96		
ΣM×DF	0	0.59	0.36	-0.36	-0.59	0
CO	0.29	0	-0.18	0.18	0	-0.29
Final Moment	10.11	20.23	-20.46	20.46	-20.25	-10.11





$\Sigma M_A = 0$ ↺+

$10.11 + 20.25 - RB \times 5 = 0$

$\boxed{RB = 6.07\ kN \uparrow}$

$\Sigma F_y = 0$

$RA + 6.07 = 0$

$\boxed{RA = -6.07 \downarrow kN}$

$\Sigma M_B = 0$ ↺+

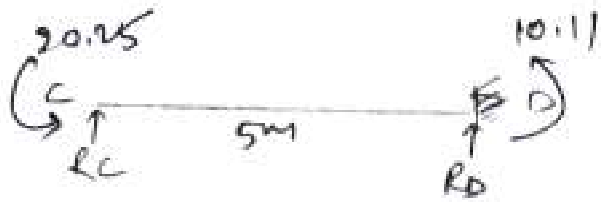
$-20.46 + 20.46 + 20 \times 4 \times 4/2 - RC \times 4 = 0$

$\boxed{RC = 40\ kN \uparrow}$

$\Sigma F_y = 0$ ↑+

$RB + 40 - 20 \times 4 = 0$

$\boxed{RB = 40\ kN \uparrow}$



$RMC = 0 \downarrow +$

$-20.25 - 10.11 - R_D \times 5 = 0$

$\boxed{R_D = -6.07\ kN \downarrow}$

$\Sigma F_y = 0 \uparrow$

$R_C - 6.07 = 0$

$\boxed{R_C = 6.07\ KN \uparrow}$

Final <u>Reaction</u> :-

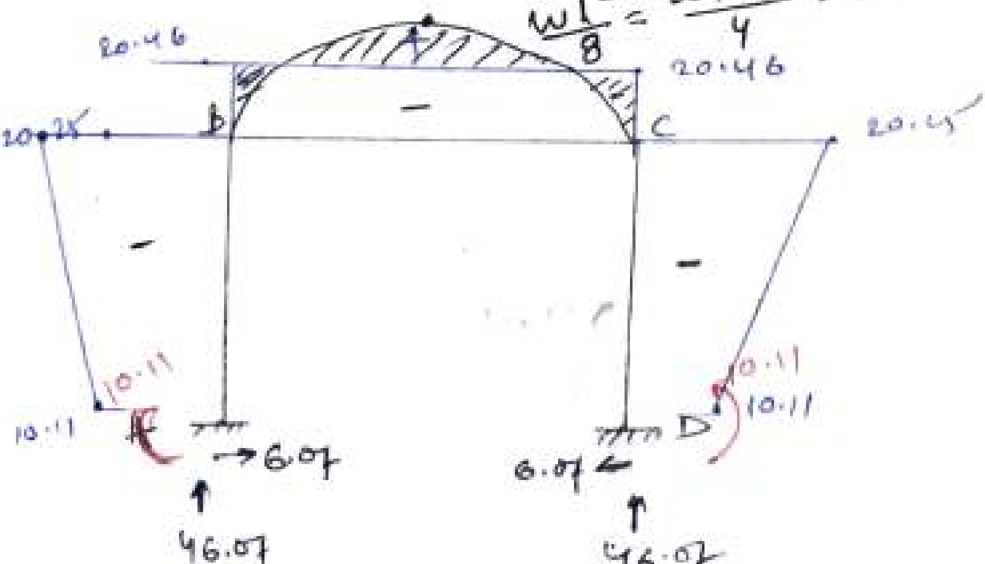
$R_A = -6.07 \downarrow kN$

$R_B = 6.07 + 40$
$= 46.07\ kN \uparrow$

$R_C = 40 + 6.07$
$= 46.07\ kN \uparrow$

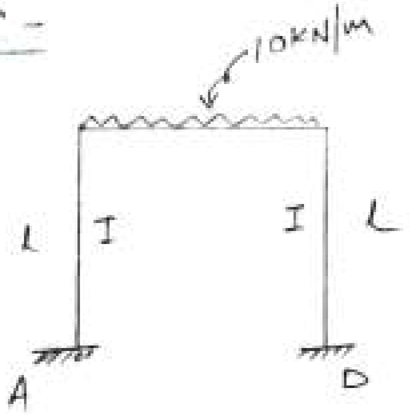
$R_D = -6.07\ kN \downarrow$

$\frac{wl^2}{8} = \frac{20 \times 4^2}{4} = 80$



Portal frames with sway → Moment Distribution :-

Concept 1 :-

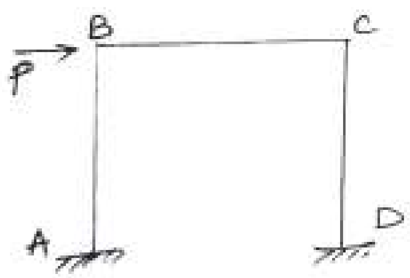


* It will not sway in any direction.

* If structure and loading are symmetric then frame will be not sway in any direction. If any of these two condition is not satisfy then frame will sway either left or right direction.

example :-

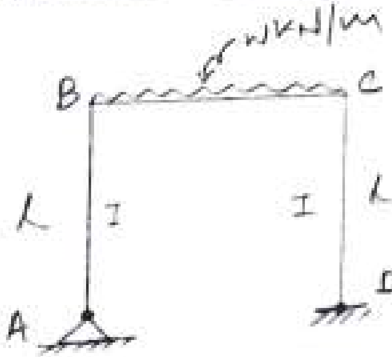
1.

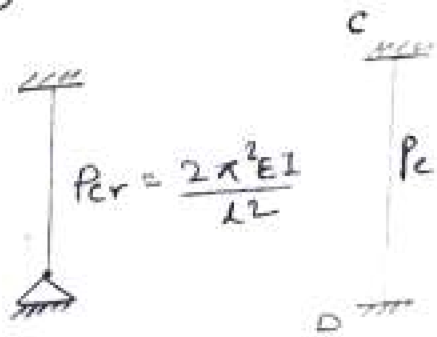


Pure sway

(frame sway to the right).

2.



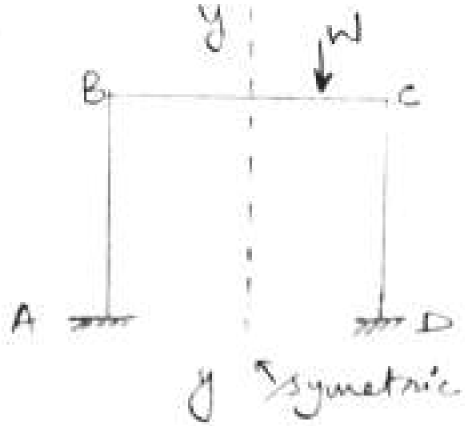


$P_{cr} = \frac{2\pi^2 EI}{L^2}$ $\quad$ $P_{cr} = \frac{4\pi^2 EI}{L^2}$

→ Loading is symmetrical frame is not symmetrical (frame sways towards weak column side)

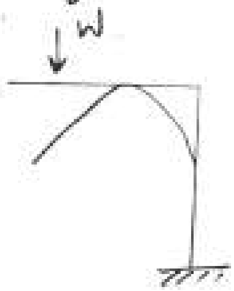
* (frame sway to the left)

(3.)

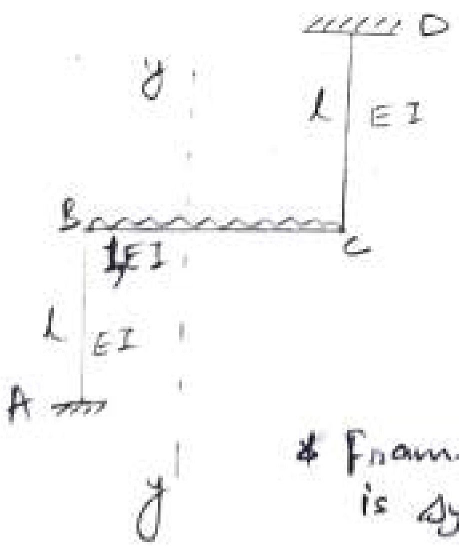


Frame is symetrical but (loading is not symetrical)

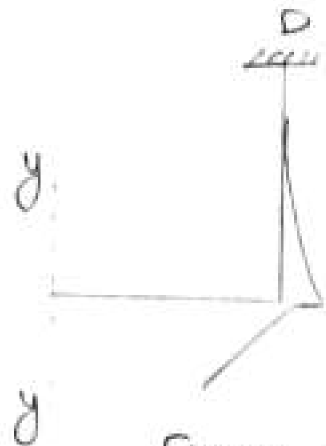
→ Cut the frame at centre. See in which direction the cut frame deflects. In that direction the frame sways.



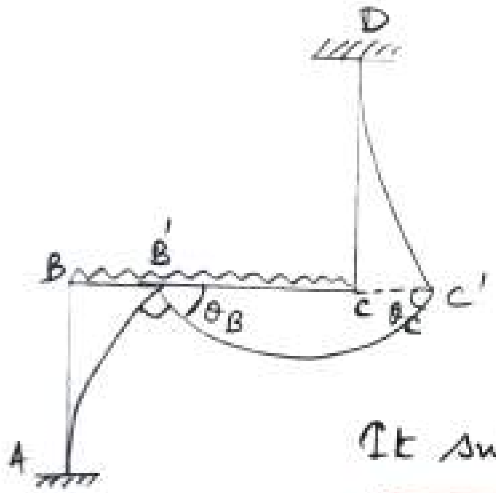
So, frames sways to the left.



* Frame is not symmetric loading is symmetrical.



Frame sway to the right.



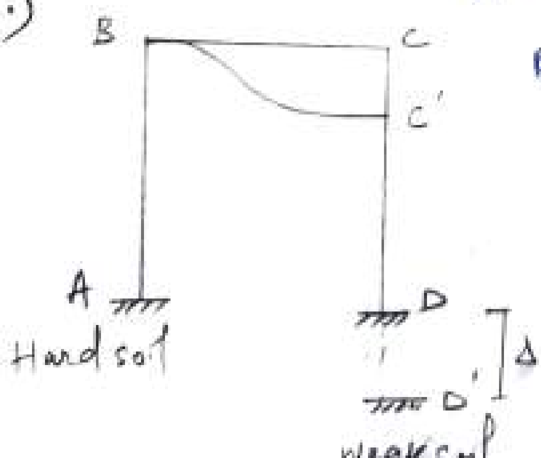
If sway 'Δ' is present.

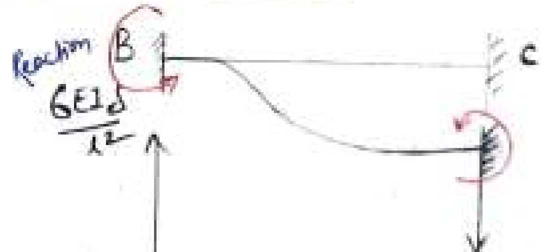
$$\boxed{\theta_B = -\theta_C}$$

Concept 2 :-

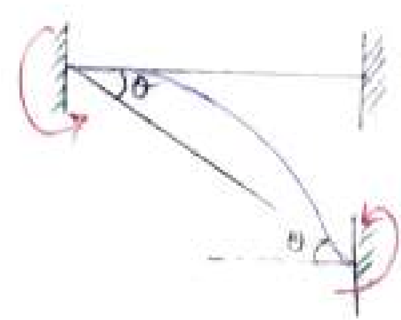
Sinking of supports :-

1.)



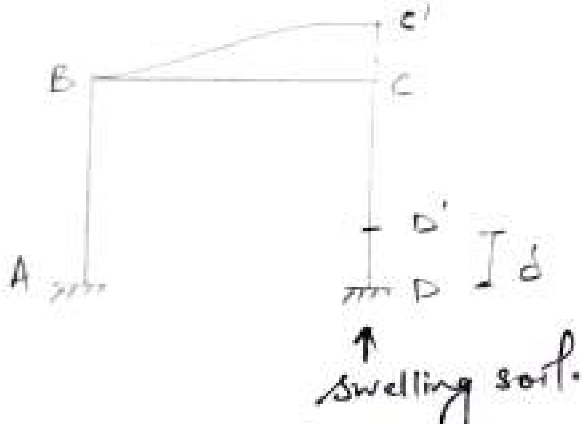


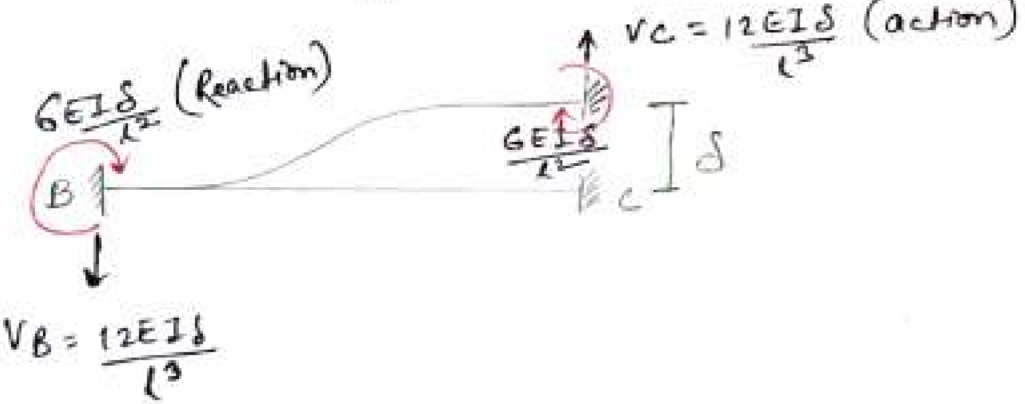
$V_B = \frac{12EI\delta}{l^3}$ $\qquad$ $V_C = \frac{12EI\delta}{l^3}$ (action)



Due to the action of $\frac{12EI\delta}{l^3}$, supports 'c' sinks down by the amount 'δ', due to this action, the reacting moment $\frac{6EI\delta}{l^2}$ are developed at 'B' & 'C' as shown in fig.

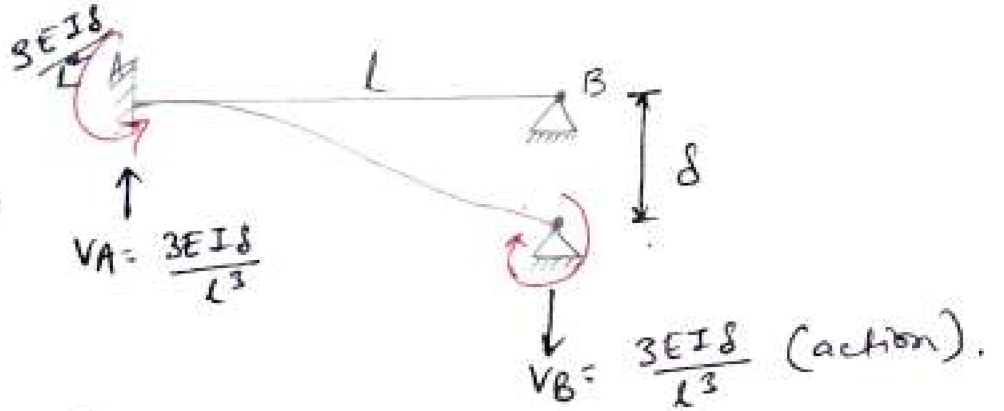
(2)

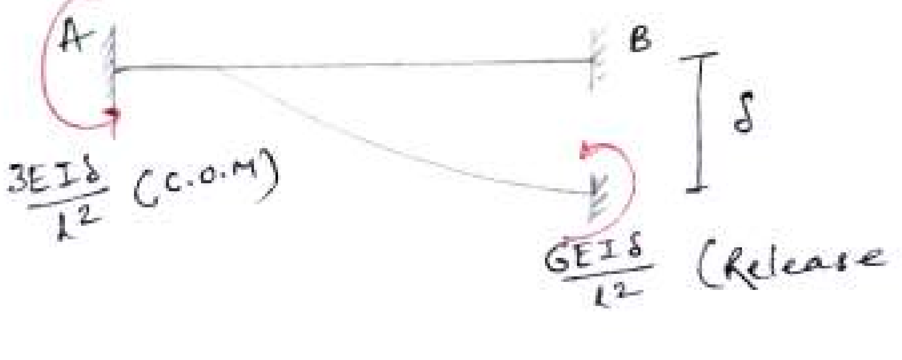




(3.)

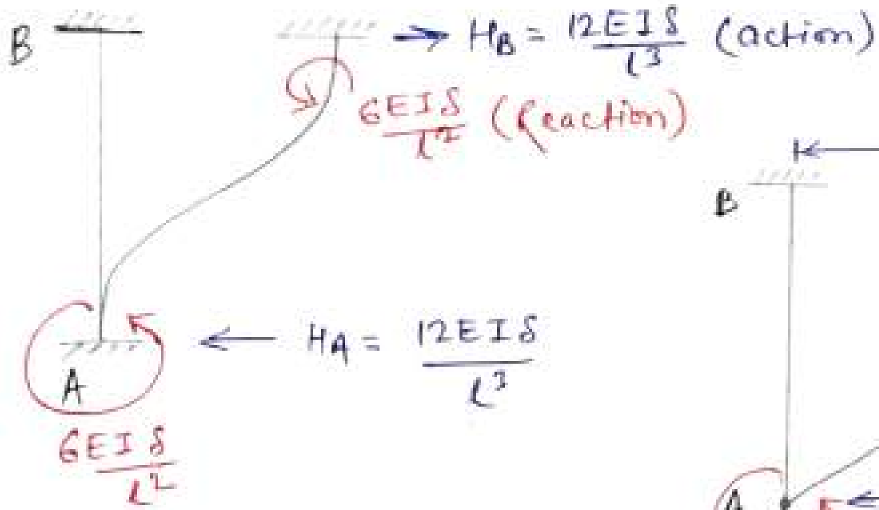
Reaction

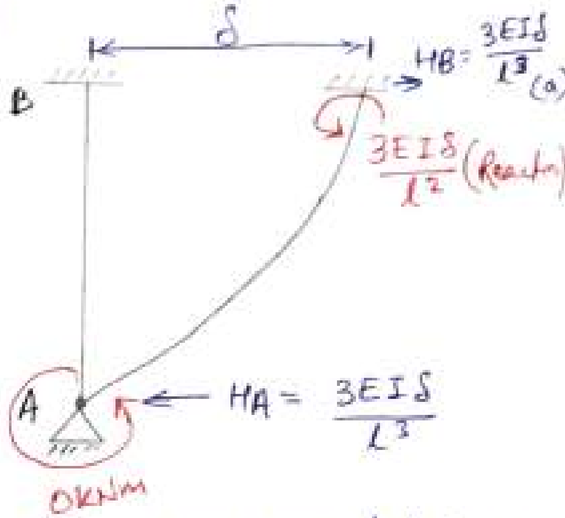




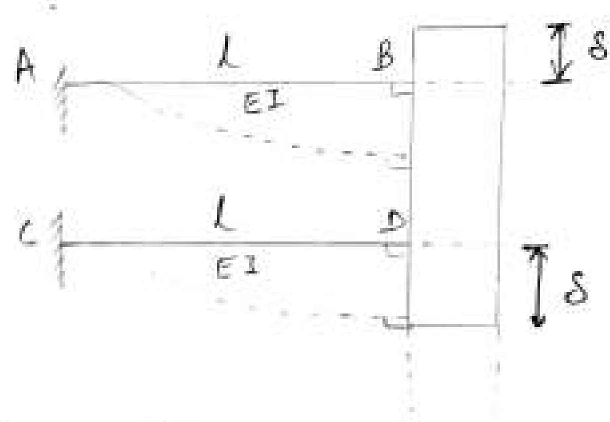
4. If any joint move vertically downwards it is called sinking.

→ If it moves horizontally, it is called sway.

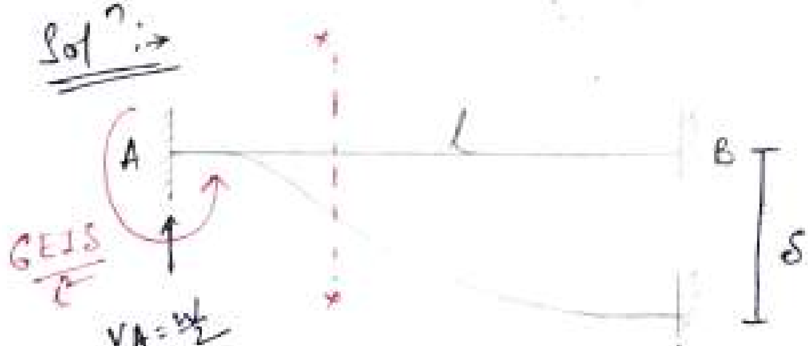




Q. Find vertical deflection 'δ' for the rigid block shown in figure.



Sol^n :→



$$V_B = \frac{w}{2} = \frac{12EI\delta}{l^3}$$

$$\boxed{\delta = \frac{wl^3}{24EI}}$$

Note :-

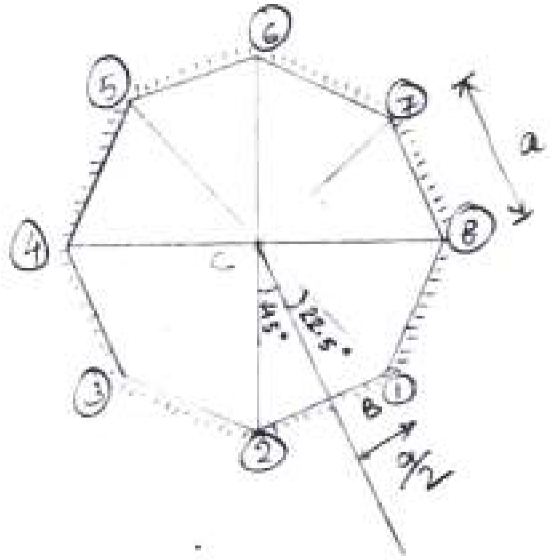
(i) Maximum S.F in the beam = (Stand at xx and look left)

$$= \boxed{+\frac{W}{2}}$$

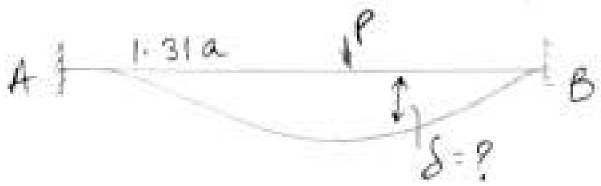
(ii) Maximum B.M in the beam $= \frac{6EI\delta}{l^2} = \frac{6EI \times \left(\frac{Wl^3}{24EI}\right)}{l^2}$

$$= \frac{Wl}{4}$$

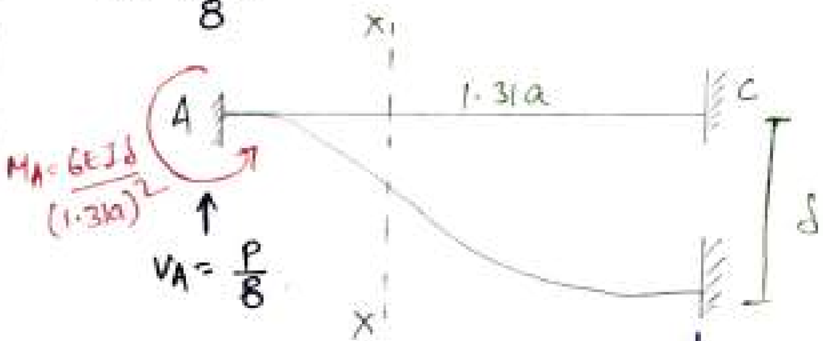
Q. For an octagonal hall of side 'a', four beams of stiffness EI are provided for supporting a fibre glass roof, the beam are fixed in the wall at corners and monolithically connected at centre 'C' as shown in fig. A verticle load 'p' ($\perp^r$ to the plane of the fig) is to be suspended at 'C'. Determine deflection at 'C' and max. B.M and S.F in the beams due to load 'p'. [20 Marks]



Sol^n: $\sin 22.5^\circ = \dfrac{a/2}{l_{BC}} \Rightarrow l_{BC} = \dfrac{a/2}{\sin 22.5^\circ} = 1.31a$.



1) 'P' is shared by 8 beams so, load on ea beam is $\frac{P}{8}$.



$V_C = \frac{P}{8} = \frac{12EI\delta}{L^3} = \frac{12EI\delta}{(1.31a)^3}$

$$\delta = \frac{P \times (1.31a)^3}{96EI} = \boxed{\frac{0.023Pa^3}{EI}}$$

2) Maximum S.F in the beam = stand at x-x and look left

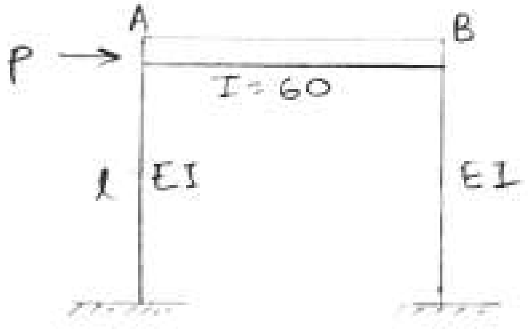
$$\boxed{S.F = \frac{P}{8}\,kN}$$

3) Maximum B.M in the beam $= M_A = \dfrac{6EI\delta}{(1.31a)^2}$

$$= \frac{6EI \times \left(\frac{0.023Pa^3}{EI}\right)}{(1.31a)^2}$$

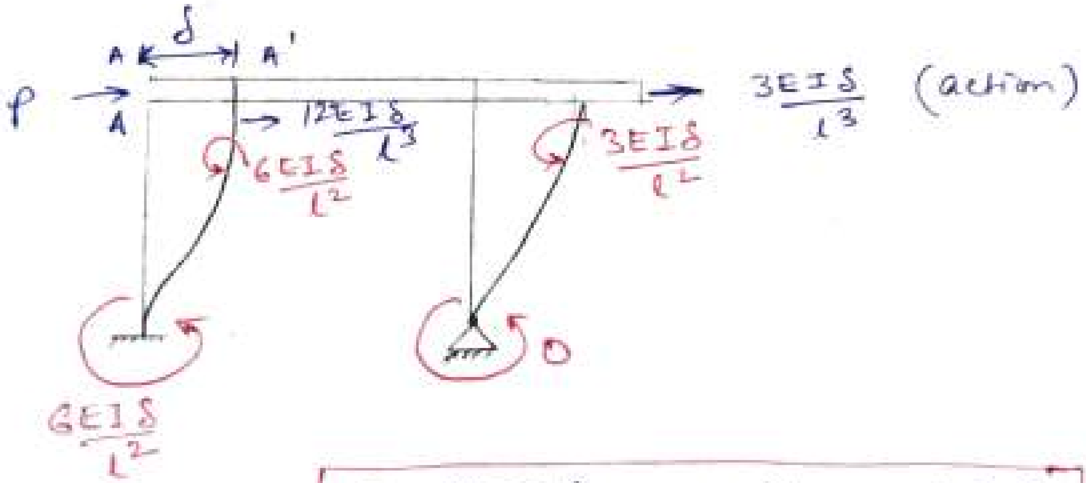
$$\boxed{M_a = 0.082\,Pa\ kN\text{-}m}$$

Q. For the frame shown in fig. force 'p' reqd. to move the girder Beam AB through Horizontal displacement 'δ' is given by? [2 Marks].



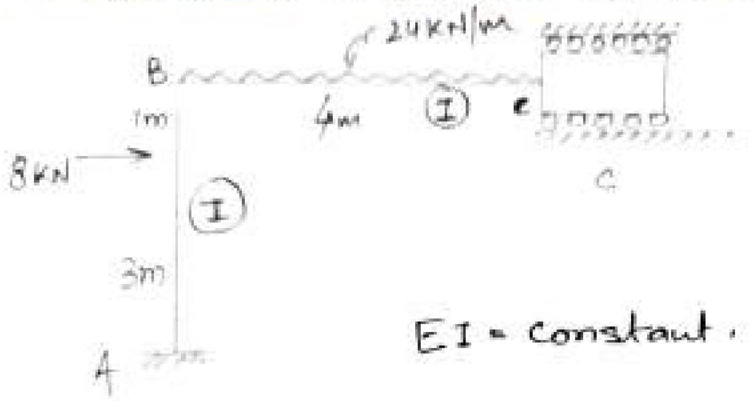
Note:-

- When compared with Icolumn if Ibeam is very large then the beam can be idealised as a beam with $\boxed{I = \infty}$. It means that beam is flexually rigid beam and it will not bend. So, the beam AB remains Horizontal after application of the load also. So, the deflection shape of the frame is as shown in figure.



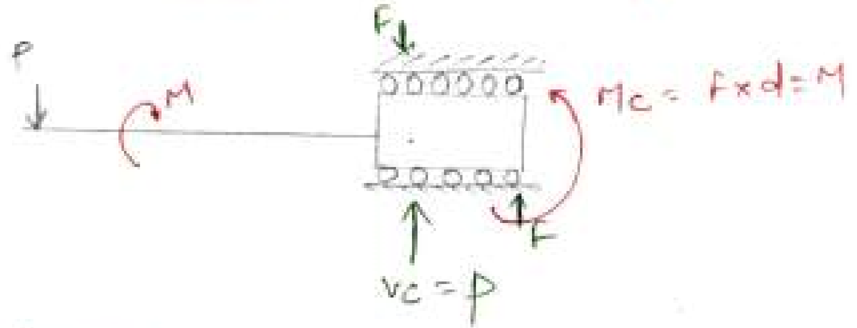
$$\boxed{P = \frac{12EI\delta}{l^3} + \frac{3EI\delta}{l^3} = \frac{15EI\delta}{l^3}}$$

Q. Analyse the frame shown in figure using Moment Distribution MTD, and Draw B.M.D.



EI = constant.

Note(i) F.B.D of guided roller support :-



Conclution :-

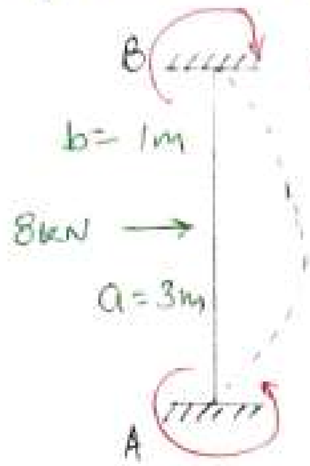
(I) From the above F.B.D, we find that at a guided roller support, Horizontal reaction is not possible but vertical reaction and Moment reaction can be developed. so, it can be idealised as a movable fixed support.

(II) Horizontal displacement at 'C' is possible, the frame sway's right due to Horizontal load.

(III) End Moment are developed due to applied loading and sway of the frame. so, from the principle of super position, add the end moments due applied loading and due to sway to get final, end Moments.

Sol^n:- **Step 1:-**

F.E.M of due to applied loading alone →



$M_{fba} = +\frac{wa^2b}{l^2}$

$M_{fab} = -\frac{wab^2}{l^2}$

$$M_{fab} = -\frac{Wab^2}{l^2} = -\frac{8\times3\times1^2}{4^2} = -1.5\ kN\text{-}m.$$

$$M_{fba} = \frac{wa^2b}{l^2} = \frac{8\times3^2\times1}{4^2} = 4.5\ kN\text{-}m.$$

$$M_{fbc} = -\frac{wl^2}{12} = \frac{-24\times4^2}{12} = -32\ kN\text{-}m$$

$$M_{fcb} = +32\ kNm.$$

Step 2:-

Joint	Member	K	ΣK	D.F = K/ΣK
B	BA	I/4	1/2	0.5
	BC	I/4		0.5

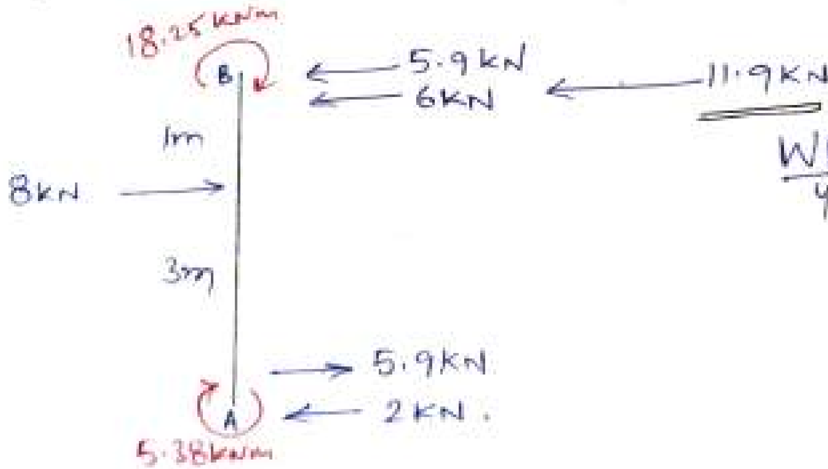
Step III. Non-sway end Moment distribution

i.e., end moment distribution due to applied loading alone.

Joint	A	B		C
D.F	1	0.5	0.5	1
F.E.M	-1.5	4.5	-32	+32
Release or Balance		+13.75	+13.75	
C.O.M	+6.88			+6.88
Final end Moments.	+5.38	+18.25	-18.25	+38.88

Note:- Joint A & C are fixed support so, don't release joint 'A' and 'C' balance means Releasing joint 'B' only -

To find sway force -



$$\frac{Wa}{4} = \frac{8 \times 3}{4} = 6\,kN$$

$$\frac{Wb}{L} = \frac{8 \times 1}{4} = 2\,kN.$$

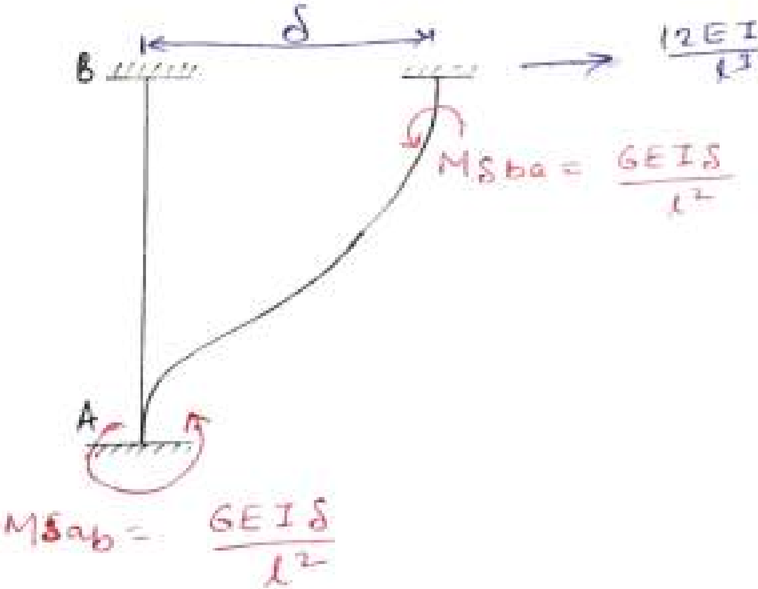
$$\frac{18.25 + 5.38}{4} = 5.9\,kN$$

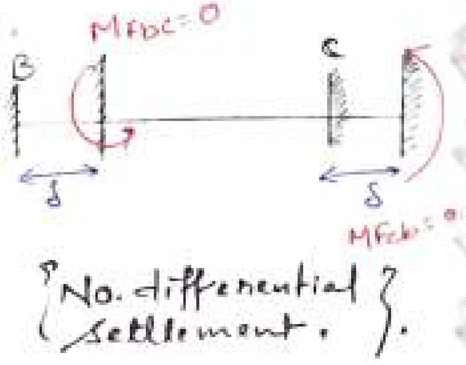
Ex at beam level = $\overleftarrow{11.9\,kN}$ (Reaction)

So, sway force at beam level = ($\overrightarrow{11.9\,kN}$). Due to this sway force the frame sways right.

(iv) Step :- Sway Analysis :-

- Sway moments distribution {end moments developed due to sway alone}.





Note :-

We can not equate $\frac{12EI\delta}{l^3} = 11.59\,km$. because, joint B is not a fixed support and it rotates, the moment $\frac{6EI\delta}{l^2}$ is distributed to two Members at 'B'. so, reaction will not be equal to $\frac{12EI\delta}{l^3}$.

Joint	A	B		C
D.F	1	0.5	0.5	1
F.E.M	-10	-10	0	0
Balance		+5	+5	
C.O.M	+2.5			+2.5
F.E.M	-7.5	-5	+5	+2.5

* -10 is assume.

To find assume Sway force

→ 3.125 kN (action)

← $\frac{-7.5-5}{4} = -3.125$ kN →

So, sway force at beam level = 3.125 kN. Due to this sway force (action), 5 kNm, 7.5 kn.m etc are developed.

But action sway force is 11.9 kN. So, from linear interpolation corrected sway moments are obtained.

So, Correction factor = $\frac{\overrightarrow{11.91 \text{ kN}}}{3.125 \text{ kN}} = +3.81$ $\left(\frac{\text{Actual}}{\text{assume}}\right)$

Same direction +ve

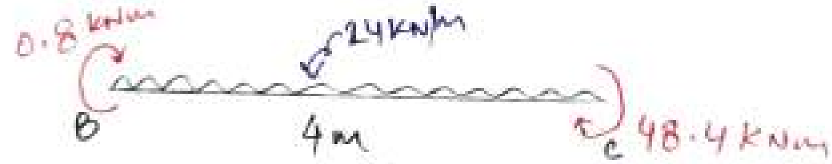
Opposite direction -ve

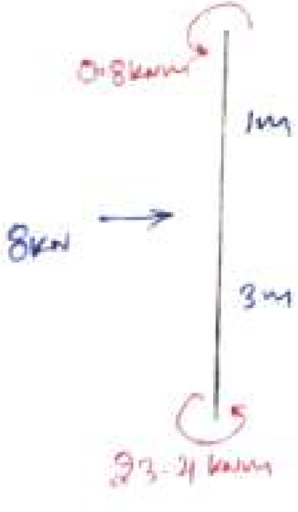
Note:- If both force are in same direction, correction factor is taken as '+ve'. If any are in opposite direction the correction factor is taken as (-ve).

(V) Final end Moments {from principle of superposition}

Joint	A		B	C
No sway end Moments	+5.38	+18.25	-18.25	+38.88
Corrected Sway	-7.5×3.81	-5×3.81		
Moments	= -28.58	-19.05	+19.05	9.82
Final end Moments	-23.21	-0.8	+0.8	48.4

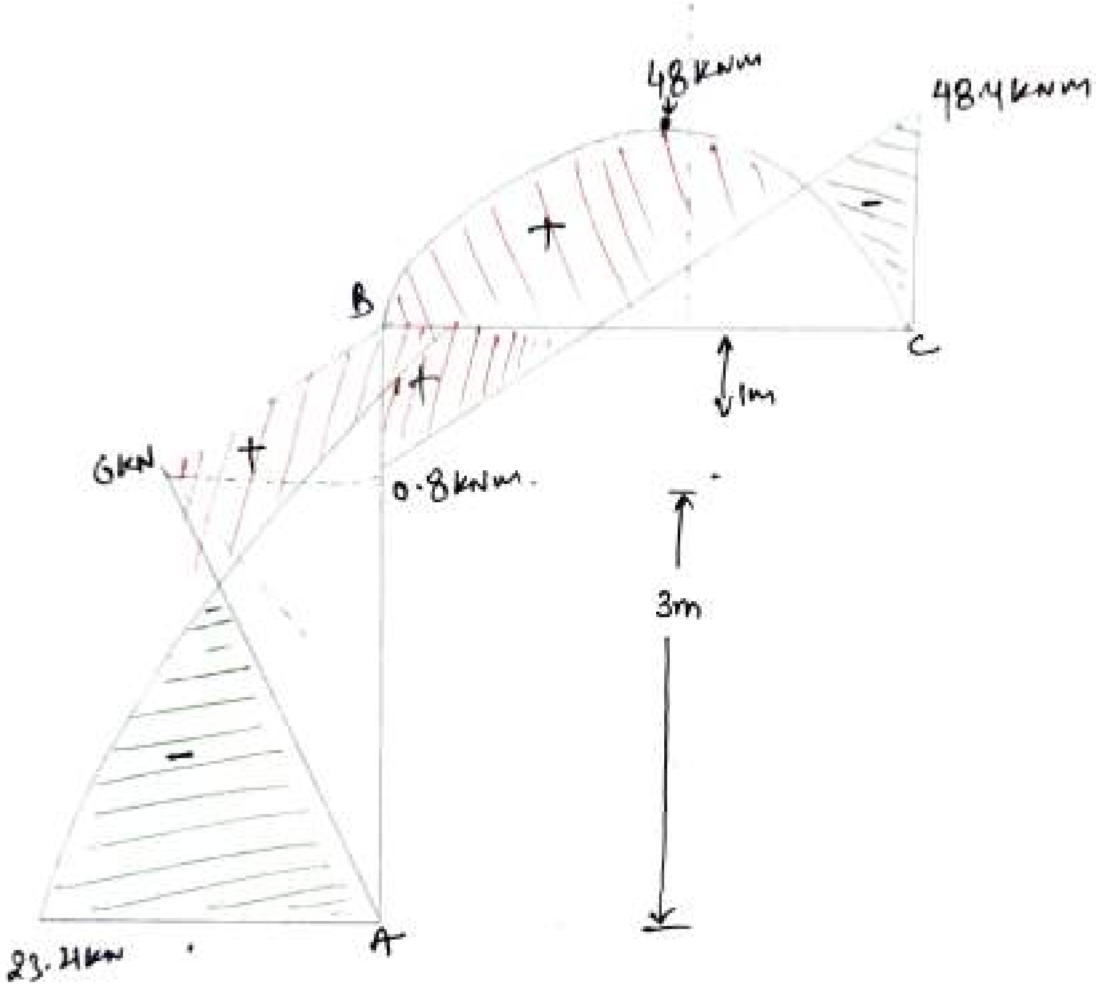
vi) Step:- SFD and BMD {Support reaction}.





$$\frac{Wab}{l} = \frac{8\times1\times3}{4} = 6\ kNm.$$

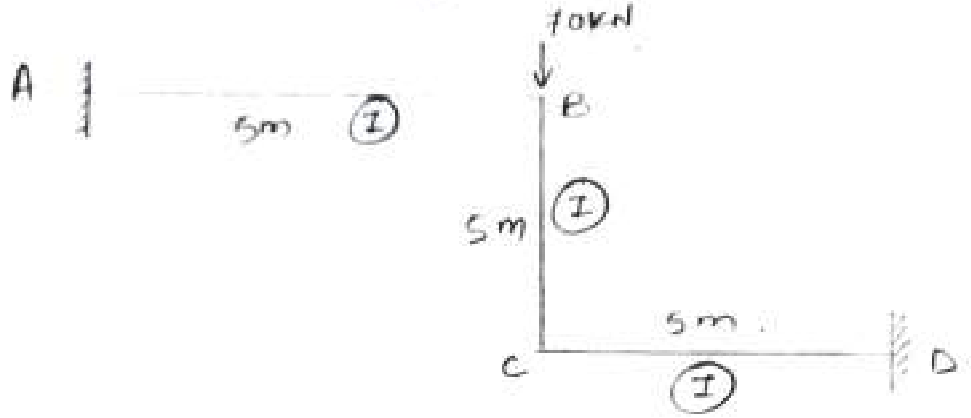
$$\frac{wl^2}{8} = \frac{24\times4^2}{8} = 48\ kNm.$$



48KNM
484KNM
+
-
B
C
+
1m
+
6KN
0.8KNM.
3m
-
23.24KN
A

{Pb based on pure sway}

Q. Analyse the frame shown in fig using M.D. MTD and draw B.M.D?



Sol^n:-

Step I. F.E.M due to applied load on the span.

$M_{Fab} = M_{Fba} = M_{Fbc} = M_{FcB} = M_{Fcd} = M_{Fdc} = 0$

{ ∵ 0 because there are no applied loads on span }.

Step II. D.F.S -

Joint	Member	K	ΣK	DF = $\frac{K}{\Sigma K}$
B	BA	½	1	0.5
	BC	½		0.5

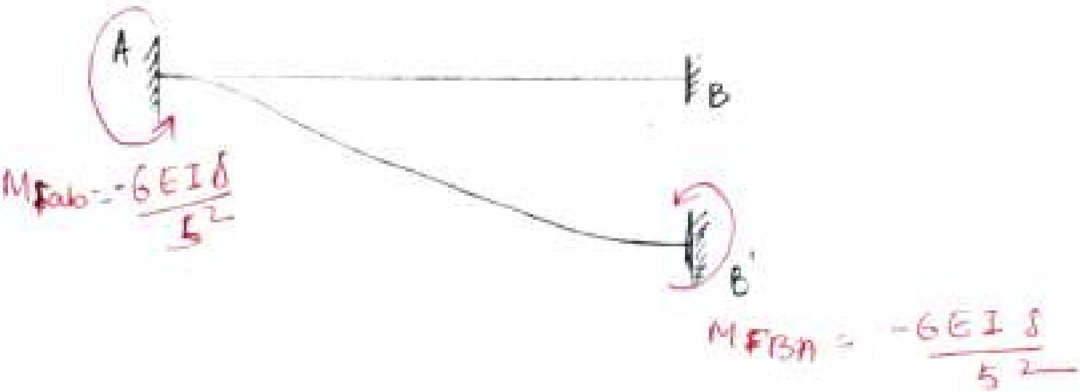
A the joint B and C.

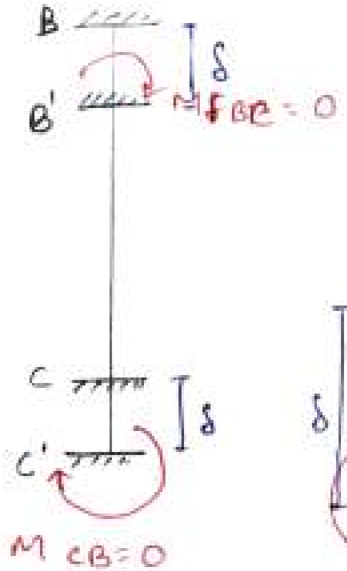
D.F = 0.5 for all members.

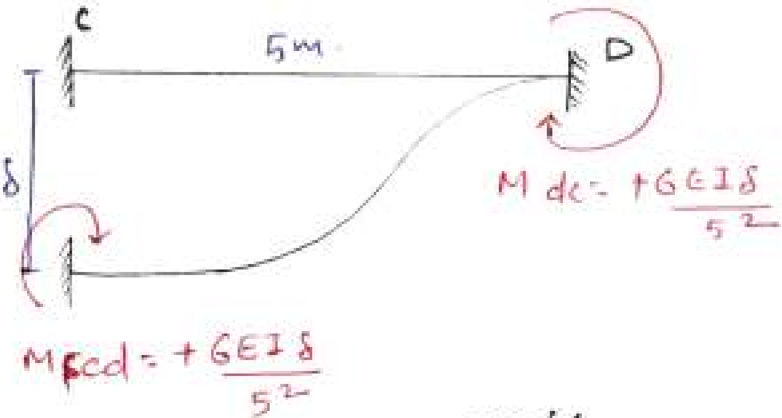
Step III. Non-sway Moment Distribution :-

- Since, there are no f.e.m.s due to applied loads, no non-sway moments are developed {that is only why it is called pure sway alone}.

Due to 70KNm force frame sinks down.







Ratio of sway moments $= \frac{M_{FBA}}{M_{Fcd}} = \frac{-6EI\delta/5^2}{+6EI\delta/5^2} = -1$

Assume $M_{\delta BA} = -10KNm$ (arbitrary value)

$$M_{\delta cd} = \frac{M_{\delta BA}}{-1} = \frac{-10}{-1} = 10KNm.$$

So, assumed sway

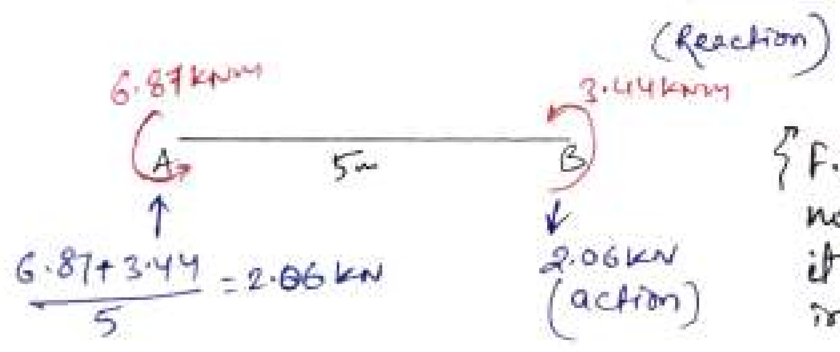
$$M_{\delta ab} = M_{\delta ba} = -10KNm.$$

$$M_{\delta bc} = M_{\delta cb} = 0$$

$$M_{\delta cd} = M_{\delta dc} = 10KN\cdot m.$$

Joint	A	B		C		D
D.F	$\boxed{1}$	$\boxed{0.5}$	$\boxed{0.5}$	$\boxed{0.5}$	$\boxed{0.5}$	$\boxed{1}$
F.E.Ms	-10	-10	0	0	+10	+10
Balance		+5	+5	-5	-5	
C.O.M	+2.5		-2.5	+2.5		-2.5
Balance		+1.25	+1.25	-1.25	-1.25	
C.O.M	+0.63		-0.63	-0.63		-0.63
Balance		+0.31	+0.31	-0.31	-0.31	
Final End Moments.	-6.87	-3.44	+3.44	-3.44	+3.44	+6.87

To find sway force :-



{F.B.D of column is not neccesary. Bcz, it will not contributing in getting sway force}.

So, sway force = 2.06 kN ↓ + 2.06 kN ↓ = 4.12 kN (↓)

Due to this sway force, sway moments 6.87 kNm, 3.44 kNm etc are developed.

Actual sway force = 70 kN (↓)

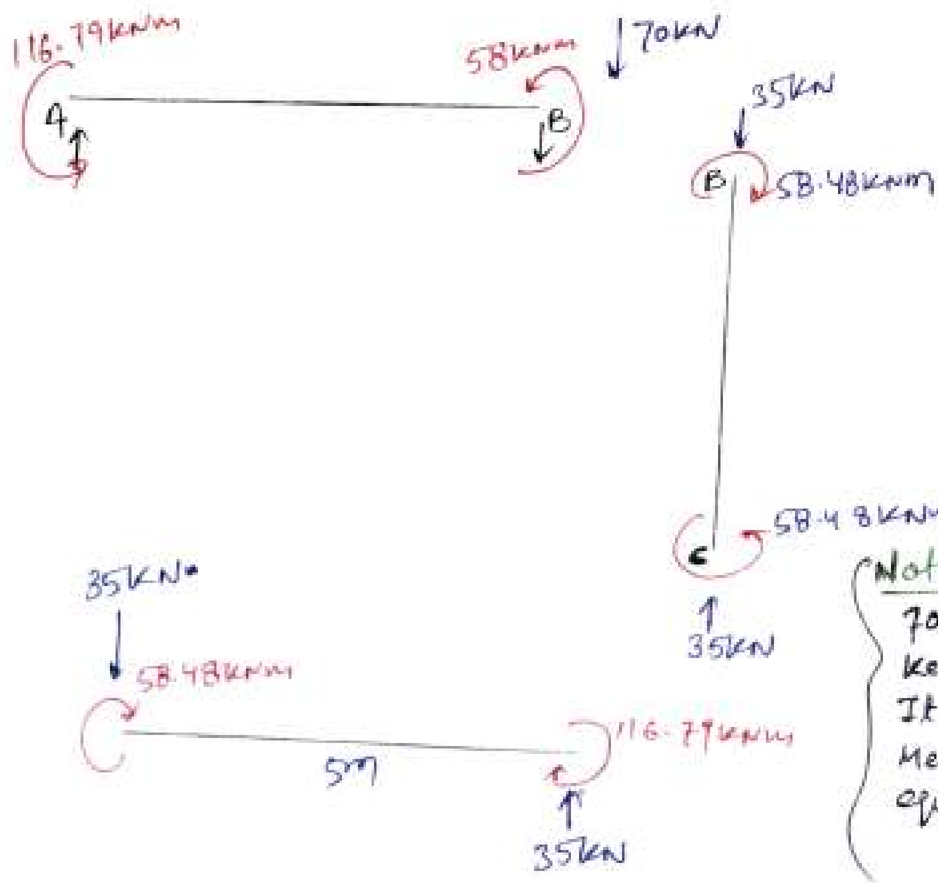
So, correction factor = $\frac{70\text{kN} (\downarrow)}{4.12\text{kN} (\downarrow)} = 17.$

Step v.) Final end Moments :—

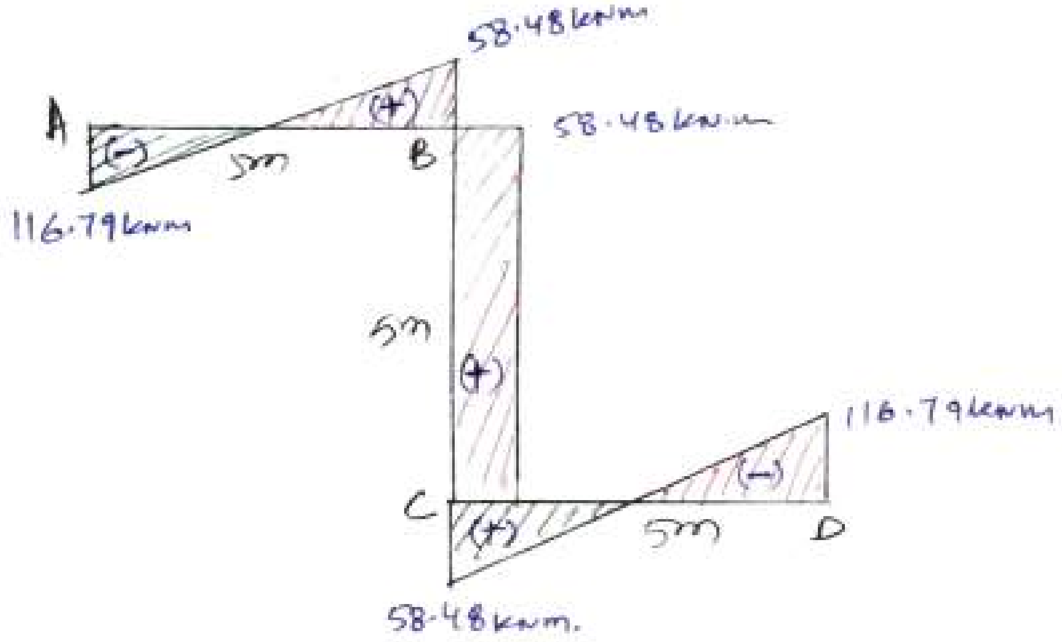
Joint	A	B		C		D
Non sway end Moments.	0	0	0	0	0	0
Corrected sway Moments.	−6.87×17 = −116.79	−3.44×17 = −58.48	+58.48	−58.48	+58.48	6.8×17 +116.79
Final end Moments	−116.79	−58.48	+58.48	−58.48	+58.48	+116.79

VI) Step BMD :-

$\frac{116.79 + 58}{5} = 35\text{kN}$

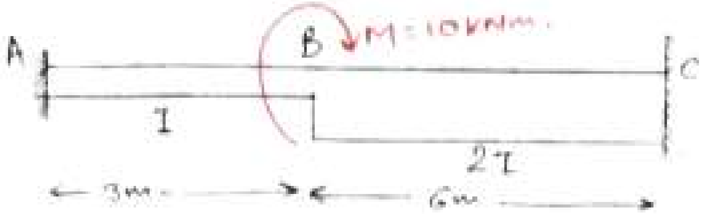


Note :-
70kN shouldnot be kept on any f.b.d. It is shared by 2 Member BA and BC equally as shown in fig.



B.M.D

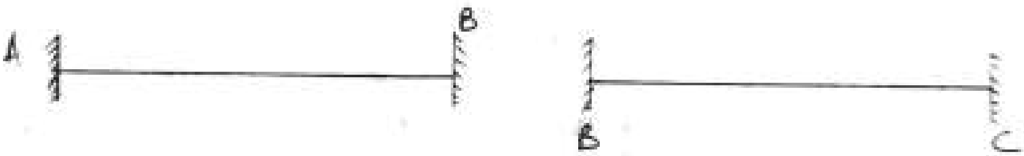
Q A fixed beam is subjected to a clockwise Moment as shown in figure. Using M.D.M, find fixed end Moments developed at the support. [10 Marks]



Note:-

Since, there is not supports at B, joint B deflects. So, find end moments due applied loading and due to sinking from the principle of superposition, add end Moments due loads and due to sinking to get final end moments.

Step 1:- F.E.M.s (due to applied loads on the spans).



$M_{fab} = M_{fba} = M_{fbc} = M_{fcb} = 0$ {∵ because there are no loads on spans}.

Step 2:- D.Fs :-

Joint	Member	k	Σk	$DF = \frac{K}{\Sigma K}$
B	BA	$I/3$	$\frac{2I}{3}$	0.5
	BC	$\frac{2I}{6} = \frac{I}{3}$		0.5

Step 3. Non sway end Moments distribution :→

Joint	A	B		C
D.F	1	0.5	0.5	1
Balance		+5	+5	
C.O.M	+2.5			+2.5
final end moments	+2.5	+5	+5	+2.5

Note. Applied 10 kNm is distributed in the ratio of Distribution factor.

To find sway force :→

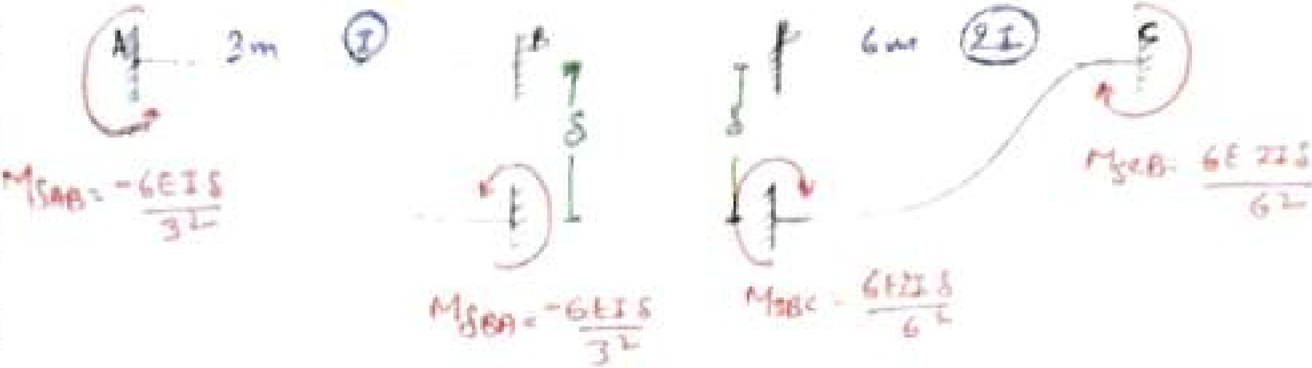
$\frac{2.5+5}{3} = \frac{7.5}{3} = 2.5$ kN (↓)	2.5 kN (↑)	$\frac{2.5+5}{6} =$ 1.25 kN (↓)	$\frac{7.5}{6} = 1.25$ kN (↑)

$V_B = 1.25$ kN (↑) {Reactions}.

So, sway force at 'B' = 1.25 kN (↓). Due to this force joint 'B' sinks downwards.

Step 4: Sway Moments Distribution :-

Joint 'B' deflects downwards.



Ratio of Sway Moments.

$$\frac{M_{sBA}}{M_{sBC}} = \frac{\frac{-6EI\delta}{3^2}}{\frac{6E2I\delta}{6^2}} = \frac{-\frac{1}{9}}{\frac{1}{18}} = -2$$

Assume, $\boxed{M_{\delta BA} = -20\ \text{kNm}}$.

$$M_{\delta BC} = \frac{M_{sBA}}{-2} = -\frac{20}{-2} = +10\ \text{kNm}.$$

So, assumed

Sway moments are $M_{\delta AB} = M_{\delta BA} = -20\ \text{kNm}$.

$M_{\delta BC} = M_{\delta CB} = +10\ \text{kNm}$.

Joint	A	B		C
DF	1	0.5	0.5	1
F.E.M	-20	-20	+10	+10
Balance		+5	+5	
COM	+2.5			+2.5
Final end moment	-17.5	-15	+15	+12.5

To find sway force :→

$\frac{17.5+15}{3} = 10.83\uparrow$	$10.83(\downarrow)$ $VB = 15.41(\downarrow)$	$4.58\,kN(\downarrow)$	$\frac{15+12.5}{6} = 4.58\,kN(\uparrow)$

So, sway force is 15.41 kN (↓). Due to this downwards sway force, 17.5 kNm, 15 kNm etc are developed.

But actual sway force = 1.25 kN (↓)

$$\text{So, Correction factor} = \frac{1.25\,kN(\downarrow)}{15.41\,kN(\downarrow)} = 0.081$$

Step V Final end Moments :→

Joint	A	B	B	C
Non sway end Moments	+2.5	+5	+5	+2.5
Corrected sway Moments	$-17.5 \times 0.081 = -1.42$			
Final end Moments	+1.08	+3.78	6.22	3.52

So, fixed Moments are :-

$M_{FAC} = +1.08\,kNm.$

$M_{FCA} = 3.52\,kNm.$

Check :→ $M_{BA} + M_{AC} = 10\,kNm.$

$3.78 + 6.22 = 10\,kNm$

So results are Corrected.

Q. Find fixed end Moments at 'A' and 'C' and deflection at centre for the beam shown in fig.

[20 Marks].

Sol^n :→ Step 1. Non sway end Moment distribution :→

Joint	A	B		C
DF	1	0.5	0.5	1
Balance		$\frac{M}{2}$	$\frac{M}{2}$	
COM	$+\frac{M}{4}$			$+\frac{M}{4}$
Final end Moments	$+\frac{M}{4}$	$\frac{M}{2}$	$\frac{M}{2}$	$\frac{M}{4}$

To find sway force :→

$\frac{0.75M}{L/2} = \frac{1.5M}{L}$ (↓) | $\frac{1.5M}{L}$(↑) $\frac{1.5M}{L}$(↓) | $\frac{1.5M}{L}$(↑)

$V_B = 0$

So, sway force = 0, It means joint 'B' will not deflect.

So, No sway Moments distribution

final end Moments are

$M_{fac} = +\frac{M}{4}$ kNm.

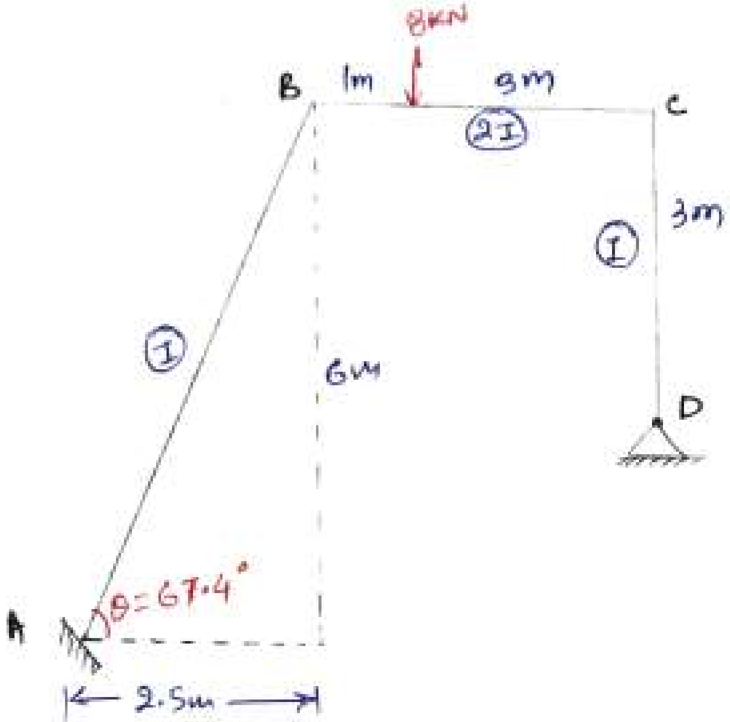
$M_{fca} = \frac{M}{4}$ kNm

$\delta_B = 0$

* Portal Frame with Inclined Member :→

Q. Analyse the frame using MDM and Draw B.M.D

[30 Marks].



Solⁿ :

$L_{AB} = \sqrt{6^2 + 2.5^2} = 6.5m$ {Pythagorous theorem}

$\tan\theta = \frac{6}{2.5} \Rightarrow \theta = 67.4°$

⇒ Non sway Analysis.

Step 1. F.E.M.s

$M_{Fab} = M_{Fba} = M_{Fcd} = M_{Fdc} = 0$

$M_{Fbc} = -\frac{8 \times 1 \times 3^2}{4^2} = -4.5 kNm.$

$M_{Fcb} = +1.5 kNm = \frac{8 \times 3 \times 1}{4^2}$

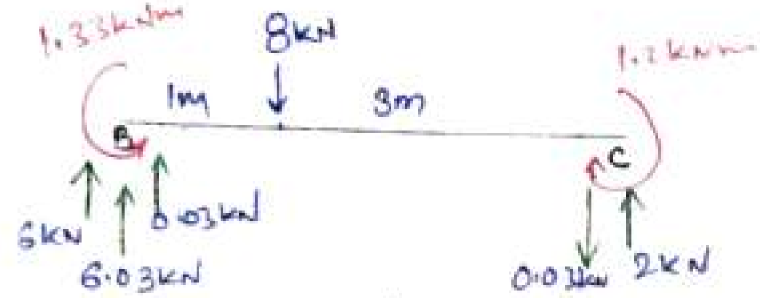
Step II · D.F.S

Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	1/6.5	0.651	0.23
	BC	2I/4		0.77
C	CB	2I/4	$\frac{3I}{4}$	0.67
	CD	$\frac{3}{4}\cdot\frac{1}{3}=\frac{1}{4}$		0.33

Step III :- Non sway end Moments distribution
{Due to applied load only}.

Joint	A	B	B	C	C	D
	1	0.23	0.77	0.67	0.33	1
F.E.M	0	0	-4.5	+1.5	0	0
Balance		+1.1	+3.4	-1.0	-0.5	
COM	0.55		-0.5	+1.7		0
Balance		+0.115	+0.385	-1.139	-0.561	
COM	0.0575		-0.5695	+0.1925		0
Balance		+0.13	+0.44	-0.128	-0.063	
F.E.M	+0.65	+1.33	-1.33	+1.12	1.12	0

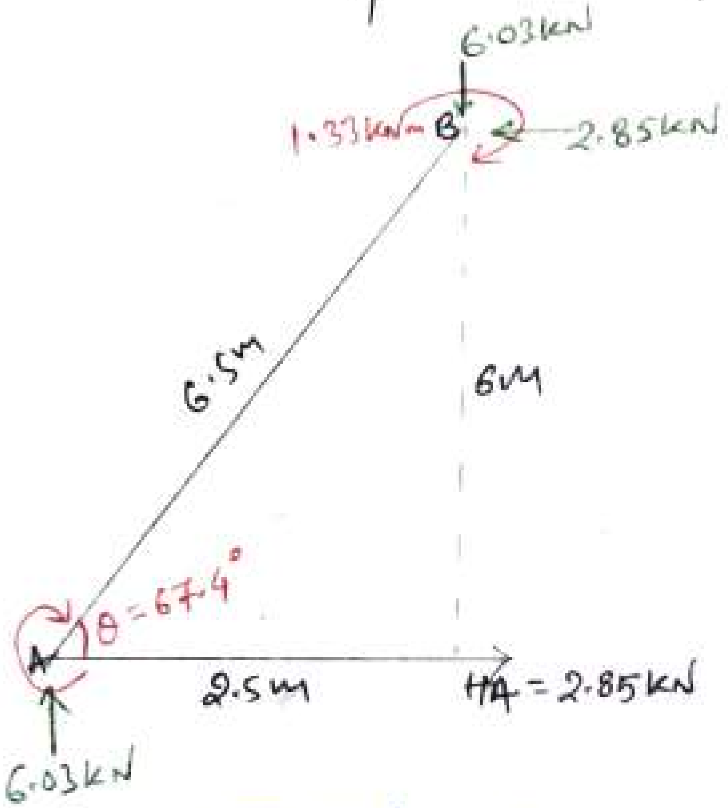
Note :- To find sway force for portal frames with inclined members, we have to draw F.B.D of all members.

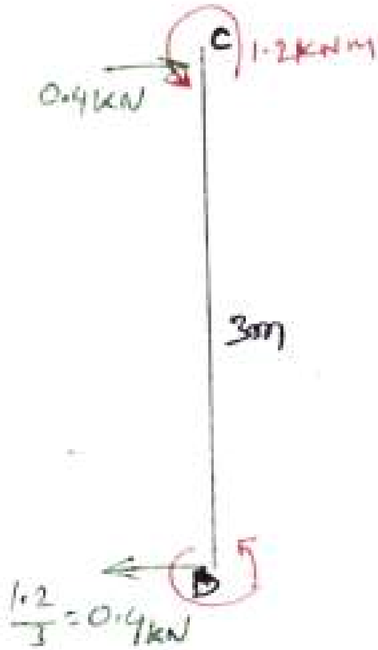


$$\frac{8\times 3}{4} = 6\,kN$$

$$= \frac{8\times 1}{4} = 2\,kN.$$

$$\frac{-1.33+1.12}{4} = -\frac{0.11}{4} = 0.03\,kN$$





To find H_B :→

$$\Sigma M_A = 0 \Rightarrow -H_B\times 6 + (-1.33) - (0.603)\times 2.5 - 0.65$$

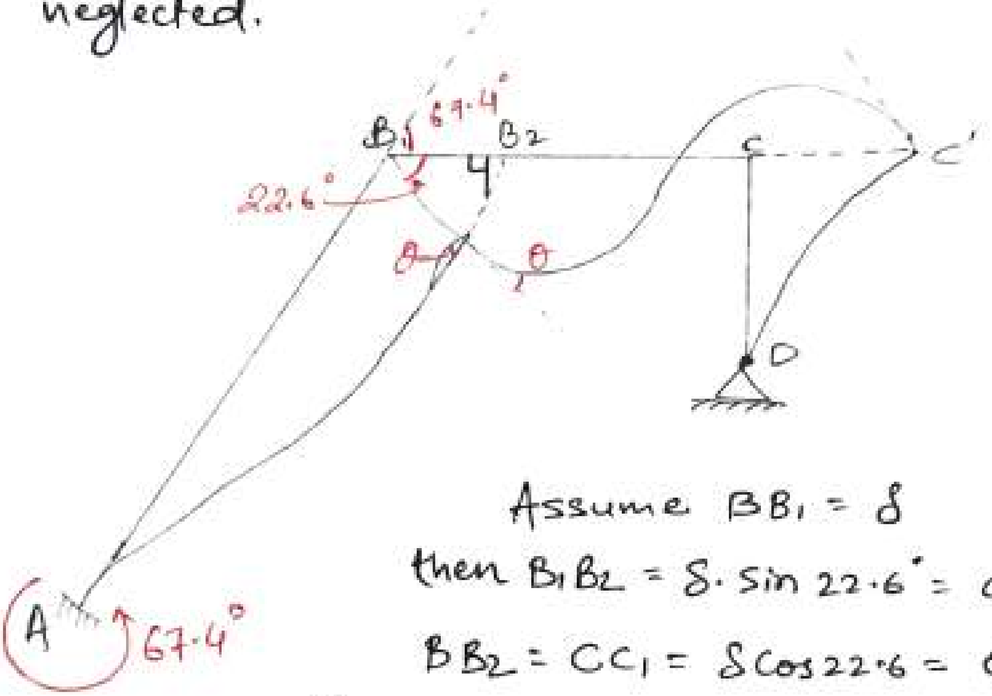
$$= 2.854\ kN\ (\rightarrow)$$

Ex. at beam level $= \overleftarrow{2.85} + \overrightarrow{0.4} = \overleftarrow{2.45}\ kN$ $\Big\{$ Net reacti[on] at beam lev[el]

So, Sway force at beam level = $\overrightarrow{2.45}$ kN. Due to this force, frame sways to the right.

Step iv :- Sway Moment distribution {End Moment due to sway alone}.

Note :- Since, Sway displacement are small, it is assumed that the displacement occur ~~on~~ perpendicular to the initial configuration of the members. and all ~~[illegible]~~ axial deformation are neglected.



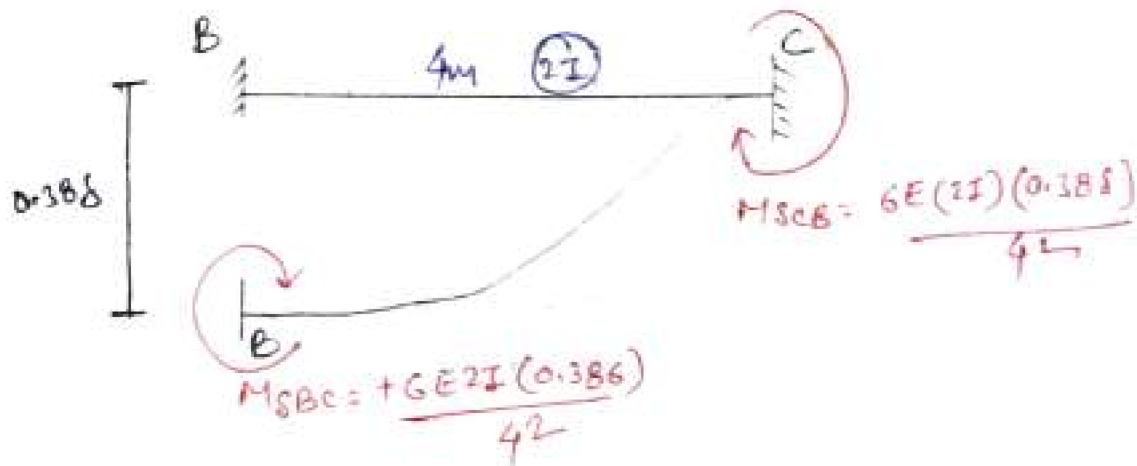
Assume $BB_1 = \delta$

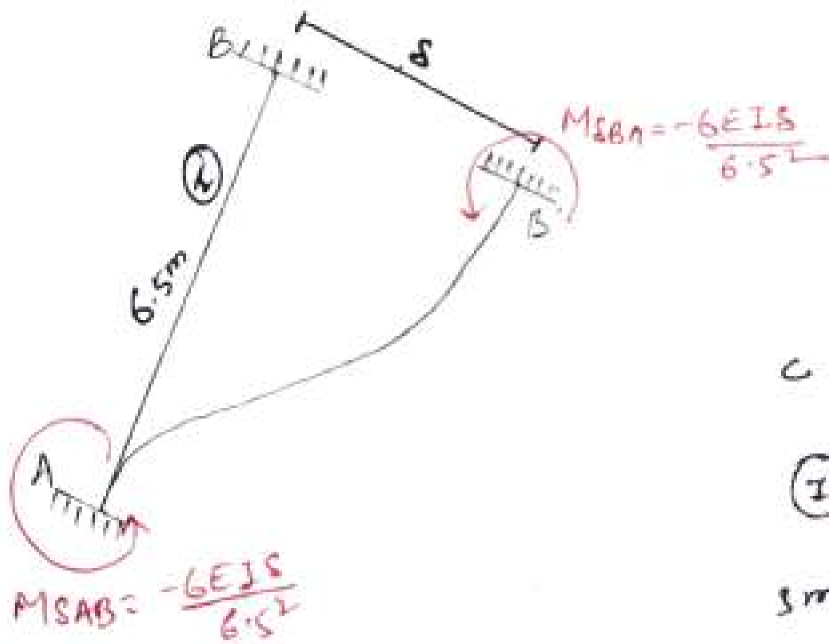
then $B_1B_2 = \delta \cdot \sin 22.6° = 0.38\delta$

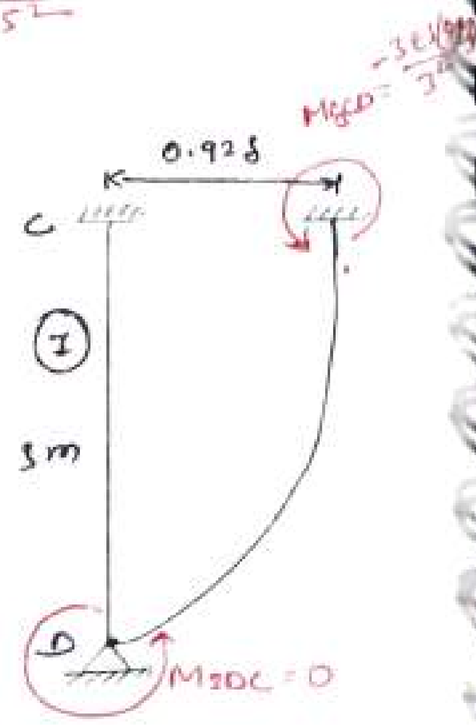
$BB_2 = CC_1 = \delta \cos 22.6 = 0.92\delta$.

{ $BB_2 = CC_1$ because axial deformation are neglected}.

→







Ratio of sway moments = MSBA = MSBC : MSCD.

$$= \frac{-6EI\delta}{6.5^2} : +\frac{6EI(2I)(0.38\delta)}{4^2} : \frac{-3EI(0.94)\delta}{3^2}$$

$= -0.14 : 0.285 : -0.307 \} \times 100$

$= -14 : 285 : 307 \} \div$ by 3

$= -4.67 : 9.5 : -10.2$

So, assumed sway Moments are :->

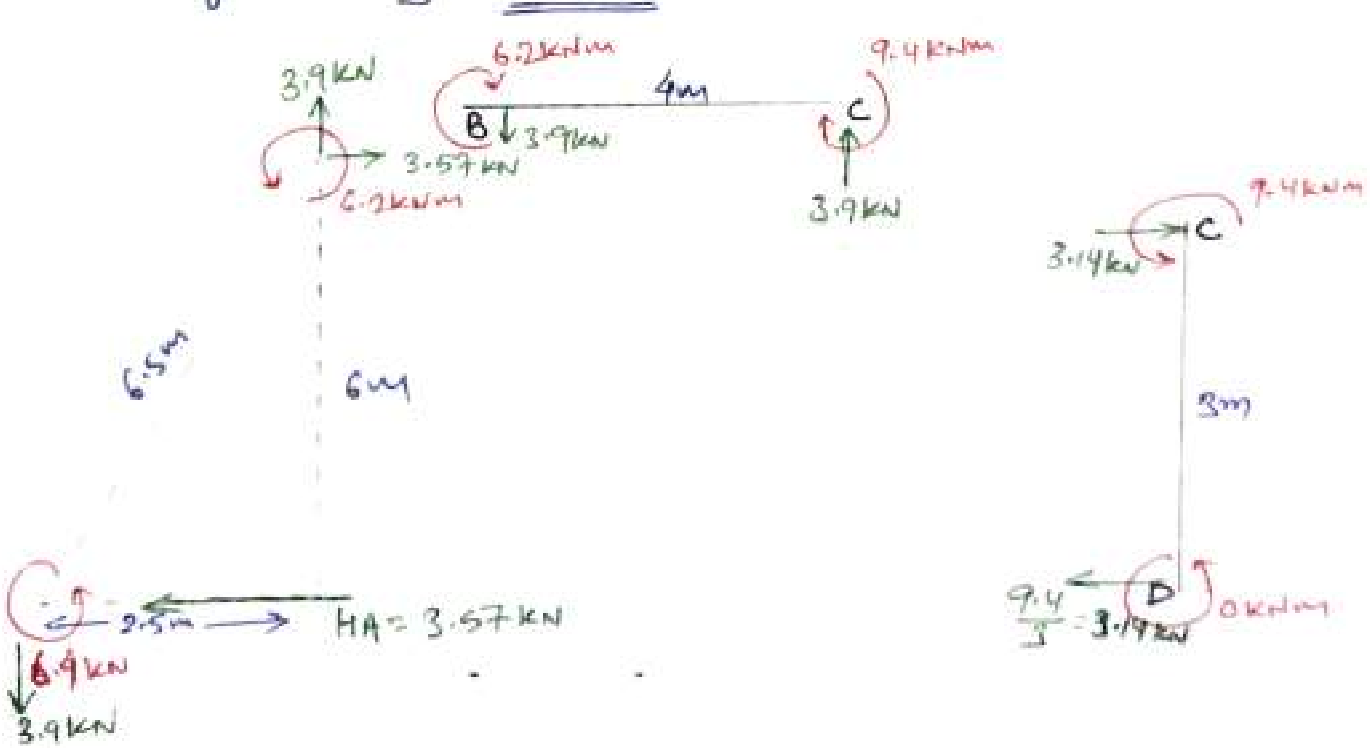
$M_{sAB} = M_{sBA} = -4.67$ kNm.

$M_{sBC} = M_{sCB} = 9.5$ kNm.

$M_{sCD} = -10.2$ kNm, $M_{sDC} = 0$.

Joint	A	B		C		D
D.F	1	0.23	0.77	0.67	0.33	1
F.E.M	-4.67	-4.67	+9.5	+9.5	-10.2	0
Balance		-1.35	-3.48	+0.47	+0.23	
COM	-0.675		+0.235	-1.74		0
Balance		-0.658	-0.181	+1.166	+0.57	
COM	-0.032	-	0.583	-0.091		
Balance		-0.763	-0.44	+0.061	+0.03	
Final end Moment	-5.4	6.2	+6.2	+9.4	-9.4	0

To find sway Force :->



To find H_A :→

$\Sigma M_B = 0 \Rightarrow \underbrace{-3.9 \times 2.5}_{\text{at A}} - \underbrace{5.4}_{\text{at A}} + H_A \times 6 - \underbrace{6.2}_{\text{at B}} = 0$

+ve -ve

$H_A = 3.57$ KN

Σx. at beam level $= \overrightarrow{3.57 \text{ KN}} + \overrightarrow{3.14 \text{ KN}}$

$= \overrightarrow{6.71 \text{ KN}}$

Due to this sway force. $\overrightarrow{6.71}$ KN,

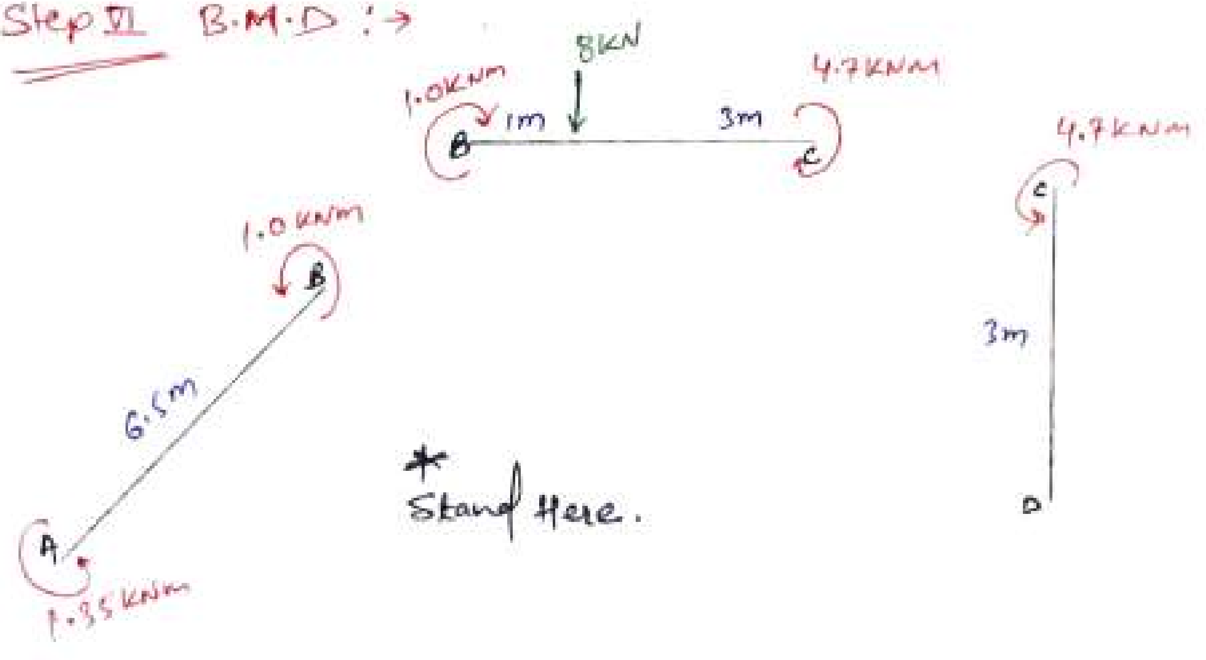
5.4KNm, 6.2 KN-m etc are developed.

So, Correction factor $= \dfrac{\overrightarrow{2.45 \text{ KN}}}{\overrightarrow{6.7 \text{ KN}}} = 0.37$ $\dfrac{\text{Actual}}{\text{assume}}$

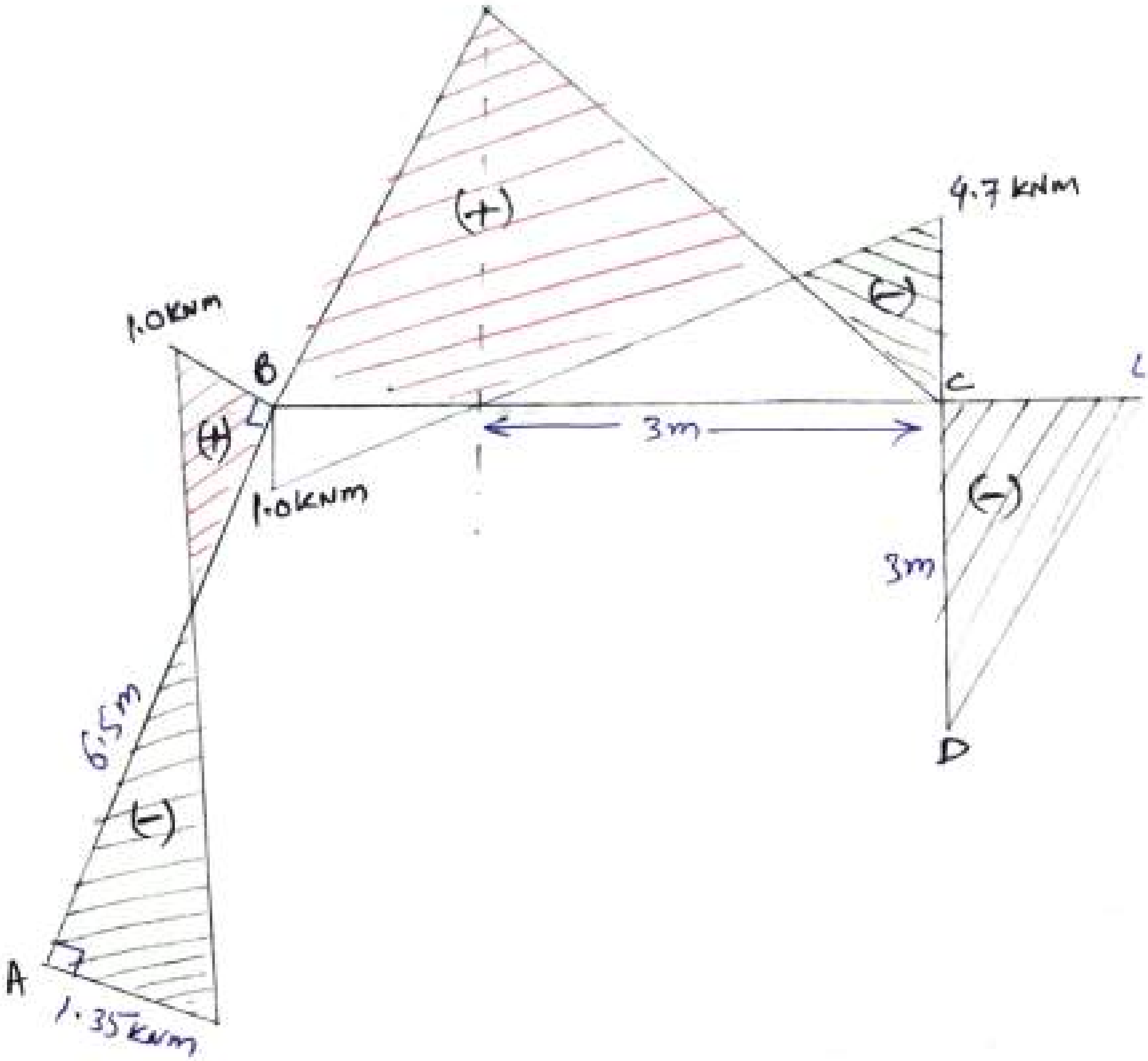
Step V :- Final end Moments {from principal of superposition}

Joint	A	B		C		D
Non sway Moments	+0.65	+1.33	-1.33	+1.2	-1.2	0
Corrected sway Moments	-5.4×0.37 = -2.0	-2.3	+2.3	+3.5	-3.5	0
final end Moment	= -1.35	-1.0	+1.0	+4.7	-4.7	0

Step VI B.M.D :→

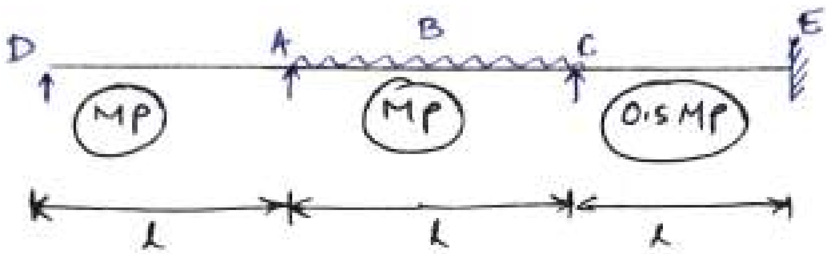


* Stand Here.



4.7 KNm
(+)
1.0KNm
B
(+)
(-)
C
L
3m
1.0KNm
(-)
3m
D
6.5m
(-)
A
1.35KNm

Q. A continous beam with plastic moment capacities is shown in fig. the correct sequence in which plastic hings will be formed in the beam is.



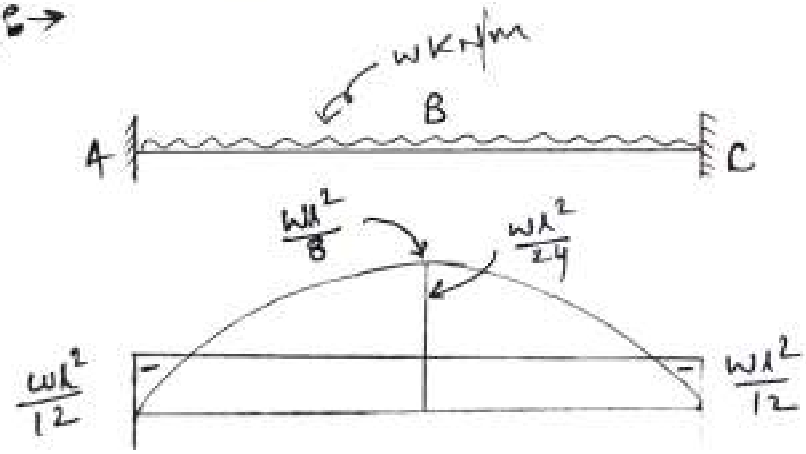
(a) C, A, B

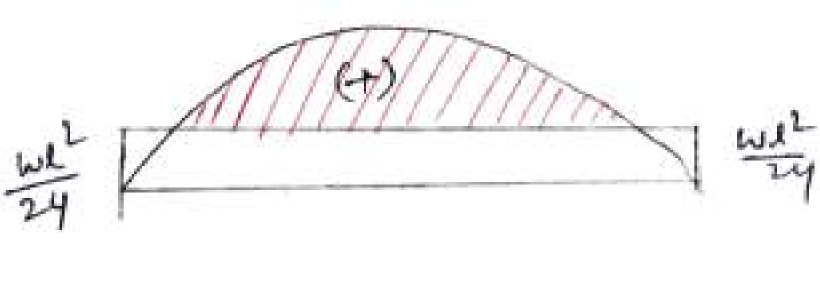
(b) A, C simultaneously, followed by B.

(c) B, C, A

(D) B, first, then A and B simultaneous.

explanation :→





- A, C simultaneous followed by (B) { If A & C are fixed support }.

- After moment distribution

  First hinge at 'B' 2nd hinge at 'C' third hinge at A.

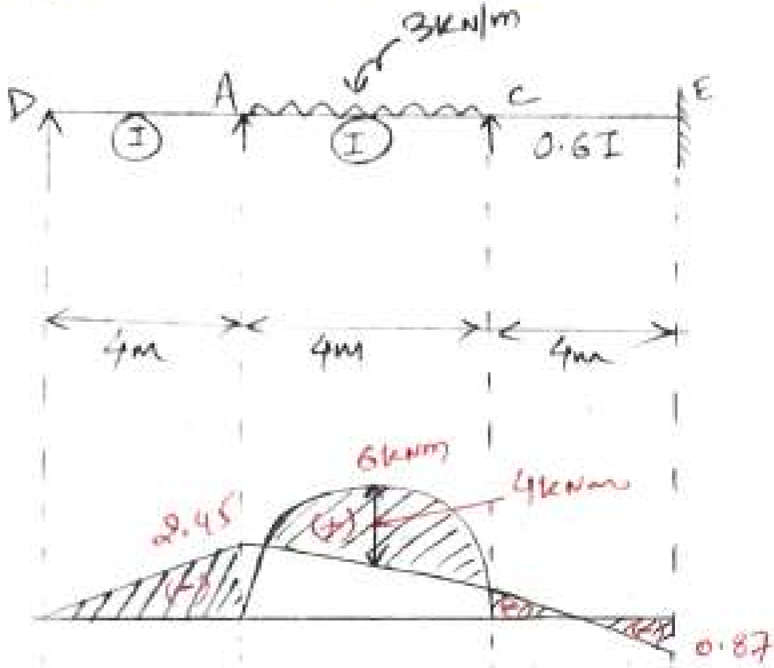
Note :-

Since, there is no holding on the span AD and CE, the fixed end Moment at A and C are distributed to the member AD and CE, also after End Moment distribution Moments. at A and C will be less and +ve B.M at 'B' will be Maximum → so, first plastic hinge is developed at 'B'.

→ 2nd plastic hinge is developed at C because 'Mp' value for the member CE is less. when the beam is loaded further.

→ Third plastic hinge is developed at 'A'.

Q. Analyse the beam shown in fig. using MDM.



BM at C = 4KNM → Mp = I hinge.

B.M at C = 1.83 KNM → 0.5Mp → 2nd Hinge.

B.M at A = 2.45 KNM → Mp → IIIrd hinge.

Step1:→

Joint	Member	K	ΣK	D.F = $\frac{K}{\Sigma K}$
A	AD	$\frac{3}{4}\cdot\frac{I}{4}$	$7I/6$	0.43
	AC	$I/4$		0.57
C	CA	$I/4$	$3I/8$	0.67
	CE	$0.5I/4$		0.33

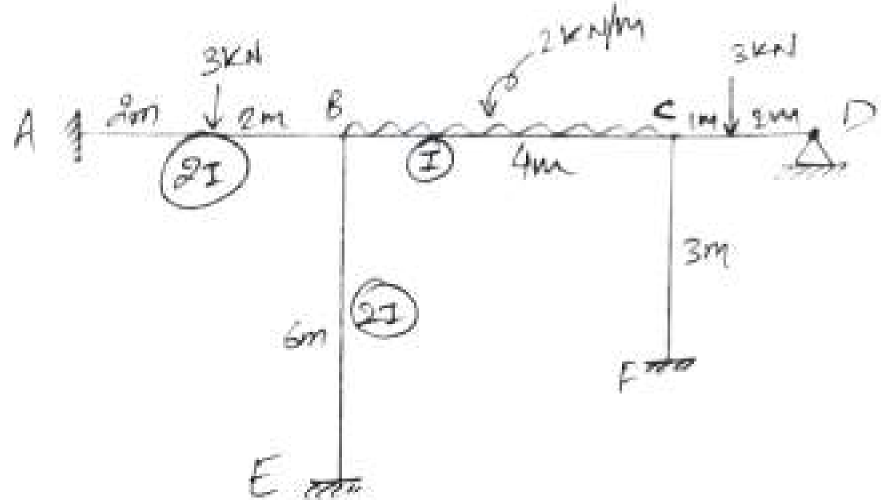
$$\frac{Wl^2}{12} = \frac{3\times4^2}{12} = 4\,KNm.$$

$$\frac{Wl^2}{8} = \frac{3\times4^2}{8} = 6\,KNm.$$

Step 2 :- End Moment Distribution :-

Joint	D	A		C		E
D.F	1	0.43	0.57	0.67	0.33	1
F.E.M	0	0	-4	+4	0	0
Balance		1.72	2.28	-2.68	-1.32	
C.O.M	0.86		-1.34	1.14		0.66
Balance		0.58	0.76	-0.76	-0.37	
C.O.M	0.29		-0.38	0.38		-0.185
Balance		0.16	0.216	-0.25	-0.12	
Final end Moments	0	+2.45	-2.45	1.83	-1.83	-0.87

Q Use M.D.M and Draw B.M.D :→



Sol^n :→ Step 1 :→ F.E.M.S :→

$$M_{fab} = -\frac{WL}{8} = \frac{-3\times4}{8} = -1.5\ kNm.$$

$$M_{fba} = +1.5\ kNm.$$

$$M_{fbc} = -\frac{2\times4^2}{12} = -2.67\ kN.$$

$$M_{fcb} = +2.67\ kNm.$$

$$M_{fcD} = -\frac{Wab^2}{L^2} = \frac{3\times1\times2^2}{3^2} = -1.33\ kNm$$

$$M_{fdc} = +\frac{3\times1^2\times2}{3^2} = +0.67\ kNm.$$

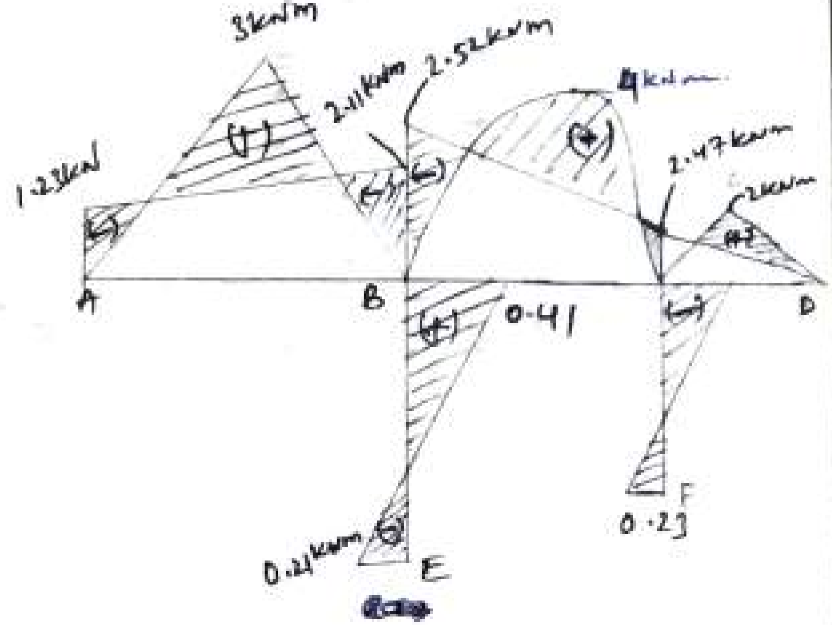
Step 2:-

Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$2I/4$	$1.083I$	0.46
	BC	$I/4$		0.23
	BE	$2I/6$		0.31
C	CB	$I/4$	$0.833I$	0.3
	CD	$\frac{3}{4}\cdot\frac{I}{3}$		0.3
	CF	$I/3$		0.4

A	B			C			D	E	F	· Joint
AB	BA	BE	BC	CB	CF	CD	DC	EB	FC	Member
1	0.46	0.31	0.23	0.3	0.4	0.3	1	1	1	DF
-1.5	+1.5	0	-2.67	+2.67	0	-1.33	+0.67	0	0	FEM
						-0.33	-0.67			Release
-1.5	+1.5	0	-2.67	+2.67	0	-1.67	0	0	0	Net FEM
	+0.54	0.36	+0.27	-0.3	-0.4	-0.3				Release
0.27			-0.15	0.14				+0.18	-0.2	COM
	0.07	0.05	-0.02	-0.04	-0.056	-0.04				Balance
0.04			-0.02	0.02			0	0.03	-0.03	COM
-1.23	2.11	0.41	-2.52	+2.47	-0.456	-2.01	0	0.21	-0.23	Final end Moment

$$\frac{wl^2}{8} = \frac{2\times 4^2}{8} = 4\,kNm$$

$$\frac{wab}{l} = \frac{3\times 1\times 2}{3} = 2\,kNm.$$



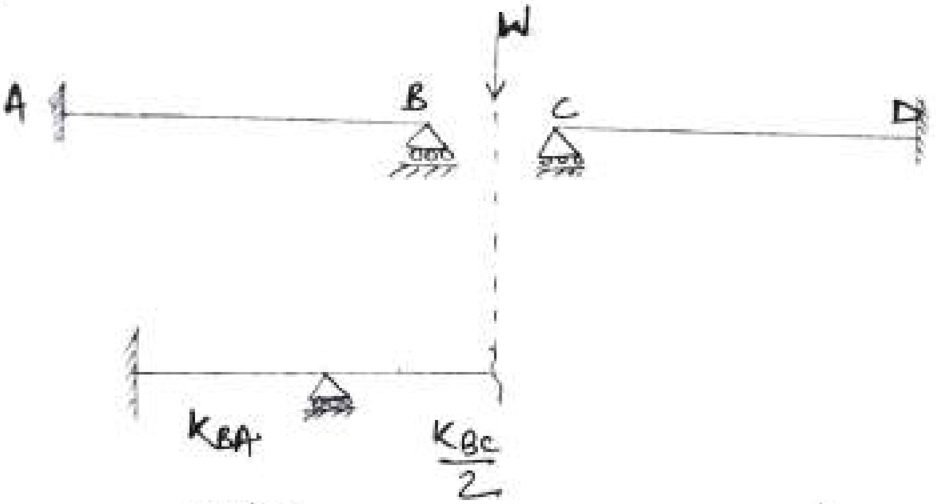
Shortcut MTD in M.D.M :→

⇒ Concept 1 :→

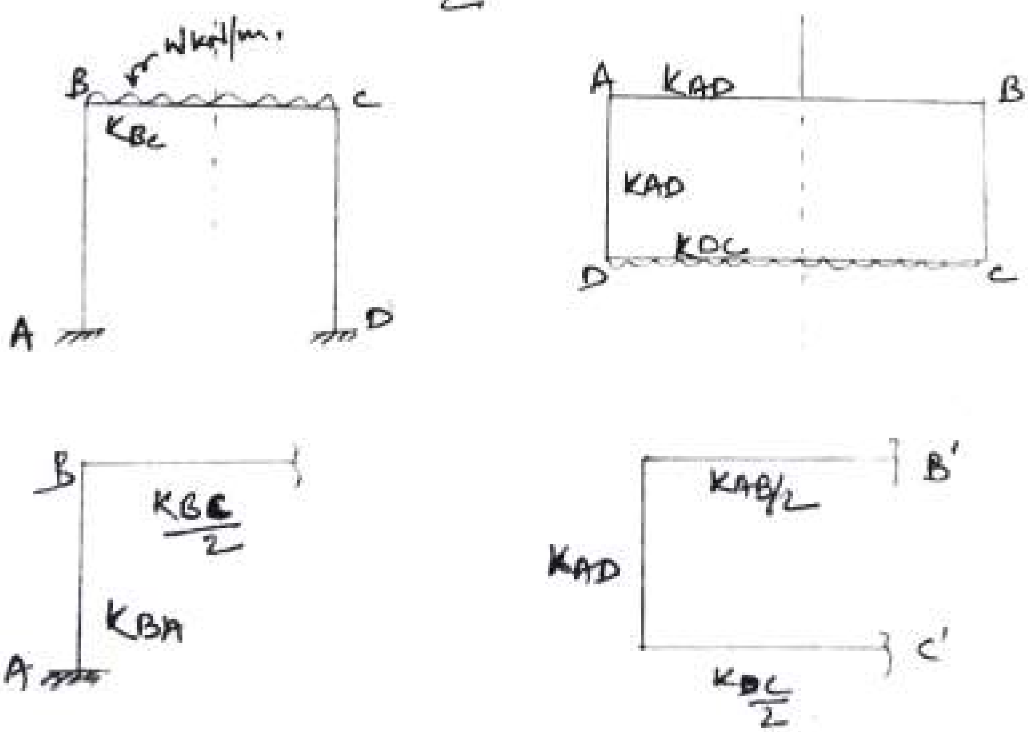
If a continous beam or frame are symmetrical and the symetric axis passes through mid-span. then Consider the stiffness of central span as half of it's original value and carry out moment distribution for half of the beam only. with opposite sign. Write all the moments for the other half of the beam or flame.

ex :→

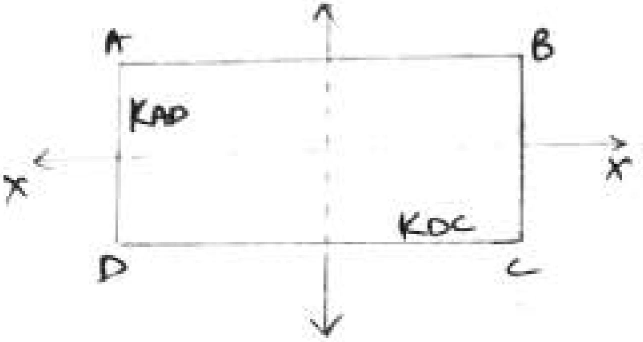
①

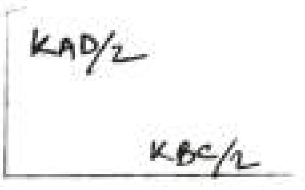


②



⇒ If the box or frame is symmetrical only about one axis then consider half of the frame with the relative stiffness as shown in the fig. and carry out the moment distribution only for the half of the frame with the opposite sign, write the moment for the other half of the frame.

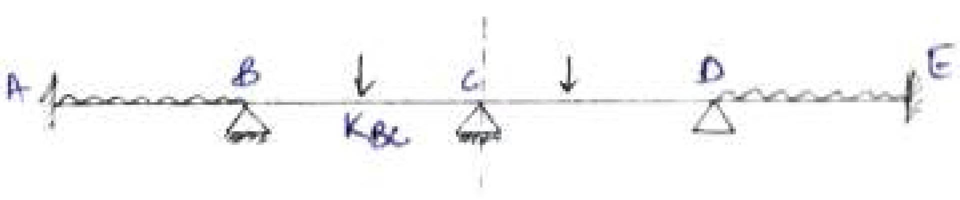




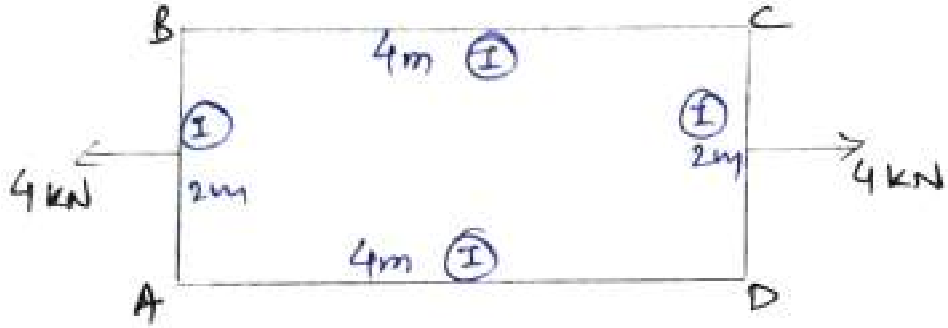
⇒ If the box frame is symmetrical about two axis then considere only quarter of the frame with the relative stiffnes as shown in the fig. carry over moment distribution at only one joint with opposite sign write the moment at all other joint.

Concept 2 :→

If the continous beam or frame is symmetrical and the symmetrical axis is passes, through the support as fixed support as carry out the moment distribution only for half of the beam, with the opposite sign write the same values for the other half of beam or frame.

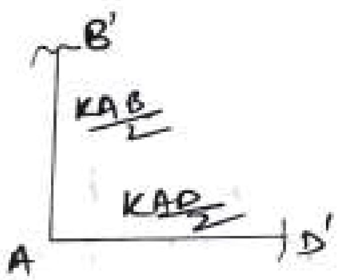


Q. Analyse the frame shown in fig. using MDM as Draw B.M.D.

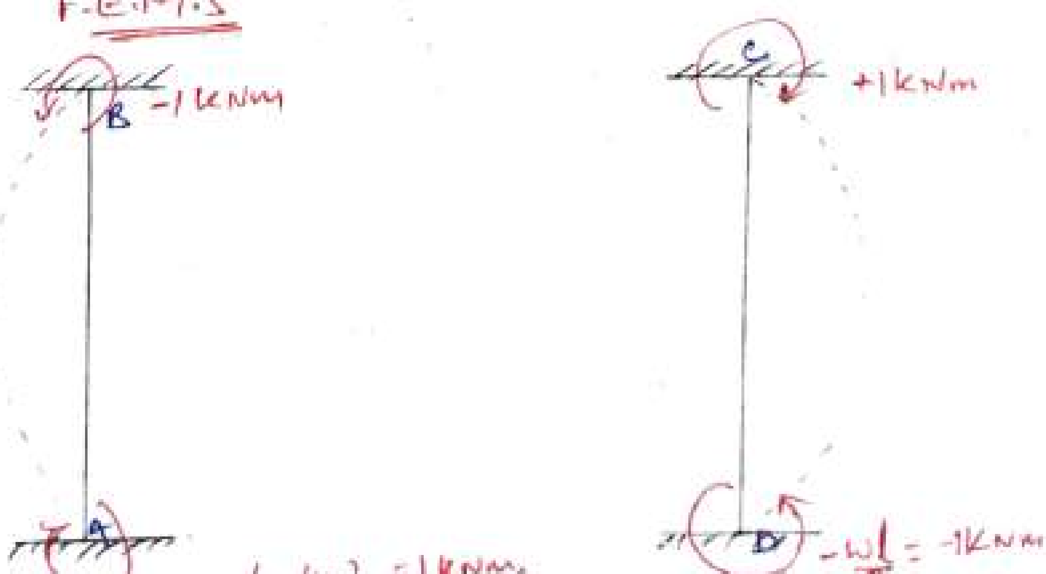


Note :-

Since, the frame is symmetrical about 'x' & 'y' axis only one quarter of the frame with relative stiffness as shown in fig. is consider. and Moment distribution is carried out only at joint A.



1st step :-> F.E.M.S



$M_{FAB} = +\frac{wl}{8} = \frac{4\times 2}{8} = 1\,kNm.$

$-\frac{wl}{8} = -1\,kNm$

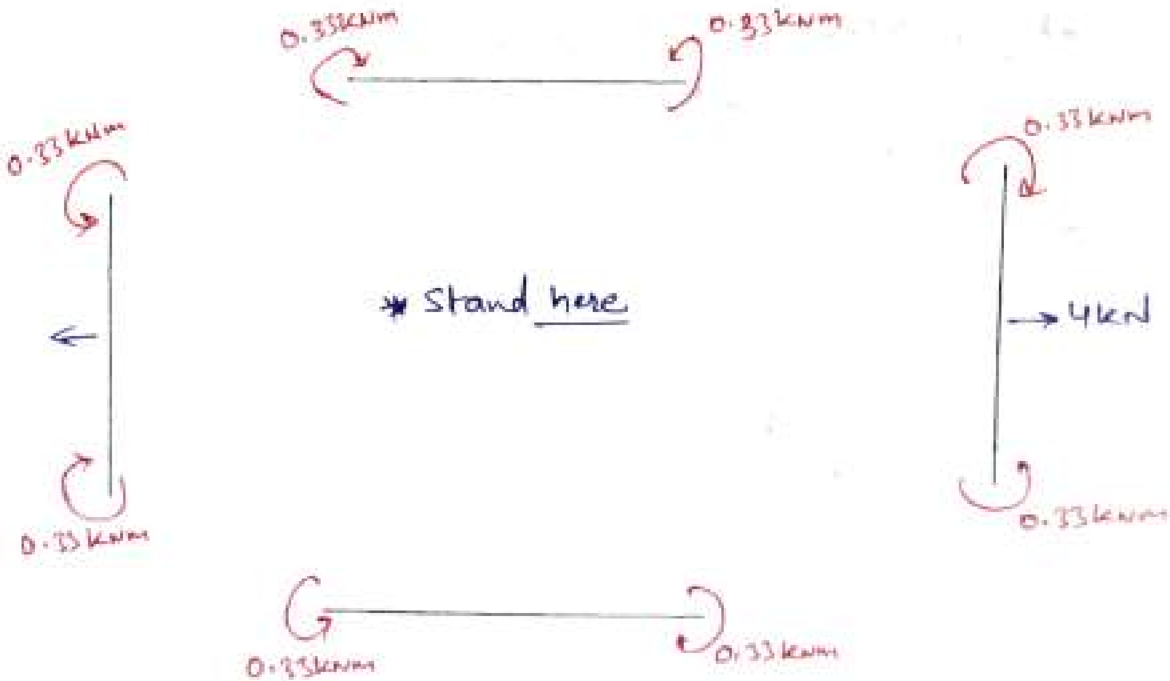
Step 2 :→ D.F's

Joint	Member	K	ΣK	Df = K/ΣK
A	AB'	$\frac{1}{2}\times\frac{I}{2}=\frac{I}{4}$	$\frac{3I}{8}$	0.67
	AD'	$\frac{1}{2}\times\frac{I}{4}=\frac{I}{8}$		0.33

Step 3 :→ End Moment Distribution :→

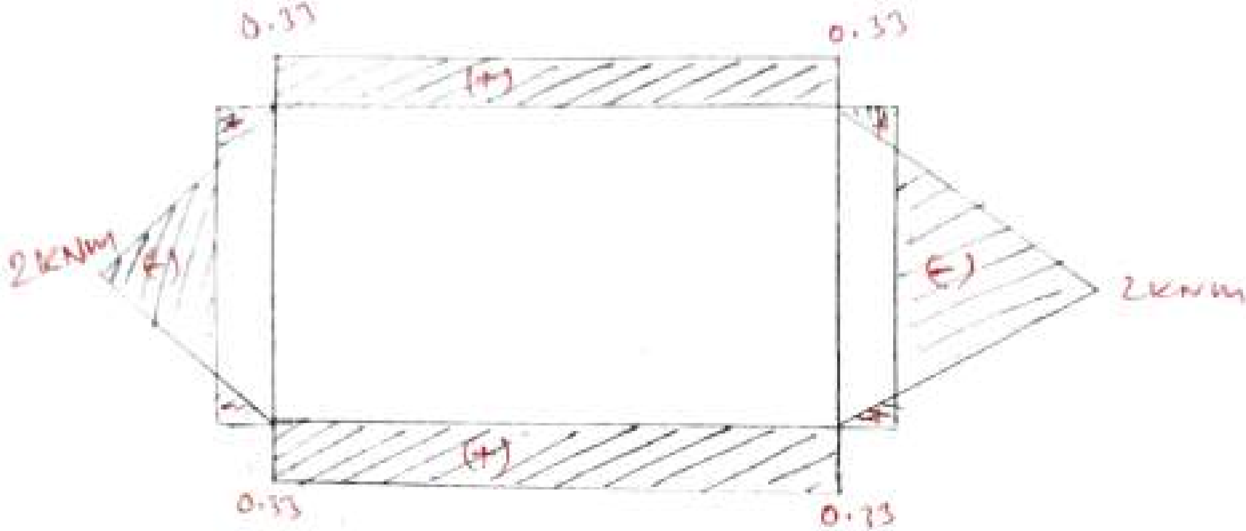
Joint	B'	A		D'
DFs	–	0.66	0.33	–
F.E.M	–	+1	0	
Release		+0.67	−0.23	
Final	–	+0.33	−0.33	–

B' and D' are not a joint.



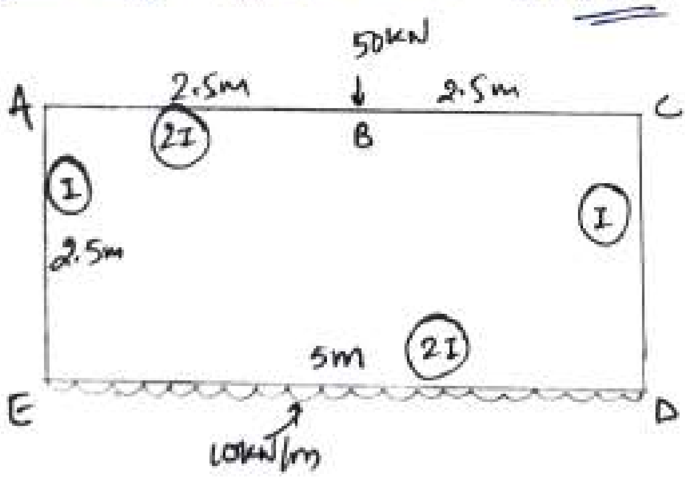
F.B.D of frame.

Step 4 :- B.M.D of frame :-

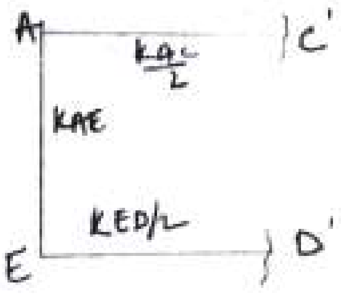


$$\frac{wl^2}{8} = \frac{4 \times 2^2}{8} = 2 \text{ kNm.}$$

Q. Analyse the ~~box~~ frame shear in fig. using Moment distribution MTD by making use of symetry. Also draw B.M.D.



Sol^n :-



Note :→ The frame is symmetrical about y-axis only, so, Consider only half of the frame with relative stiffness as shown in fig. carry out Moment distribution only for Half of the frame with opposite sign write the moment for the other half of the frame.

Step 1 :→ F.E.M.S :→

$M_{Fac} = \frac{-50 \times 5}{8}$

$= -31.25$ kNm

$M_{Fca} = \frac{+50 \times 5}{8} = +31.25$ kNm.

$M_{Fed} = +\frac{wl^2}{12}$

$= \frac{+10 \times 5^2}{12}$

$= 20.83$ kNm

$M_{Fde} = -20.83$ kNm.

Step 2 :→ D.F.S :→

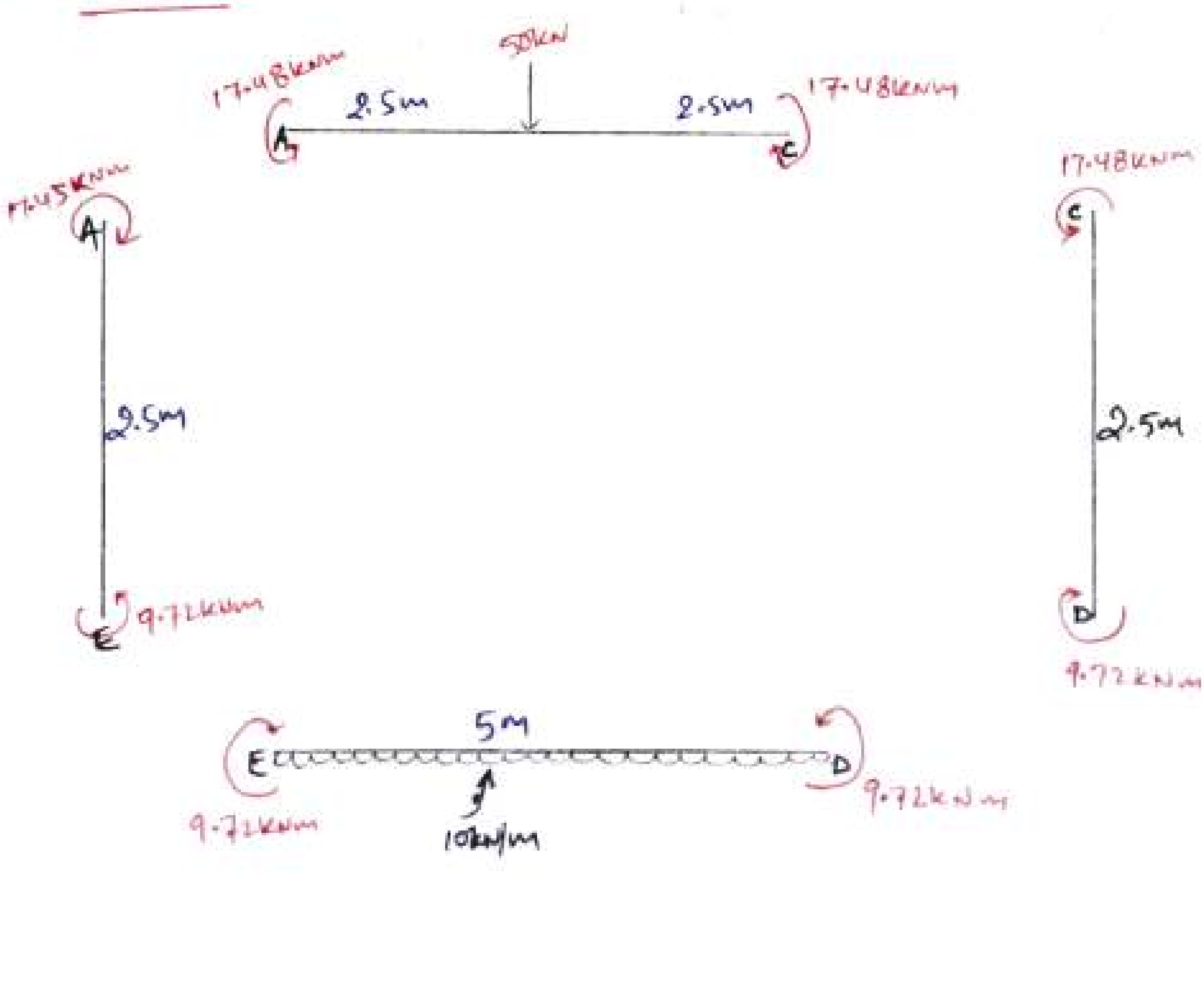
Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
A	AC'	$\frac{1}{2} \times \frac{2I}{5} = I/5$	$\frac{3I}{5}$	0.33
	AE	$\frac{I}{2.5} = \frac{2I}{5}$		0.67
E	EA	$\frac{I}{2.5} = 2I/5$	$\frac{3I}{5}$	0.67
	ED'	$\frac{1}{2} \cdot \frac{2I}{5} = \frac{I}{5}$		0.33

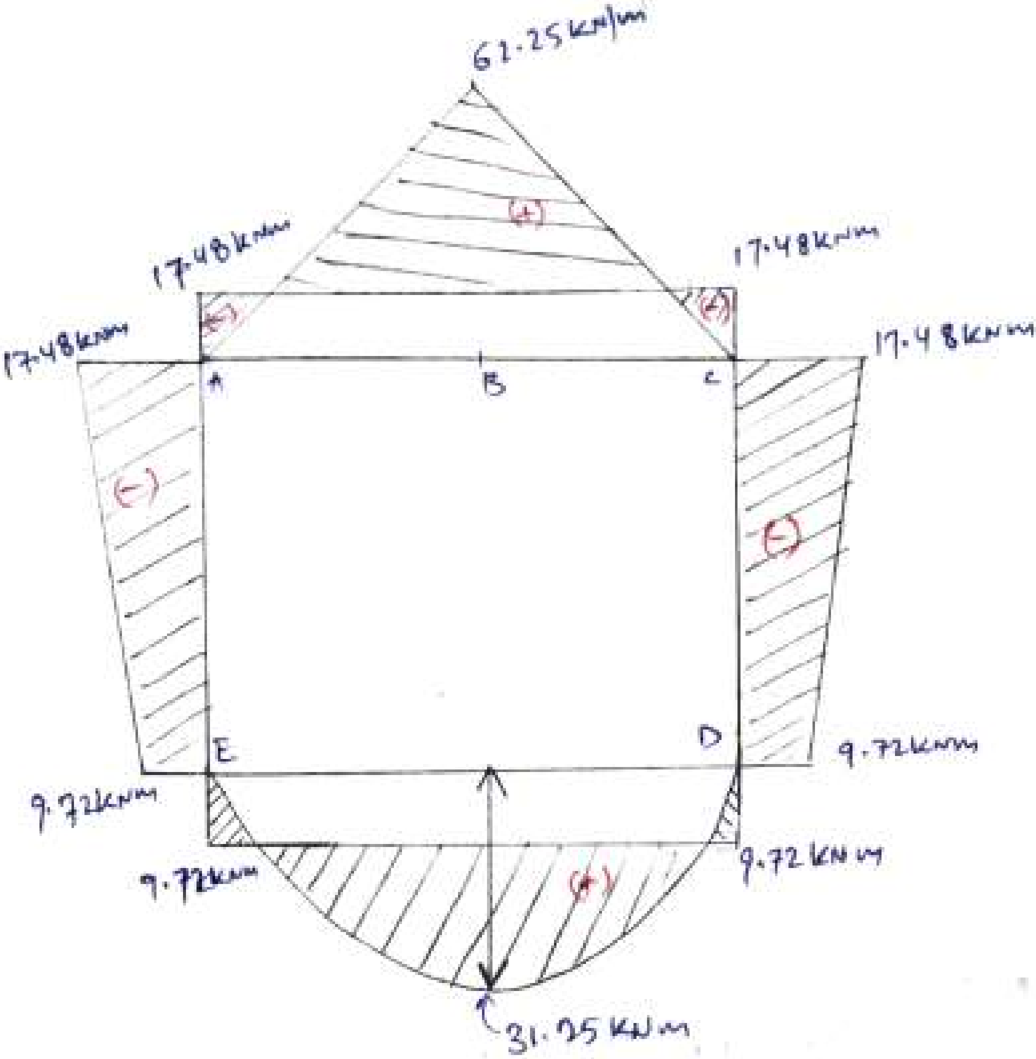
(III) End Moment Distribution :->

C'AD' are not joint.

Joint	C'	A		E		D'
D.F	-	$\boxed{0.33}$	$\boxed{0.67}$	$\boxed{0.67}$	$\boxed{0.33}$	-
F.E.M	-	-31.25	0	0	+20.83	-
Balance		+10.31	20.93	-13.95	-6.87	
COM	-		-6.98	10.47		-
Balance		+2.3	4.67	-7.01	-3.45	
COM	-		-3.5	2.34		-
Balance		+1.16	+2.35	-1.57	-0.77	
Final end Moment	-	-17.48	+17.48	-9.72	+9.72	-

(IV) B.M.D :->

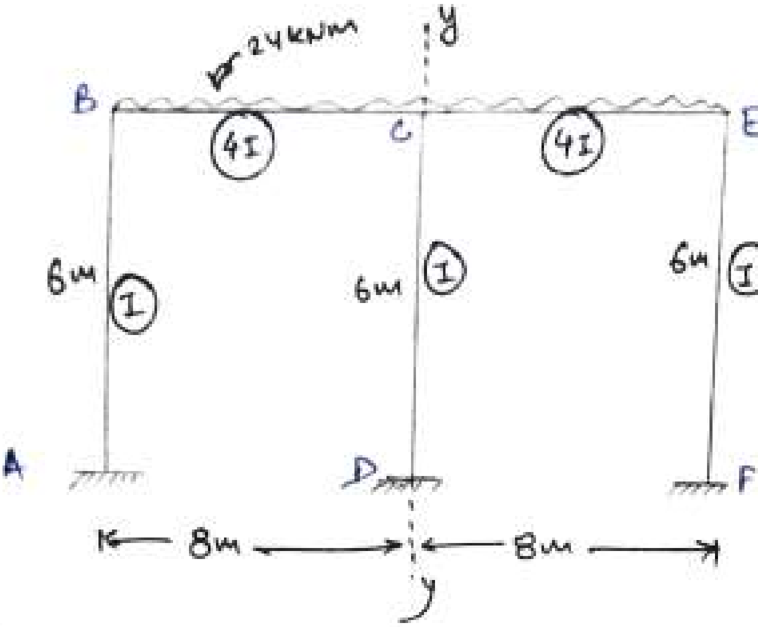




$$\frac{Wl}{4} = \frac{50 \times 5}{4} = 62.5\ \text{KNm.}$$

$$\frac{wl^2}{8} = \frac{10 \times 5^2}{8} = 31.25\ \text{KNm.}$$

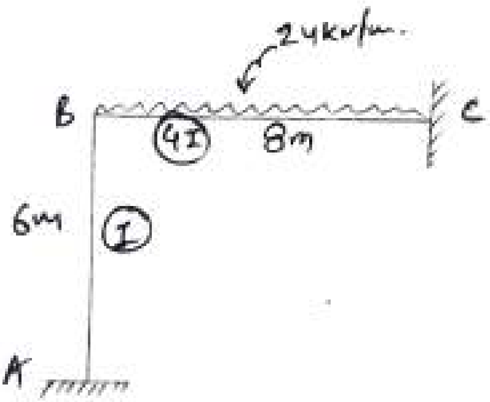
Q. Consider symmetry of the rigid frame the magnitude of movement at c is?



Sol^n:-

Note:->

Since, the frame is symmetrical about y-axis and the symmetric axis is passing through the support considered only half of the frame by taking 'c' as a fixed support as shown in fig.



Step I :- D.F :-

Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BC	$\frac{4I}{8} = 3I/6$	$\frac{4I}{6}$	0.75
	BA	$\frac{1}{6}$		0.25

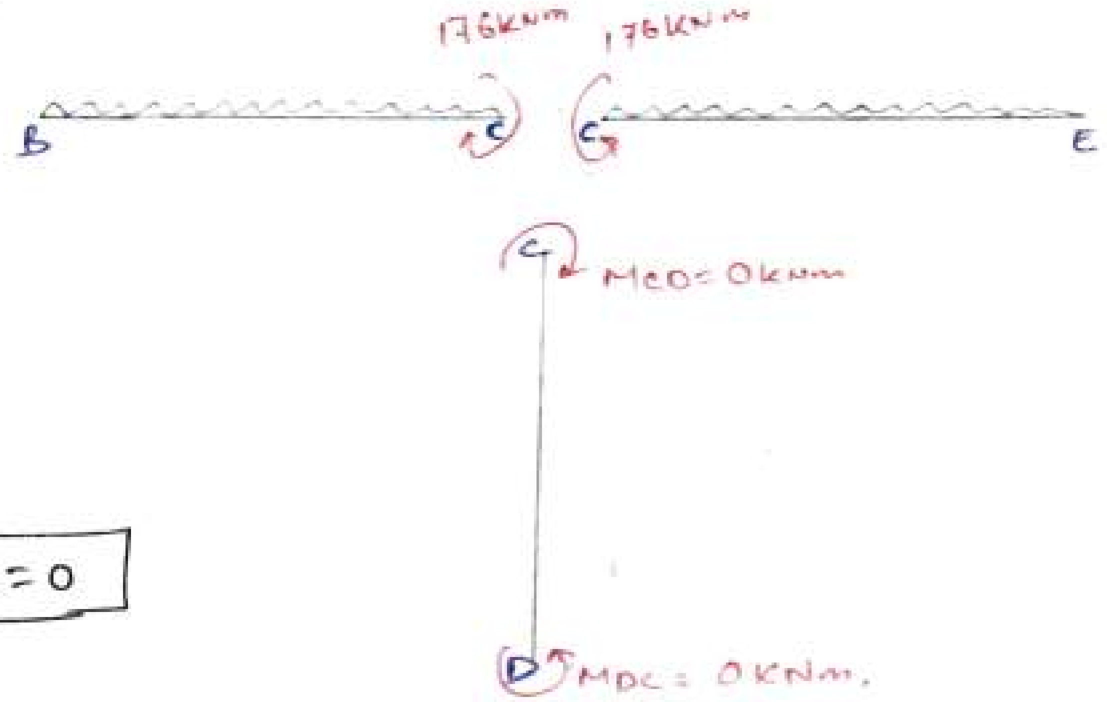
Step II End Moment distribution :-

Joint	A	B	B	C
D.F	1	0.25	0.75	1
F.E.M	0	0	−128	+128
Balance		+32	+96	
C.O.M	+16			48
Final end moment	+16	+32	−32	+176

So, Moment at 'c' = 176 KNm .

Q For the above Pb what is the Moment at 'c' for the column 'CD' is.

Sol^n :-



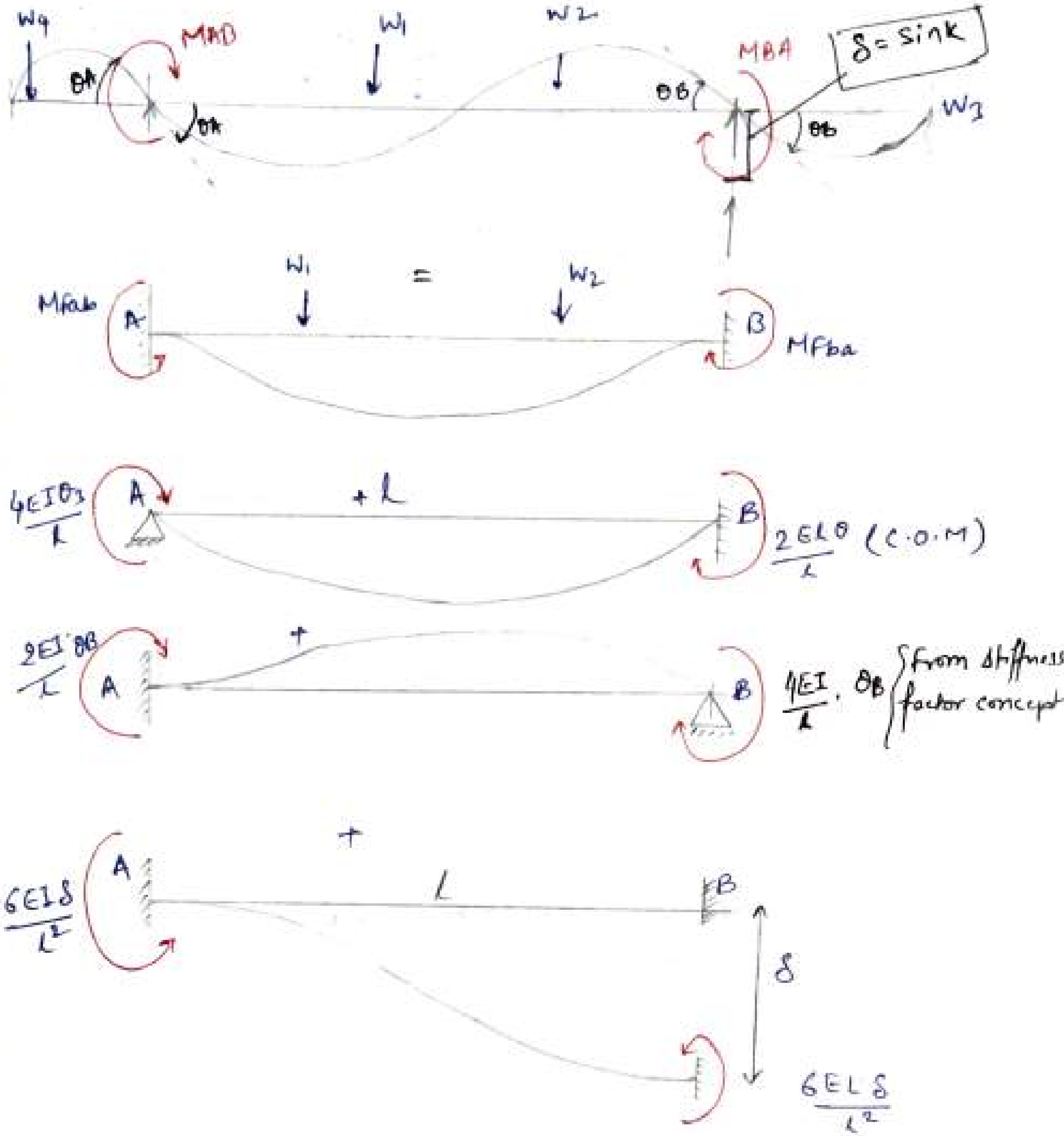
$\boxed{M_{CD} = 0}$

Conclusion :-

Because of symmetry, M_{CB} & M_{CE} are equal so, Column CD is axially loaded Member only, Moment in the colume 'CD' is zero.

II topic :-

Slope deflection MTD (Deflection eq^n) :-



MAB = End Moment at A for the span AB

$$= -M_{fab} + \frac{4EI}{l}\theta_A + \frac{2EI}{l}.\theta_B - \frac{6EI\delta}{l^2}$$

MBA = End Moment at B for the span BA.

$$= +M_{fba} + \frac{2EI}{l}\theta_A + \frac{4EI}{l}\theta_B - \frac{6EI}{l^2}\delta.$$

→

Rearranging :-

$$M_{AB} = M_{Fab} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$M_{BA} = M_{Fab} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

slope deflection equation.

Note :- From the principle of superposition if we add four deflected shapes of the beam, we get final deflected shape. Similarly by adding four end moments, we get final end moments for the span 'AB'.

Concept ② :-

Many used the concept of stiffness factor and carry over moment to develope slope deflection equation. i.e, why it is also called stiffness MTD of Analysis.

Concept ③ :-

If far end is force, i.e overhang beam slope deflection eq^n. can not be written because free end cannot be fixed to get slope deflection eq^n.

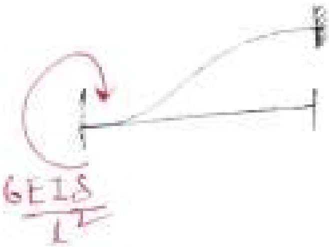
Concept ④ :-

Sign conventions in S.D.MTD :-

(i) Clockwise end moment's are taken as +ve and anticlockwise end moments are taken as '-ve' {this sign convention is used while writting slope deflection eq^n. only}.

(ii) Sagging B.M is taken as '+ve' and hogging B.M is taken as '-ve'. this sign convention is used while drawing B.M.D only.

(iii) Clockwise rotation taken as '-ve' and anticlockwise rotations are taken as '-ve' this sign conventions are used while drawing deflection profile.



(iv) with respect to left support, if the right support sinks downwards then 'δ' is taken as '+ve'.
w.r. to left support, if the Right supports moves upwards the 'δ' is taken as '-ve'.

{to suit to the end moment sign convension}.

Concept (5) :-

In slope deflection MTD also, axial forces and corresponding axial deformation are to neglected. so, in the analysis of beam, members are assumed to be in extensible.

Concept (6) :-

In slope deflection MTD, the unknow are slopes and deflection {i.e., independent displacement} so, if kinamatic indeterminacy or Degree of freedom (DK), is less, then use S.D MTD to find end moment quickly.

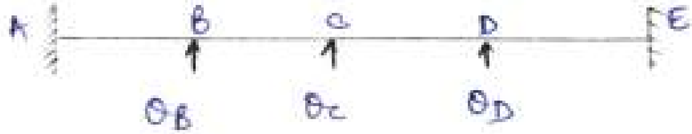
Concept (7) :- Kinametic indeterminacy (D_K) :-

It is the no. of indeterminacy displacements at all joint in a structure.

Concept (8) :-

No. of simultaneous eq^n. reqd. to find the unknown in S.D.MTD = DK.

Q: The no. of simultaneous eqn. reqd for four span. Continous beam. with both ends fixed are :-



$\boxed{D_K = 3}$ $(\theta_B, \theta_C, \theta_D)$

Since, $D_K = 3$, we required 3 simultaneous eqn. to find them.

So, 3 joints equillibrium eqn. are.

$M_{BA} + M_{BC} = 0$ —— (i)

$M_{CB} + M_{CD} = 0$ —— (ii)

$M_{DC} + M_{DE} = 0$ —— (iii)

Note :-

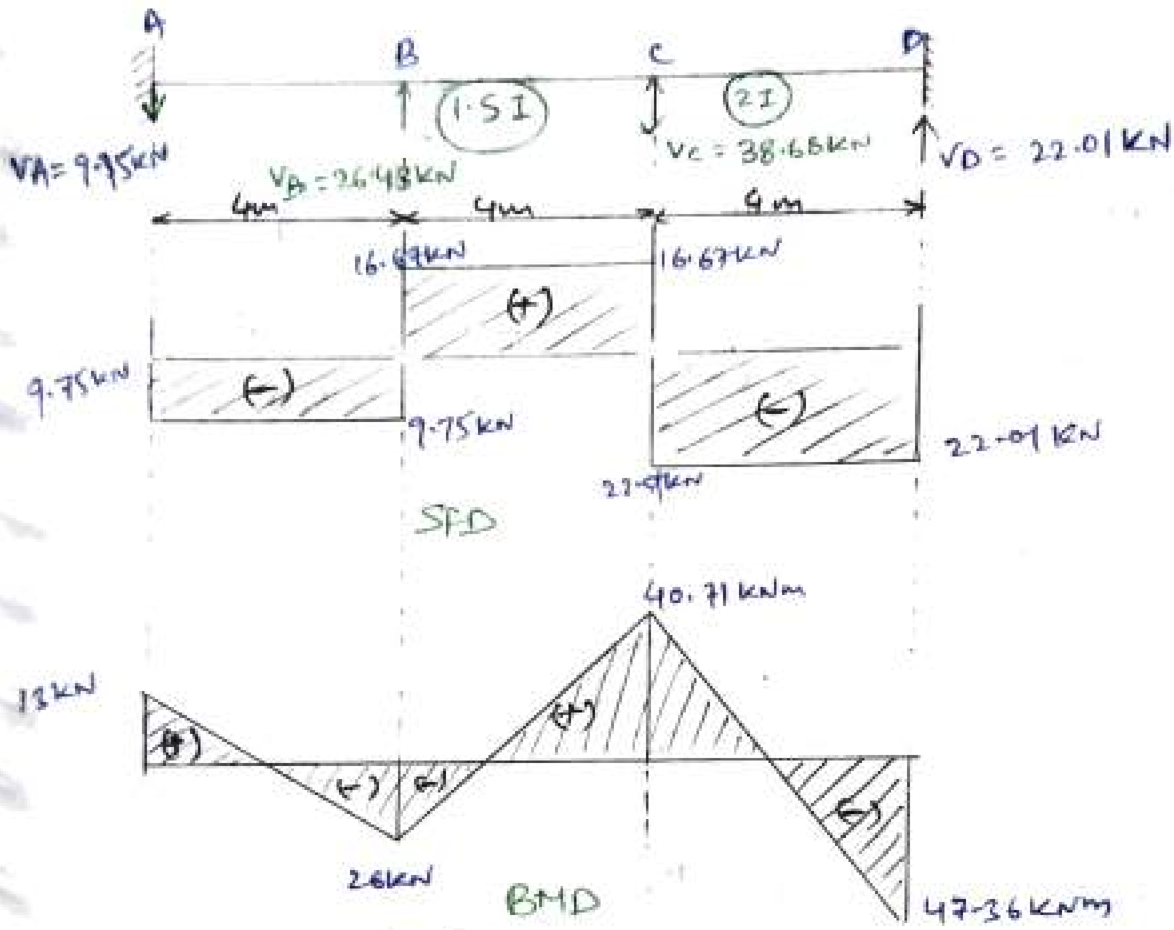
Slope deflection MTD is also called

(1) Stiffness MTD because stiffness factor are used.

(2) Displacement MTD, because displacement are unknown.

(3) Equillibrium, MTD, because joints equillibrium eqn. are used to find unknowns.

Q. Analyse the continous beam shown in fig. if supports 'c' sinks by 10mm. take $E = 2\times10^5 N/mm^2$, $I = 3.6\times10^7 mm^4$. use S.D.MTD. Draw S.F.D and B.M.D.

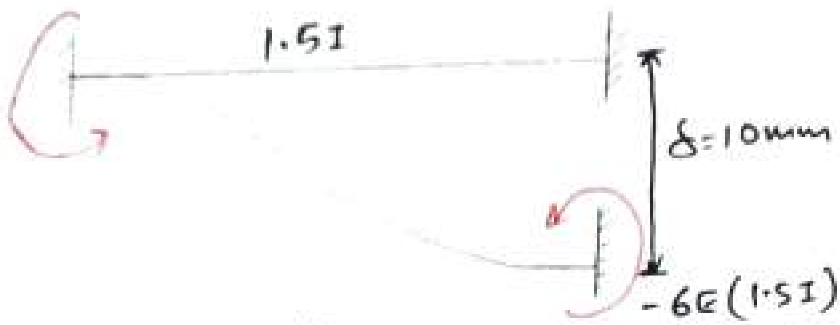


Step 1:

F.E.M.S {due to applied load and sinking}.

From the principle of superposition, the F.E.M due to applied load and Due to sinking of supports can be added to get final fixed end moments. {if δ is Known}.

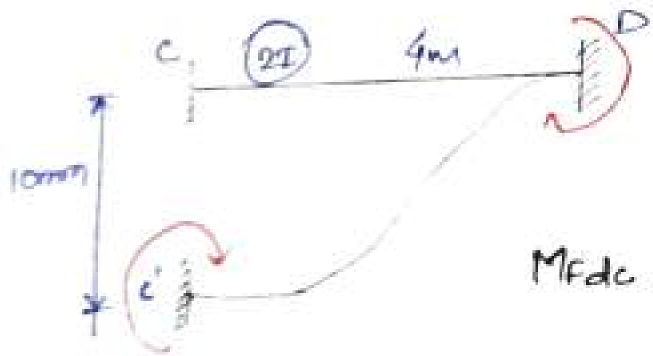
$M_{Fab} = M_{Fba} = 0$ [No. loads and no. sinking].



$$M_{Fbc} = \frac{-6E(1.5I)\delta}{l^2}$$

$$= \frac{-6\times2\times10^5\times(1.5\times3.6\times10^7)\times10}{4000^2} = -40.5 \text{ kNm.}$$

$M_{Fcb} = -40.5$ kNm.



$$M_{Fdc} = +\frac{6E(2I)\delta}{l^2}$$

$$M_{Fcd} = +\frac{6E(2I)\delta}{l^2}$$

$$= \frac{6\times2\times10^5\times(2\times3.6\times10^7)\times10}{4000^2} = +54 \text{ kNm}$$

Step 2 :- Slope deflection equations :-

$$M_{AB} = \cancel{M_{Fab}}^{0} + \frac{2EI}{l}\left(\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{l}}^{0}\right) \quad \{\theta_A = \delta = 0 \text{ for Span AB}\}$$

$$M_{AB} = \frac{2E(2I)}{4}(\theta_B) = \boxed{EI\theta_B}$$

$$M_{BA} = \cancel{M_{Fba}}^{0} + \frac{2EI}{l}\left(2\theta_B + \cancel{\theta_A}^{0} - \cancel{\frac{3\delta}{l}}^{0}\right)$$

$$M_{BA} = \frac{2E(I)(2\theta_B)}{4} = \boxed{2EI\theta_B}$$

$$M_{BC} = M_{Fbc} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{l}}\right)$$ {take $\delta = 0$, because it is considered in M_{Fbc}}.

$$= -40.5 + 2E\frac{(1.5I)}{4}(2\theta_B + \theta_C).$$

$$M_{BC} = \boxed{-40.5 + 1.5EI\theta_B + 0.75EI\theta_C}$$

$$M_{CB} = M_{Fcb} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \cancel{\frac{3\delta}{l}}\right)$$ {$\delta = 0$, bcz it is considered in M_{Fcb}}.

$$= -40.5 + 2E\left(\frac{1.5I}{4}\right)(2\theta_C + \theta_B).$$

$$\boxed{M_{CB} = -40.5 + 1.5EI\theta_C + 0.75EI\theta_B}$$

$$M_{cd} = M_{Fcd} + \frac{2EI}{l}\left(2\theta_C + \cancel{\theta_D}^{0} - \cancel{\frac{3\delta}{l}}^{0}\right)$$ {$\delta = 0$, because is considered in M_{Fcd}}.

$$= +54 + \frac{2E(2I)}{4}(2\theta_C) = \boxed{54 + 2EI\theta_C}$$

$$M_{dc} = M_{Fdc} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \cancel{\frac{3\delta}{l}}\right) = 54 + \frac{2E(2I)}{4}\cdot\theta_C$$

$$= 54 + EI\theta_C$$

Step III :-

We joint equillibrium eq^n :⇒
to find unknown displacements.

$$M_{BA} + M_{BC} = 0 \quad \text{—— (I)}$$

$$M_{CB} + M_{CD} = 0 \quad \text{—— (II)}$$

So {$x = 2\ (\theta_B, \theta_C)$
we required two eq^n. to find them.}

from ①,

$$M_{BA} + M_{BC} = 0$$

$$\underbrace{[2EI\theta_B]}_{M_{BA}} + \underbrace{[-40.5 + 1.5EI\theta_B + 0.75EI\theta_c]}_{M_{BC}} = 0$$

$$\boxed{3.5EI\theta_B + 0.75EI\theta_c = 40.5} \quad —①$$

from ②

$$M_{CB} + M_{CD} = 0$$

$$\Rightarrow \underbrace{[-40.5 + 1.5EI\theta_c + 0.75EI\theta_B]}_{M_{CB}} + \underbrace{[54 + 2EI\theta_c]}_{M_{CD}} = 0$$

$$\boxed{3.5EI\theta_c + 0.75EI\theta_B = -13.5} \quad —②$$

Solving ① and ②

$$\boxed{\begin{aligned} EI\theta_B &= 13 \\ EI\theta_c &= -6.64 \end{aligned}}$$

$M_{AB} + M_{BA} = 13 + 26 = 39$

Step iv :- Find end Moments :-

$M_{AB} = EI\theta_B = 13kNm.$

$M_{BA} = 2EI\theta_B = 2 \times 13 = 26kNm$

$M_{BC} = -40.5 + (1.5 \times 13) + 0.75(-6.64) = -26kNm.$

$M_{CB} = -40.5 + 1.5(-6.64) + 0.75(13) = -40.71kNm.$

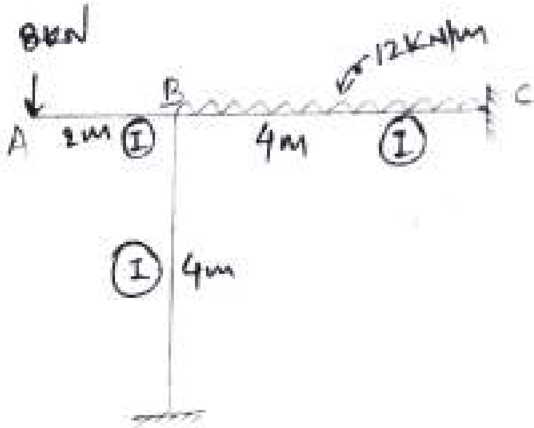
$M_{CD} = +54 + 2 \times (-6.64) = 40.71kNm.$

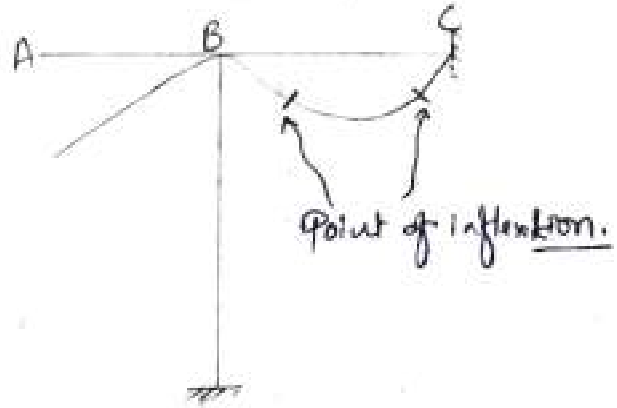
$M_{DC} = 54 + EI\theta_c = 54 + (-6.64) = 47.36kNm.$

Step V: Support reaction SFD and BMD :-

Reaction due to load:	0	0	0	0	0	0
Reaction due to end Moments	$\frac{39}{4} = 9.75 (\downarrow)$	$9.75 (\uparrow)$	16.67 kN $(\uparrow)$	16.67 kN $(\downarrow)$	22.01 kN $(\downarrow)$	22.01 kN $(\uparrow)$
Final Reaction	$V_A = 9.75$ kN $(\downarrow)$	$V_B = 26.43$ kN $(\uparrow)$		$V_C = 38.68$ kN $(\downarrow)$		$V_B = 22.01$ kN $(\uparrow)$

① For the frame shown in fig. slope at 'B' is.





Sol^n:-

$D_K = 1$ (θ_B)

So, we required ① eq^n. to find θ_B

Step 2 :- Joint equilibrium eqn. :-

$$M_{BA} + M_{BC} + M_{CD} = 0$$

$M_{Fba} = M_{BA} = +16\ \text{kNm}$ (Slope deflection eqn. cannot be written if far end is free).

$$M_{BA} = +16\ \text{kNm}$$

$$M_{BC} = M_{Fbc} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C}^{\,0} - \cancel{\frac{3\Delta}{L}}^{\,0}\right)$$

$$= \left[\frac{-12 \times 4^2}{12}\right] + \frac{2EI}{4}(2\theta_B) = \boxed{EI\theta_B - 16}$$

$$M_{BA} + M_{BC} + M_{BD} = 0$$

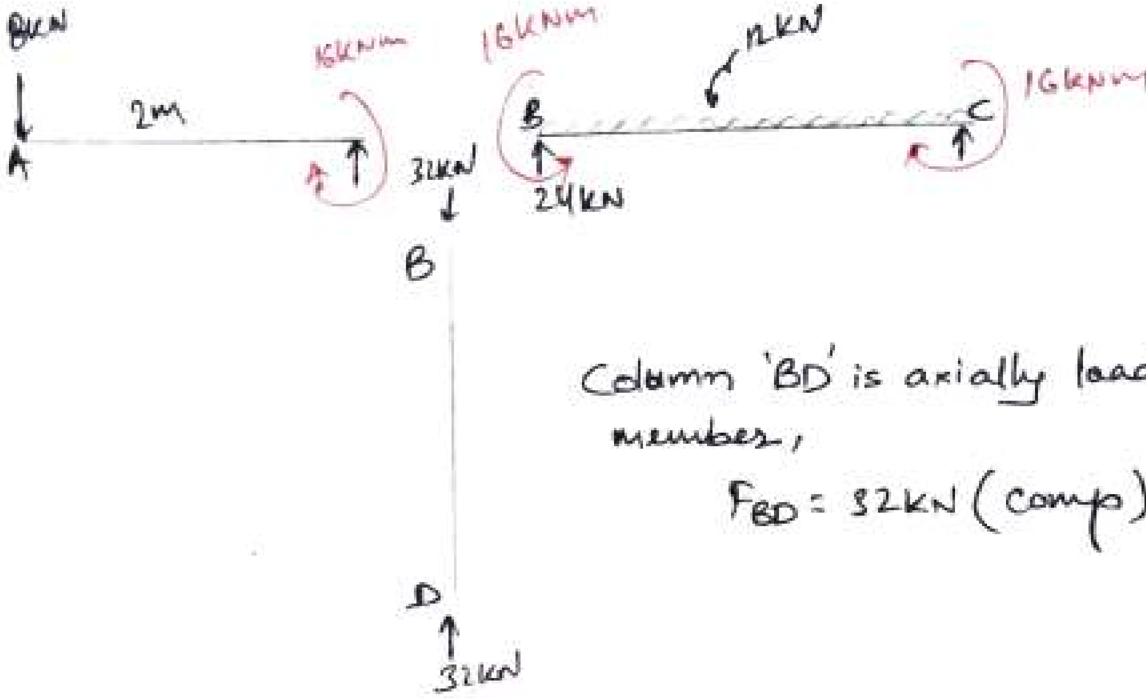
$$[+16] + [EI\theta_B - 16] + [EI\theta_B] = 0$$

M_{BA}

$$\boxed{\theta_B = 0}$$

Note :-

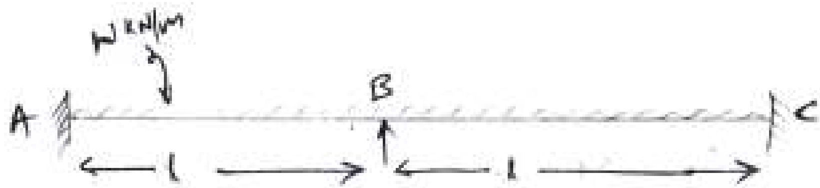
- If sum of fixed end moment are zero at a joint the slope at that joint will be zero.
- Compressive force in the member BD is



Column 'BD' is axially loaded members,

$F_{BD} = 32\,kN$ (comp).

Q.



Sol^n:- $\boxed{\theta_B = 0}$

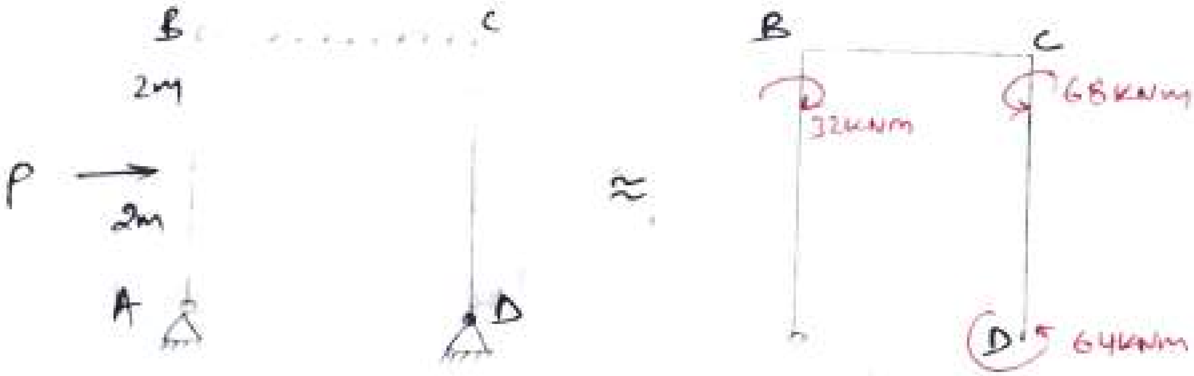
Sum of F.E.M.s at B

$= M_{fba} + M_{fbc}$

$= +\frac{wl^2}{12} - \frac{wl^2}{12} = 0$

So, $\boxed{\theta_B = 0}$

Q. The portal frame shown in fig. is analysed the ~~portal~~ because final column moments are found to be as shown in fig. the value of 'P' is



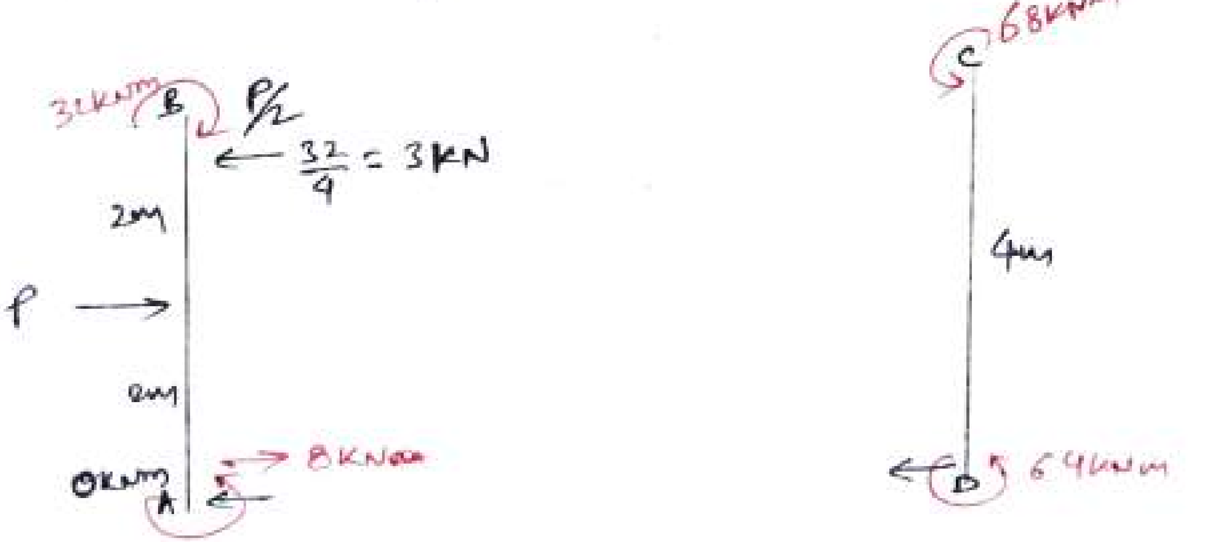
Sol^n:

① From equillibrium of entire structure –

$$\Sigma x = 0 \Rightarrow \boxed{+P - H_A - H_B = 0} \quad \text{—} ①$$

[$\xrightarrow{}$ +ve $\xleftarrow{}$ –ve]

② from F.B.D of coloumn AB & CD.



$$H_D = \frac{64+68}{4} = 33\text{ kN}$$

$$\Sigma x = 0 \Rightarrow \underset{\text{at } E}{+P} - \underset{\text{at } A}{\frac{P}{2}} + \underset{\text{at } A}{8} - \underset{\text{at } D}{33} = 0$$

$\boxed{P = 50\text{ kN}}$ Ans.

Note :-

DK for the frame $= 4(\theta_A, \theta_B, \theta_C, \delta)$ → sway.

we require 4 equation to find them.

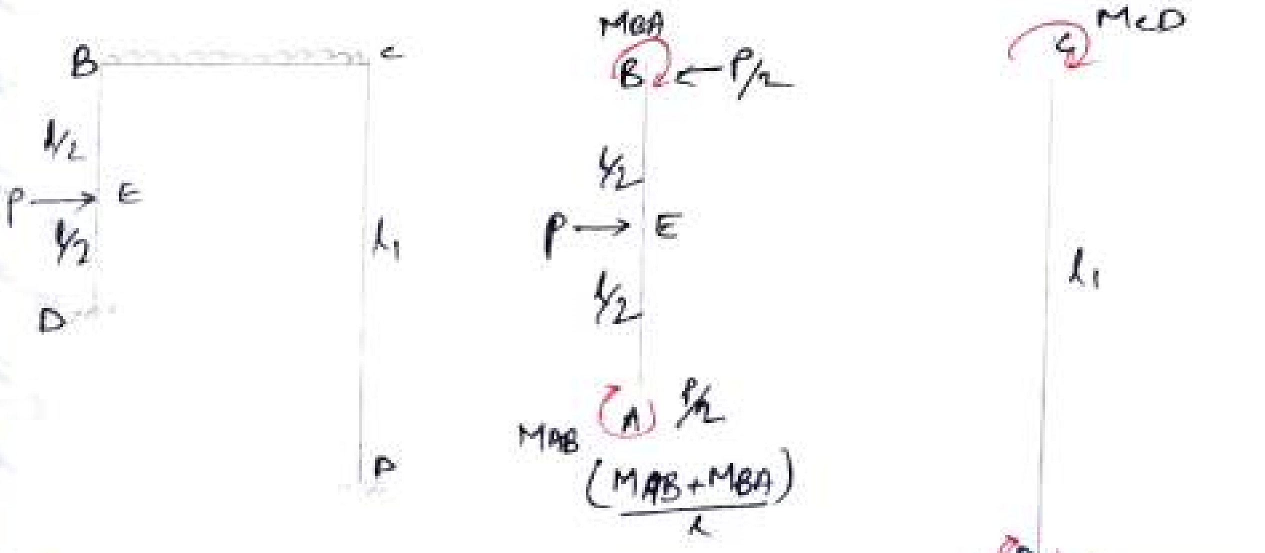
$$M_{AB} = 0 \quad \text{——} (1)$$

$$M_{BA} + M_{BC} = 0 \quad \text{——} (2)$$

$$M_{CB} + M_{CD} = 0 \quad \text{——} (3)$$

$$\Sigma x = 0 \Rightarrow +P - H_A - H_D = 0 \quad \text{——} (4)$$

Horizontal shear eqn. for the frame is ? {Σx = 0} is ?



$\Sigma x = 0 \Rightarrow$ Horizontal shear eqn.

$$\frac{M_{CD} + M_{DC}}{l_1}$$

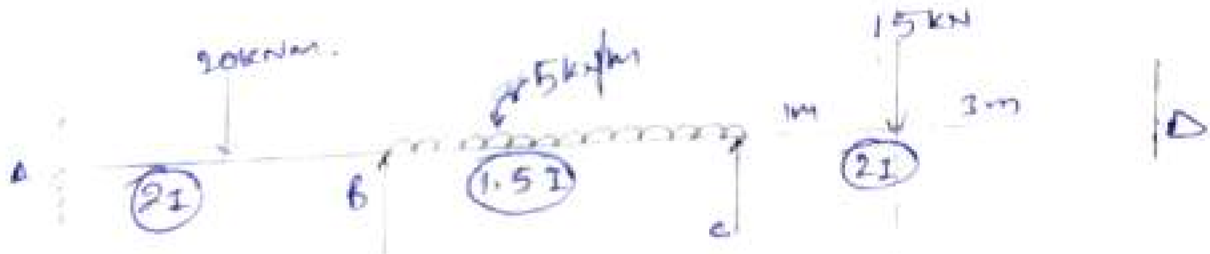
$$\Rightarrow \underset{\text{at } E}{+P} - \underset{\text{at } A}{\frac{P}{2}} + \underset{\text{at } A}{\frac{M_{AB} + M_{BA}}{l}} + \underset{\text{at } D}{\left(\frac{M_{CD} + M_{DC}}{l_1}\right)} = 0$$

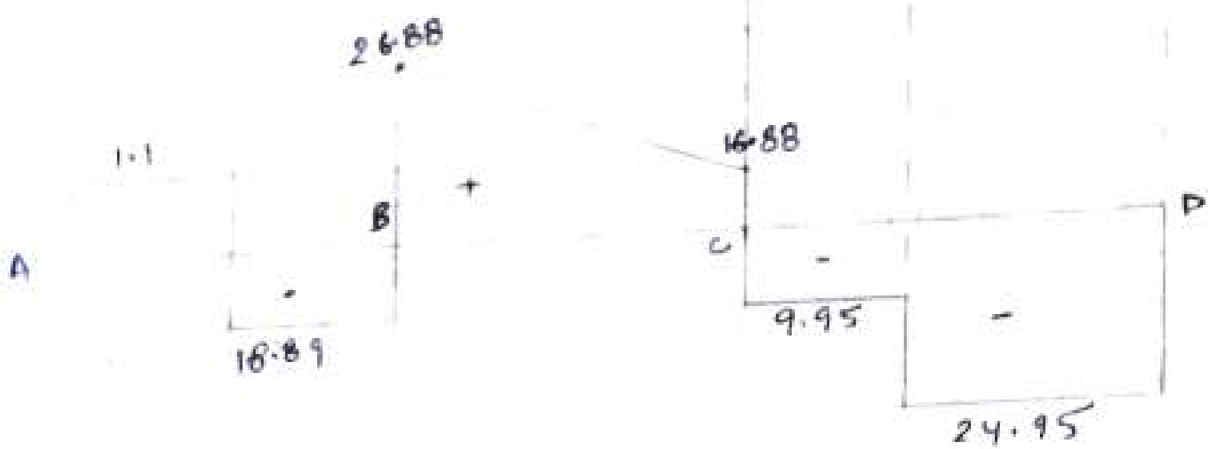
Q. Analyse the continuous beam shown in fig. if support C sink by 10mm take

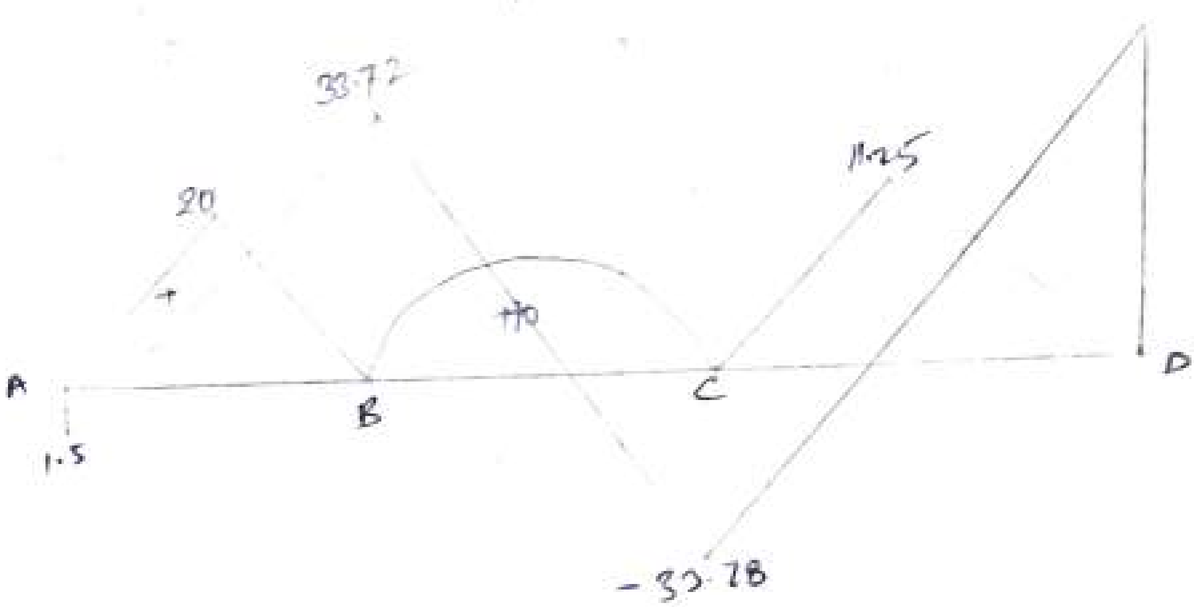
$E = 2\times10^5\ N/mm^2$

$I = 3.6\times10^7\ mm^4$.

Use S.D MTD and MDMTD. Draw S.F.D and B.M.D







(I) Go Through slope deflection MTD :-

Step-I Fixed end Moment

$M_{AB} = -20 \times 4/8 = -10\ kN \cdot m.$

$M_{FBA} = 10\ kNm.$

$M_{FBC} = -\frac{5 \times 4^2}{12} - \frac{6 \times 2 \times 10^5 \times 3.6 \times 10^7 \times 10 \times 1.5}{4000^2 \times 10^6}$

$M_{FBC} = -6.67 - 40.5$

$= -47.17\ kNm.$

$M_{FCB} = +6.67 - 40.5$

$= -33.83\ kNm.$

$M_{FCB} = +6.67 - 40.5$

$= -33.83\ kNm$

$M_{FCD} = -15 \times \frac{1 \times 3^2}{4^2} + \frac{6 \times 2 \times 10^5 \times 2 \times 3.6 \times 10^7 \times 1.0}{4000^2 \times 10^6}$

$= -8.44 + 54$

$= +45.56\ kNm.$

$M_{FDC} = 2.81 + 54$

$= +56.8\ kNm.$

∵ Hence, DH = 2

In we required only two type eq^n. to find the unknown.

Now,

Eq^n

→

Step 2 :- Slope deflection eq^n :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\cancel{\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{l}}^{0}\right)$$

$$M_{AB} = -10 + EI(\theta_A - \theta_B = 0)$$

$$[M_{AB} = -10 + EI\theta_B] \quad \text{— (i)}$$

$$M_{BA} = 10 + \frac{2E.I}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$M_{BA} = 10 + EI(2\theta_B)$$

$$[M_{BA} = 10 + 2EI\theta_B] \quad \text{— (ii)}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{l}}^{0}\right)$$

{ $\delta = 0$ bcz it constant in M_{FBC} }

$$M_{BC} = -47.17 + \frac{2 \times 1.5 \times EI}{4}(2\theta_B + \theta_C - 0)$$

$$[M_{BC} = -47.17 + 1.5EI\theta_B + 0.75EI\theta_C] \quad \text{— (iii)}$$

$$M_{CB} = -33.83 + \frac{2 \times E \times 1.5I}{4}(2\theta_C + \theta_B - 0)$$

$$[M_{CB} = -33.83 + 1.5EI\theta_C + 0.75EI\theta_B] \quad \text{— (iv)}$$

$$M_{CD} = 45.56 + \frac{2 \times E \times 2 \times 7}{4}(2\theta_C + 0 - 0)$$

$$[M_{CD} = 45.56 + 2EI\theta_C] \quad \text{— (v)}$$

$$M_{DC} = 56.8 + EI(0 + \theta_C - 0)$$

$$[M_{DC} = 56.8 + EI\theta_C] \quad \text{— (vi)}$$

Step III :> we apply joint equillibrium eq^n at B & C

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$10 + 2EI\theta_B - 47.17 + 1.5EI\theta_B + 0.75EI\theta_C = 0$$

$$3.5EI\theta_B + 0.75EI\theta_C = 37.17 \quad \text{— (A)}$$

$$-33.83 + 1.5EI\theta_C + 0.75EI\theta_B + 45.56 + 2EI\theta_C = 0$$

$$3.5EI\theta_C + 0.15EI\theta_B = -11.75$$

or $\left[0.75EI\theta_B + 3.5EI\theta_C = -11.73\right]$ — (B)

Solve eq^n. A and B.

we get

$$EI\theta_B = 11.88$$

$$EI\theta_C = -5.89.$$

Step IV) find END Moment :>

$M_{AB} = -10 + 11.88 = 1.88$ KNm.

$M_{BA} = 10 + 2\times 11.88 = 33.76$ KNm

$M_{BC} = -47.17 + 1.5\times 11.88 + [0.75\times(-5.89)]$

$M_{BC} = -33.77$ KNm

$M_{CB} = -33.83 - (1.5\times 5.89) + (0.75\times 11.88)$

$M_{CB} = -33.76$ KNm.

$M_{CD} = 45.56 - (2\times 5.89)$

$= 33.77$ KNm.

$M_{DC} = 56.8 - 5.8$

$= 51$ KNm.

Solve the eq^n. by MD MTD :→

Sol^n:→ Similarly we find fixed end Moment first

$\therefore M_{AB} = -10 KNm$

$M_{BA} = 10 KNm$

$M_{BC} = -47.17 KNm$

$M_{CB} = -33.83 KNm.$

$M_{CD} = 45.56 KNm$

$M_{DC} = 86.8 KNm.$

Step 2 :→

Joint	Member	K	ΣK	D.A
B	BA	$\frac{2I}{4} = 2I/4$	$3.5I/4$	0.57
	BC	$\frac{1.5I}{4}$		0.43
C	CB	$1.5I/4$	$3.5I/4$	0.43
	CD	$2I/4$		0.57

Step 3 :- END Moment :-

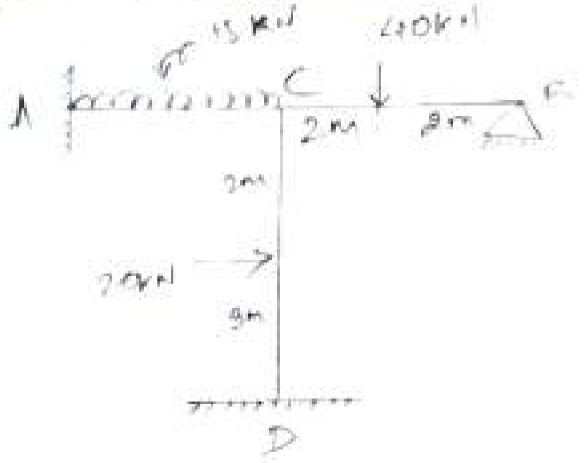
Joint	A	B		C		D
D.F	1	0.57	0.43	0.43	0.57	1
F.E.A	-10	10	-47.17	-33.83	45.56	56.8
Balance		+21.19	+15.98	-5.04	-6.69	
COM	+10.595		-2.52	+7.99		+3.34
Balance		+1.44	1.08	-3.44	-4.55	
COM	+0.72		-1.72	+0.54		-2.28
Balance		+0.98	0.74	-0.23	-0.31	
COM	+0.99		+0.12	+0.23		-0.18
Balance		+0.068	-0.052	+0.18	-0.21	
COM	+0.034		-0.08	+0.03		-0.1
Balance		+0.046	+0.034	-0.013	-0.017	
T.M	+1.84	+33.72	-33.72	-33.78	+33.78	50.6

Step 4 :- Reaction force calculation :-

Member	1.8m A 20kN 4m 33.72 B		B 5kN/m C		C 1m 15kN 3m D	
Reaction due to Load	10↑	10↑	10↑	10↑	11.75↑	3.35↑
Reaction due to moment	8.89↓	8.89↑	16.88↑	16.88↓	21.2↓	21.2↑

VA = 1.11 kN   VB = 45.77 kN   VC = -16.13 kN   VD = 24 kN

Q. Analyse the frame and Draw BMD by slope deflection Method.



Sol^n:- Find fixed end Moments :-

$$M_{FAC} = \frac{-wl^2}{12} = \frac{-15\times 2^2}{12} = -5\,kN.m.$$

$$M_{FCA} = \frac{wl^2}{12} = \frac{15\times 2^2}{12} = 5\,kNm.$$

$$M_{FCB} = -\frac{wl}{8} = -\frac{40\times 4}{8} = -20\,kNm.$$

$$M_{FBC} = +\frac{wL}{8} = 20\,kNm.$$

$$M_{FDC} = -\frac{wab^2}{l^2} = \frac{-20\times 3\times 2^2}{5^2} = -9.6\,kN\,m.$$

$$M_{FCD} = \frac{wa^2b}{L} = \frac{20\times 3^2\times 2}{5^2} = 14.4\,kNm.$$

Apply slope Deflection equation on each span :-

$$M_{AC} = M_{AC}^f + \frac{2EI}{L}(2\theta_A + \theta_C)$$

$$M_{AC} = -5 + \frac{2EI}{2}(2\times 0 + \theta_C) \quad \left\{ \text{Note } \theta_A = 0 \text{ fixed} \right\}.$$

$$M_{AC} = -5 + EI\theta_C. \quad \text{--- (1)}$$

$-\frac{3\delta}{L} = 0$
Not given.

$$M_{CA} = M_{CA}^{f} + \frac{2EI}{L}(2\theta_C + \theta_A)$$

$$M_{CA} = 5 + \frac{2EI}{2}(2\theta_C + 0)$$

$$M_{CA} = 5 + 2EI\theta_C \quad —(11)$$

for span CB

$$M_{CB} = M_{FCB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$M_{CB} = -20 + \frac{2EI}{4}(2\theta_C + \theta_B)$$

$$M_{CB} = -20 + \tfrac{1}{2}EI\theta_B + EI\theta_C \quad —(3)$$

$\{\because \theta_B$ is not fixed$\}$.

$$M_{BC} = +M_{FBC} + \frac{2EI}{L}(2\theta_B + \theta_C).$$

$$M_{BC} = 20 + \frac{2EI}{4}(2\times\theta_B + \theta_C)$$

$$M_{BC} = 20 + \tfrac{1}{2}EI\theta_C + EI\theta_B. \quad —(4)$$

for span CD

$$M_{CD} = M_{CD}^{f} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$\{\theta_D = 0$ fixed$\}$.

$$M_{CD} = 14.4 + \frac{2EI}{5}(2\theta_C + 0)$$

$$M_{CD} = 14.4 + \tfrac{4}{5}EI\theta_C \quad —(5)$$

$$M_{DC} = M_{fDC} + \frac{2EI}{l}(2\theta_D + \theta_C)$$

$$M_{DC} = -9.6 + \frac{2EI}{5}(2\times 0 + \theta_C)$$

$$M_{DC} = -9.6 + \tfrac{2}{5}EI\theta_C \quad —(6)$$

→

Apply condition of equilibrium :->

$M_{CA} + M_{CD} + M_{CB} = 0$ — (i)

$\boxed{M_{BC} = 0}$ — (ii)

$5 + 2EI\theta_C + 14.4 + \frac{4}{5}EI\theta_C + (-20) + \frac{1}{2}EI\theta_B + EI\theta_C$

$\frac{1}{2}EI\theta_B + 2 + \frac{4}{5} + 1(EI\theta_C) = -5 - 14.4 + 20$ — (a)

$M_{BC} = 20 + EI\theta_B + \frac{1}{2}EI\theta_C = 0$

~~20 + EIθB~~

$EI\theta_B + \frac{1}{2}EI\theta_C = -20$ — (b)

solve eq^n. (a) and (b), we get

$EI\theta_B = -21.49$

$EI\theta_C = 2.98$

Step (iv). Final moment- put $EI\theta_B$ and $EI\theta_C$ in (1), (2), (3), (4), (5) and (6), we get.

$EI\theta_B = -21.49$

$EI\theta_C = 2.98$

eq^n (1)

$M_{AC} = -5 + EI\theta_C$

$M_{AC} = -5 - 2.98$

$\boxed{M_{AC} = -2.02 \text{ KNm.}}$

eq^n (2)

$\boxed{M_{CA} = 5 + 2(2.98) = 10.96 \text{ KNm.}}$

eq^n (3)

$M_{CB} = -20 + \frac{1}{2}(21.49) + 2.98$

$\boxed{M_{CB} = -27.76 \text{ kNm}}$

eqⁿ ④

$$\boxed{M_{BC} = 20 + (-21.49) + \tfrac{1}{2}(2.98) = 0}$$

eqⁿ ⑤

$$M_{CD} = 14.4 + \tfrac{4}{5}(EI\theta_C)$$

$$M_{CD} = 14.4 + \tfrac{4}{5}(2.98)$$

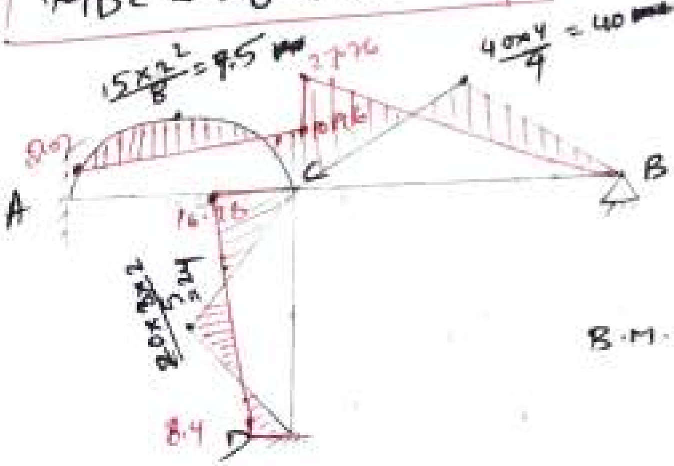
$$\boxed{M_{CD} = 16.78\, kNm}$$

eqⁿ ⑥

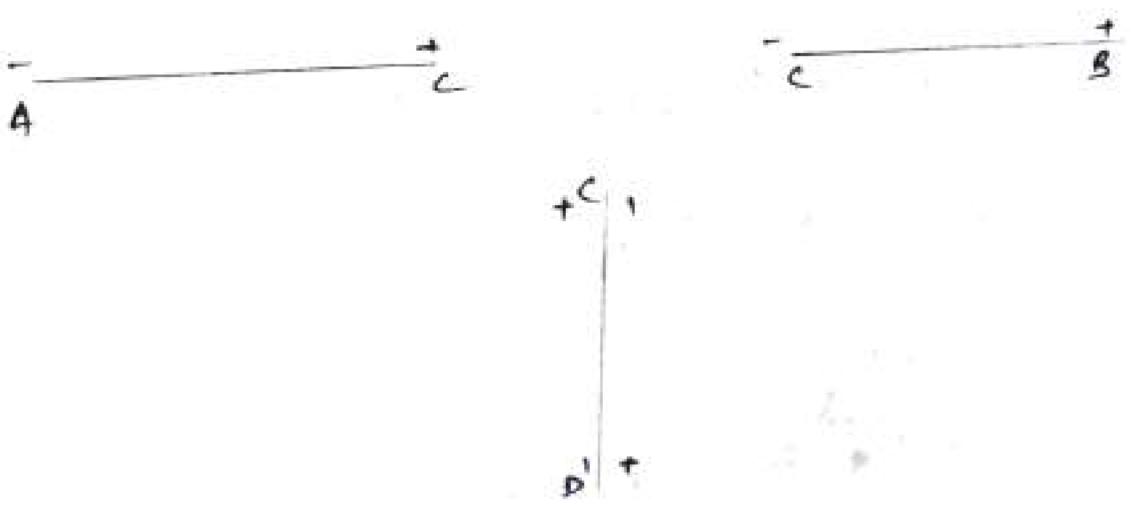
$$M_{DC} = -9.6 + \frac{2EI\theta_C}{5}$$

$$M_{DC} = -9.6 + \frac{2}{5}(2.98)$$

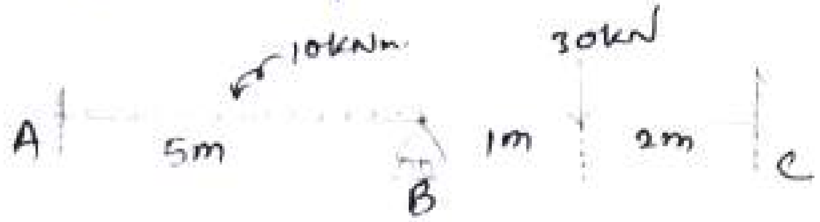
$$\boxed{M_{DC} = -8.4\, kNm}$$



B.M.D.



Q. Using slope deflection method Analyse the frame loaded and support as shown. Also, draw BMD & Deflected shape.



1) Fixed End Moment –

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-10 \times 5^2}{12} = -20.833\,kNm.$$

$$M_{FBA} = \frac{wL^2}{12} = 20.833\,kNm$$

$$M_{FBC} = -\frac{Wab^2}{L^2} = \frac{-30 \times 1 \times 2^2}{3} = -13.33\,kNm.$$

$$M_{FCB} = \frac{Wa^2b}{L^2} = \frac{-30 \times 1^2 \times 2}{3} = 6.667\,kNm.$$

Step 2 :- Apply SDM for each span :->

$$M_{AB} = M_{AB}^F + \frac{2EI}{L}\left(2\theta A + \theta B - \cancel{\frac{3\delta}{L}}^{\,0}\right)$$

$$M_{AB} = -20.833 + \frac{2EI}{5}\left(2 \times 0 + \theta B - 0\right)$$

$\left\{ \theta = 0 \text{ ; fixed} \right.$

$$M_{AB} = -20.833 + \frac{2}{5} EI\theta_B. \longleftarrow (1)$$

$$M_{BA} = M_{BA}^F + \frac{2EI}{L}\left(2\theta B + \theta A - \cancel{\frac{3\delta}{L}}^{\,0}\right)$$

$$M_{BA} = 20.833 + \frac{2EI}{5}\left(2\theta B + 0 - 0\right)$$

$$M_{BA} = 20.833 + \frac{4}{5} EI\theta B. \longleftarrow (2)$$

SDM for BC :→

$$M_{BC} = M_{BC}^f + \frac{2EI}{L}\left(2\theta_B + \overset{0}{\theta_C} - \cancel{\frac{3\delta}{L}}\right)$$

$$M_{BC} = -13.33 + \frac{2EI}{3}\left(2\theta_B + 0 - 0\right)$$

$$M_{BC} = -13.33 + \frac{4}{3}EI\theta_B \quad \text{— (3)}$$

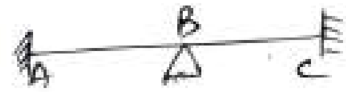
$$M_{CB} = M_{CB}^f + \frac{2EI}{L}\left(2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}\right)$$

$$M_{CB} = 6.667 + \frac{2EI}{3}\left(2\times 0 + \theta_B - 0\right)$$

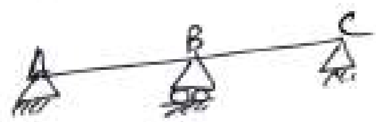
$$M_{CB} = 6.667 + \frac{2}{3}EI\theta_B \quad \text{— (4)}$$

step 3 :→ Apply condition of Equillibrium :→

$\boxed{M_{BA} + M_{BC} = 0}$ ⟶



$\boxed{M_{AB} = 0 \; ; \; M_{CB} = 0}$ ⟷



$$M_{BA} + M_{BC} = 0$$

$$\left[20.833 + \frac{4}{5}EI\theta_B\right] + \left[-13.33 + \frac{4}{3}EI\theta_B\right]$$

$$EI\theta_B = -3.57$$

step 4 : find final moment

Put $EI\theta_B$ in eqⁿ (1), (2), (3), (4), (5), & (6)

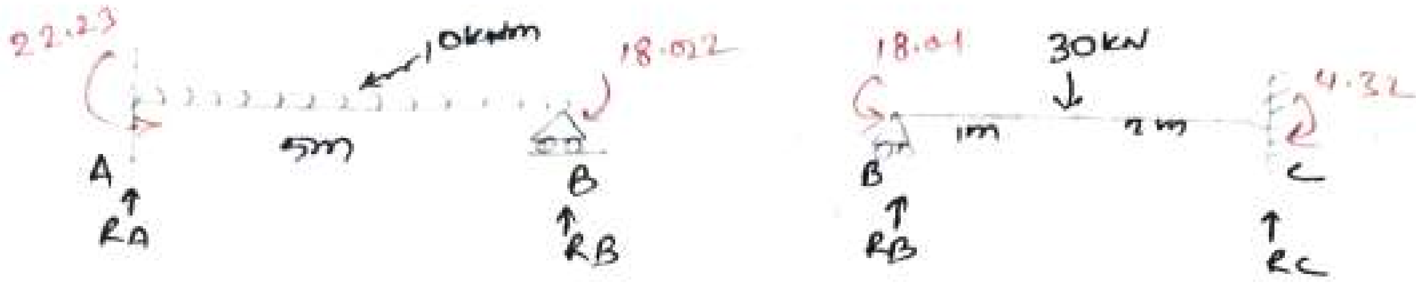
$$M_{AB} = -20.833 + \frac{2}{5}\times(-3.57) = -22.23 \text{ kNm}$$

$$M_{BA} = 20.833 + \frac{4}{5}(-3.57) = 18.022 \text{ kNm}$$

$$M_{BC} = -13.33 + \frac{4}{3}(-3.57) = -18.01 \text{ kNm}$$

$$M_{CB} = 6.667 + \frac{2}{3}(-3.57) = 4.32 \text{ kNm.}$$

⑤ Find support Reaction



$\Sigma M_A = 0$

$= -22.23 + 10 \times 5 \times \frac{5}{2} + 18.022 - R_B \times 5 = 0$

$\boxed{R_B = 24.15 \text{ kN}}$

$\Sigma F_y = 0 \uparrow +$

$R_A + R_B - 10 \times 5 = 0$

$R_A + 24.15 - 50 = 0$

$\boxed{R_A = 25.85 \text{ kN}}$

$\Sigma M_B = 0$

$-18.01 + 30 \times 1 - R_C \times 3 + 4.32 = 0$

$\boxed{R_C = 5.43 \text{ kN}}$

$\Sigma F_y = 0 \uparrow$

$R_B + 5.43 - 30 = 0$

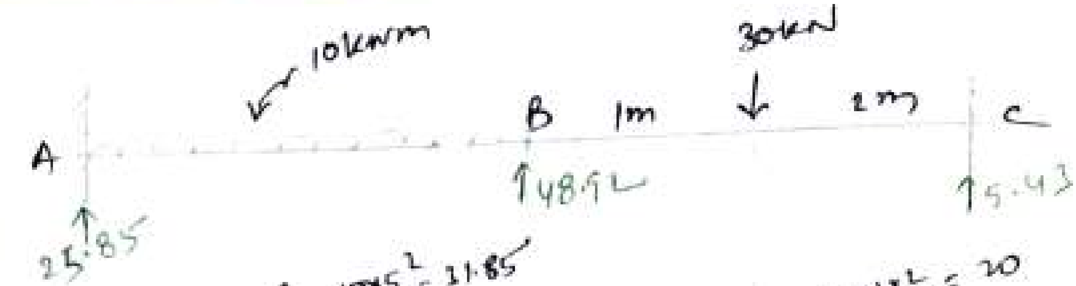
$\boxed{R_B = 24.51}$

Final support Reaction :-

$R_A = 25.85 \text{ kN}$

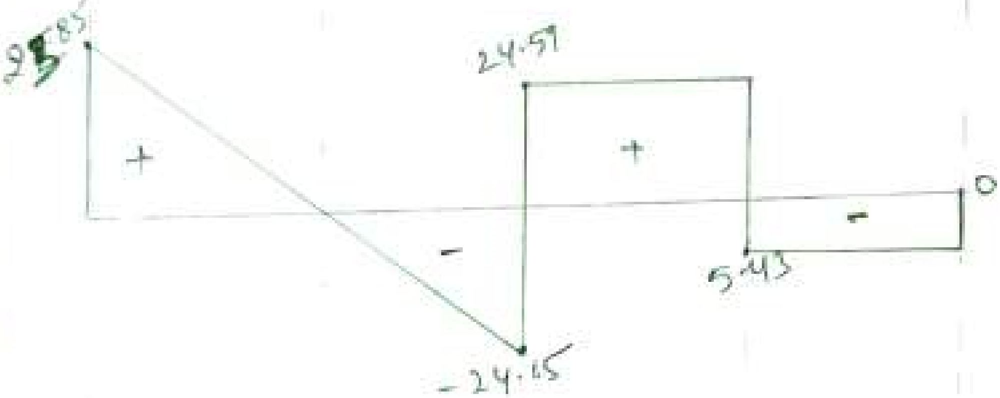
$R_B = 24.15 + 24.57 = 48.72 \text{ kN}$

$R_C = 5.43 \text{ kN}$



$\frac{wl^2}{8} = \frac{10\times5^2}{8} = 31.85$

$\frac{Wab}{L} = \frac{30\times1\times2}{3} = 20$



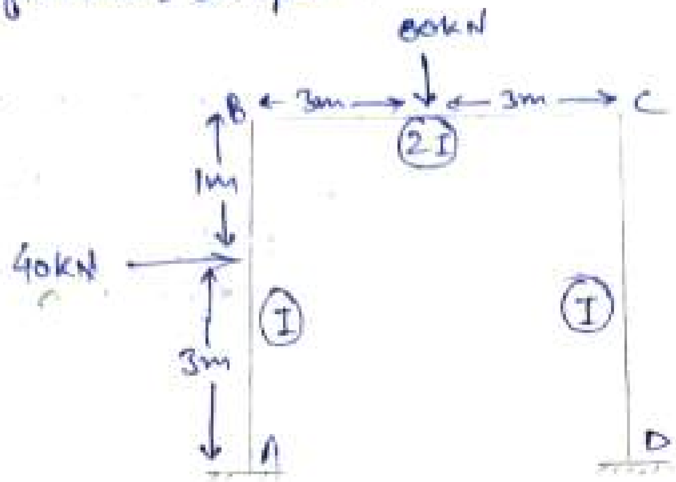
$22.85 - 10\times5 = -24.15$

$-24.15 + 48.92 = 24.57$

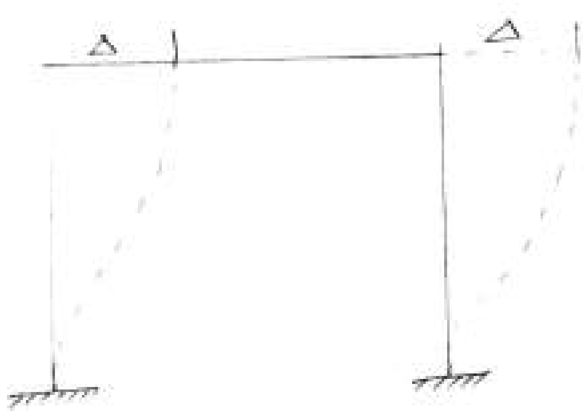
$24.57 - 30 = -5.43$

$-5.43 + 5.43 = 0$

Q. Using slope deflection method Analyse the frame loaded & supported as shown also, draw BMD. and deflection shape.



Solⁿ:



Find End Moment :→

$$M_{FAB} = -\frac{40\times3\times1^2}{4^2} = -7.5\,kNm.$$

$$M_{FBA} = \frac{40\times3^2\times1}{4^2} = 22.5\,kNm.$$

$$M_{FBC} = -\frac{80\times6}{8} = -60\,kNm.$$

$$M_{FCB} = 60\,kNm.$$

$$M_{FCD} = M_{FDC} = 0.$$

$M_{AB} = -7.5 + \frac{2EI}{4}\left(2\cancel{\theta_A}^{0} + \theta_B - \frac{3\delta}{4}\right)$

$M_{AB} = -7.5 + \frac{EI}{2}\left(\theta_B - \frac{3\delta}{4}\right)$

$M_{BA} = 22.5 + \frac{2EI}{4}\left(2\theta_B + \theta_A - \frac{3\delta}{4}\right)$

$M_{BA} = 22.5 + \frac{EI}{2}\left(2\theta_B - \frac{3\delta}{4}\right)$

$M_{BC} = -60 + \frac{2E(2I)}{6}\left[2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}^{0}\right]$ { Beam related displace=0

$M_{BC} = -60 + \frac{4EI}{6}(2\theta_B + \theta_C)$

$M_{CB} = 60 + \frac{2EI}{3}\left(2\theta_C + \theta_B - \cancel{\frac{3\delta}{l}}^{0}\right)$

$M_{CD} = \cancel{M_{FCD}}^{0} + \frac{2EI}{4}\left(2\theta_C + \cancel{\theta_D}^{0} - \cancel{\frac{3\delta}{l}}^{0}\right)$

$M_{DC} = \frac{EI}{2}\left(2\cancel{\theta_D}^{0} + \theta_C - \frac{3\delta}{l}\right)$

θ_B, θ_C & δ = three unknown joint.

Equillibrium at B; $M_B = 0$

$\Rightarrow 22.5 + \frac{2EI}{4}\left(2\theta_B - \frac{3\delta}{4}\right) + (-60) + \frac{4EI}{6}(2\theta_B + \theta_C) = 0$

$\Rightarrow 22.5 + EI\theta_B - 0.375EI\delta - 60 + 1.33EI\theta_B + 0.67EI\theta_C = 0$

$\Rightarrow 2.33EI\theta_B + 0.67EI\theta_C - 0.375EI\delta = 37.5$ — (I)

$\boxed{M_C = 0}$

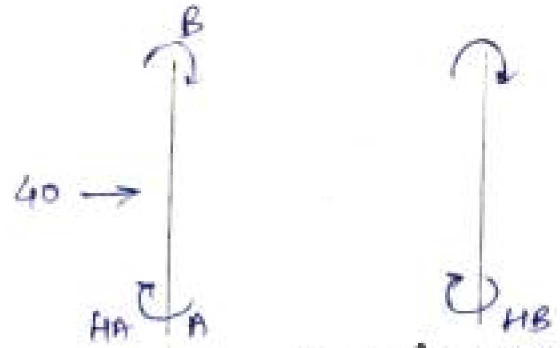
$60 + \frac{4EI}{6}(2\theta_C + \theta_B) + \frac{2EI}{4}\left(2\theta_C - \frac{3\delta}{4}\right) = 0$

$\Rightarrow 60 + 2.33EI\theta_C + 0.67EI\theta_C - 0.375EI\delta = 0$ — (II)

$0.67EI\theta_C + 2.33EI\theta_C - 0.375EI\delta = -60$

$\Sigma H = 0$

$H_A + H_D + 40 = 0$



$H_A + H_D + 40 = 0$

$\Rightarrow 22.5 + 0.5EI(2\theta_B - 0.75\delta)$

$+ \dfrac{7.5 + 0.5EI(\theta_B - 0.75\delta)}{4} + \dfrac{0.5EI(2\theta_C - 0.75\delta) + 0.5EI(\theta_C - 0.75\delta)}{4}$

$+ 40 = 0$

$\Rightarrow \boxed{EI\theta_B + EI\theta_C - 1.5EI\delta = -90.}$

$EI\theta_B = 39.25$

$EI\theta_C = -19.25$

$EI\delta = 110$

By putting this value SD eq^n.

$M_{AB} = -29.12$ KNm

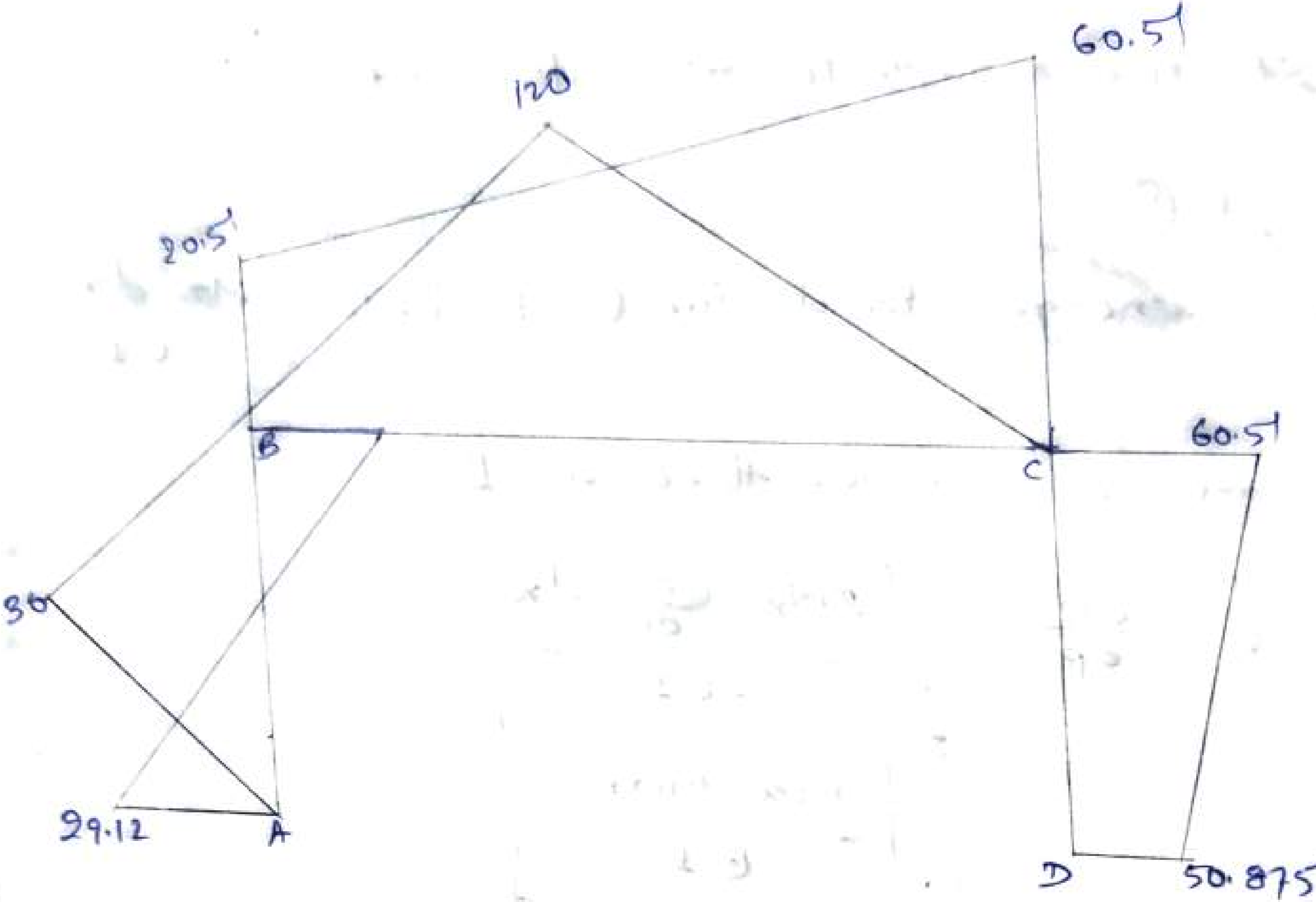
$M_{BA} = 20.5$ KNm

$M_{BC} = -20.5$ KNm

$M_{CB} = 60.57$ KNm

$M_{CD} = -60.57$ KNm

$M_{DC} = -50.875$ KNm



60.51
120
20.51
B
C
60.51
30
29.12
A
D
50.875

Deflections in frame :-

{Strain energy MTD (or) Unit load MTD} :-

Concept ① :-

Strain energy stored due to $\boxed{B.M = U = \int \frac{M_x^2 \, dx}{2EI}}$

② From Castigliano's theorem II

$$\delta = \frac{\partial U}{\partial P} = \int \frac{2M_x \cdot \frac{\partial M}{\partial P} \cdot dx}{2EI}$$

$$= \boxed{\int \frac{M_x \cdot M \, dx}{EI}}$$

Where,

M_x = B.M at any section xx due to applied load.

$M = \frac{\partial M_x}{\partial P}$ = B.M at x-x due to unit load.

{applied at the point where we want to find deflection.}.

③ From find slope at any point,

$$\theta = \frac{\partial U}{\partial M} = \int \frac{2 \cdot M_x \times \frac{\partial M_x}{\partial M} \cdot dx}{2EI}$$

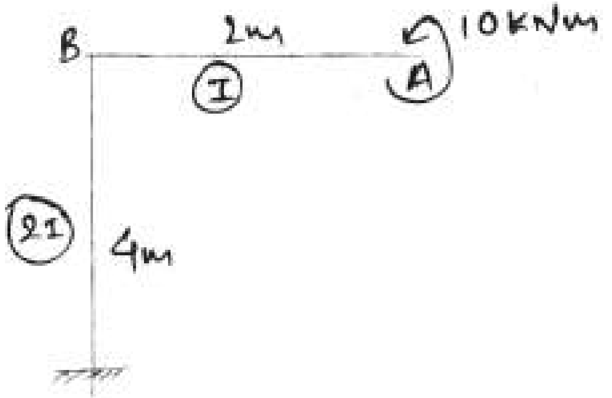
$$= \int \frac{M_x \cdot M \cdot dx}{EI}$$

Where, M_x = B.M at x-x due to applied loads.

$M = \frac{\partial M_x}{\partial M}$ = B.M at x-x due to unit Couple.

{applied at the point where we want to find slope}.

Q. For the cantilever frame shown in fig. slope of 'A' is.



Note :-

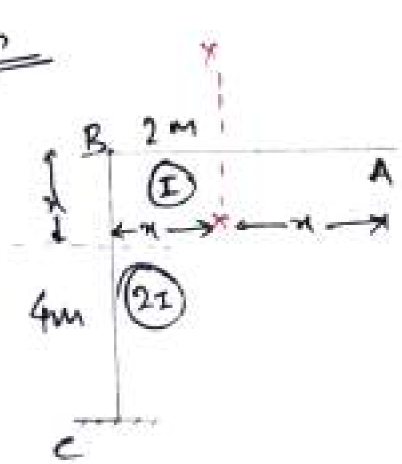
$$\theta_A = \int \frac{M_x \cdot M \cdot dx}{EI}$$

where,

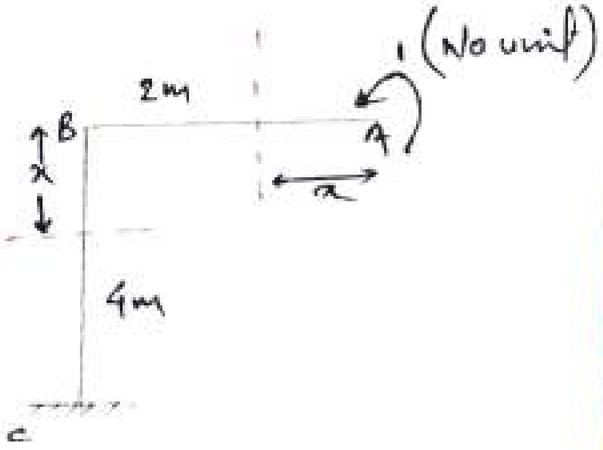
M_x = B.M at x-x due to applied loads

M = B.M at xx due to unit couple at 'A'.

Sol^n :-



M_x value



'm' values.

Member	M_x	M	$\theta_A = \int \frac{M_x \cdot m \cdot dx}{EI}$
AB	+10	+1	$\int_0^2 \frac{10 \times 1 \times dx}{EI} = \frac{10 \times 2}{EI} = \frac{20}{EI}$
BC	+10	1	$\int_0^4 \frac{10 \times 1 \times dx}{2EI} = \frac{10 \times 4}{2EI} = \frac{20}{EI}$

$$\boxed{\theta_A = \frac{40}{EI}}$$

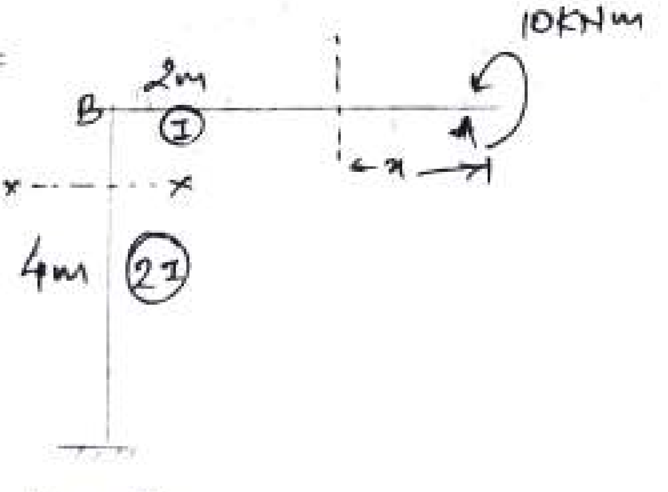
Q. In the above Pb find vertical deflection of point 'A'.

Note :- $\delta_{VA} = \int \frac{M_x \cdot m \cdot dx}{EI}$

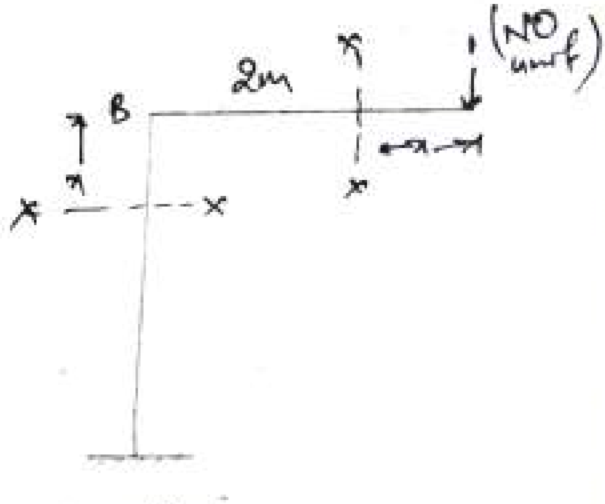
M_x = B.M at xx due to applied loads.

M = B.M at xx due to unit verticle load 'A'.

Sol^n



Mx value



m value

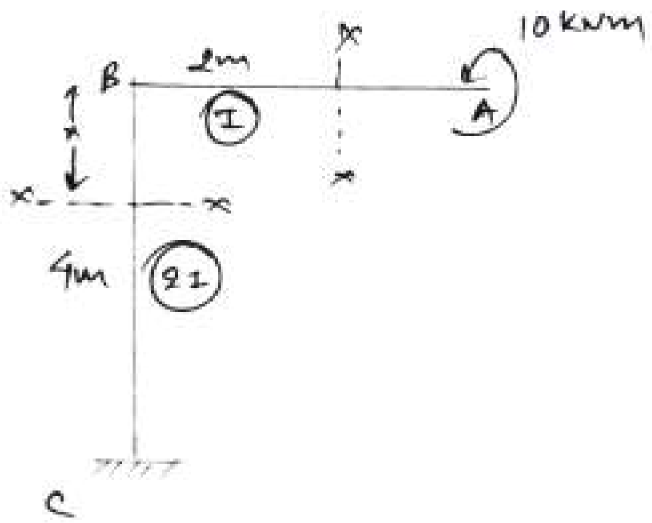
Member	M_x	M	$\delta_{VA} = \int \frac{M_x \cdot M \cdot dx}{EI}$
AB	+10	$-1 \times x$	$\int_0^2 \frac{10 \times (-x)\,dx}{EI} = \frac{-10 \times 2^2}{2EI} = \frac{-20}{EI}$
BC	+10	-1×2	$\int_0^4 \frac{10 \times (-2)\,dx}{E(2I)} = \frac{-20 \times 4}{2EI} = \frac{-40}{EI}$

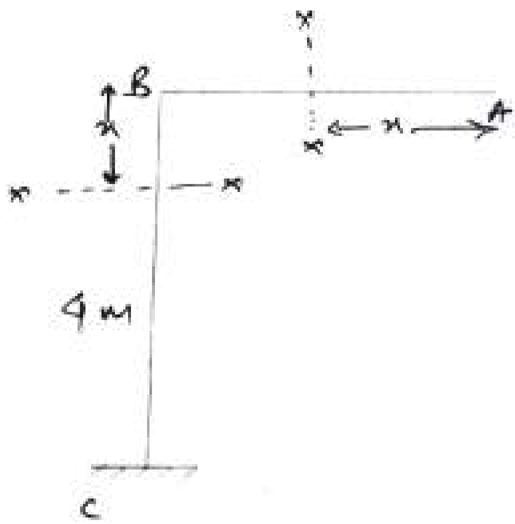
$$\delta_{VA} = \frac{-60}{EI}$$ {-ve sign indicates deflection opposite to the diretion of unit loads. i.e., δ_{VA} is upward}

Q. In the above Pb find Horizontal deflection of point 'A'.

Note :- $\delta_{HA} = \int \frac{M_x \cdot m \cdot dx}{EI}$

where, m = B.M at x-x due to unit Horizontal load at A.





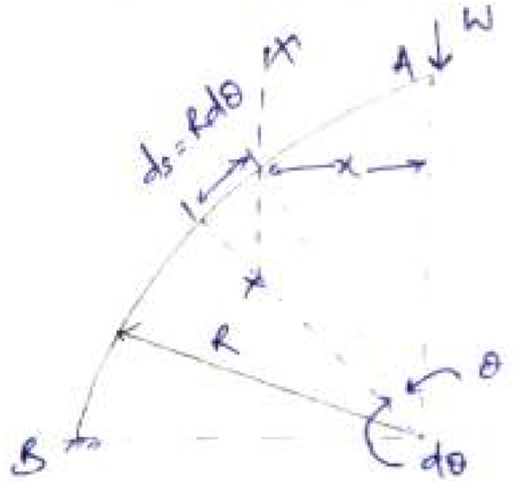
M_x value .. .. m value.

Member	M_x	m	$\delta_{HA} = \int_0^l \frac{M_x \cdot m \cdot dx}{EI}$
AB	+10	0	0
BC	+10	$(-1 \times x)$	$\int_0^4 \frac{10 \times (-x)\,dx}{E(2I)} = \frac{-10 \times 4^2}{2 \cdot E(2I)} = \frac{-40}{EI}$

$\delta_{HA} = \frac{-40}{EI}$ {-ve sign indicates Horizontal deflection is opposite to unit load, so δ_{HA} is leftwards.}.

Q. IES 2014 [Marks].

Find vertical deflection of 'A'.



Note :- $\delta_{VA} = \int \frac{M_x \cdot m\, ds}{EI}$

{ ds → bcz strain energy stored in Curved Member should be considered }.

$x = R\sin\theta$

$M_x = -10 \times x$

$= -W \times R\sin\theta$

$m = -1 \times x = -1 \times R\sin\theta$

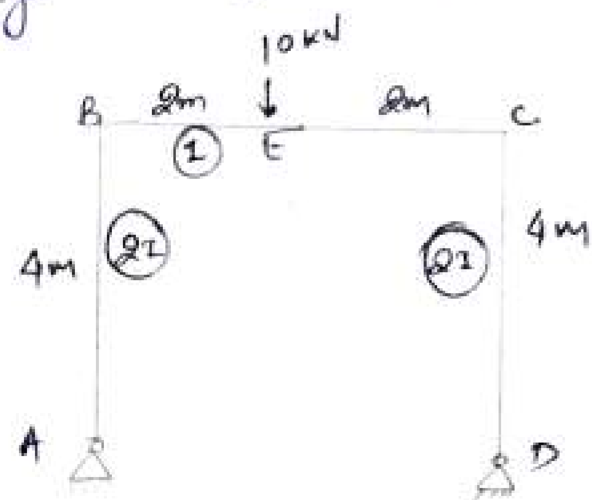
$ds = R\,d\theta$

$$\delta_A = \int \frac{M_x \cdot m \cdot ds}{EI} = \int_0^{\pi/2} \frac{(-W \cdot R\sin\theta)(-1 R\sin\theta) R\,d\theta}{EI}$$

$$= \frac{WR^3}{EI} \int_0^{\pi/2} \sin^2\theta \, d\theta.$$

$$= \frac{WR^3}{EI} \left[\pi/4\right].$$

Q: Find the Horizontal deflection of joint 'D' as shown in fig.



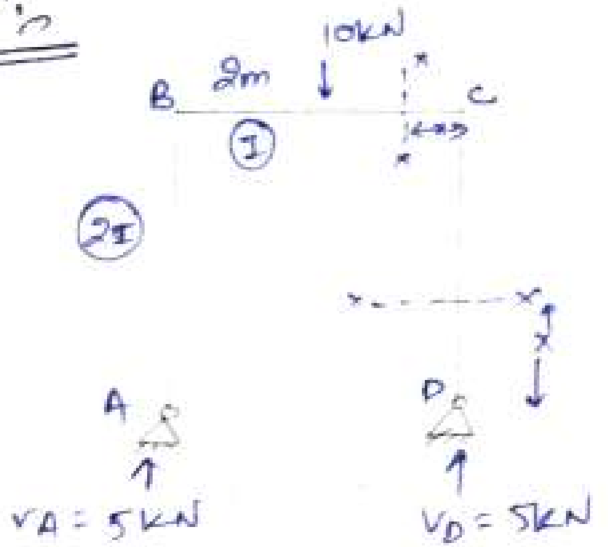
Note:-

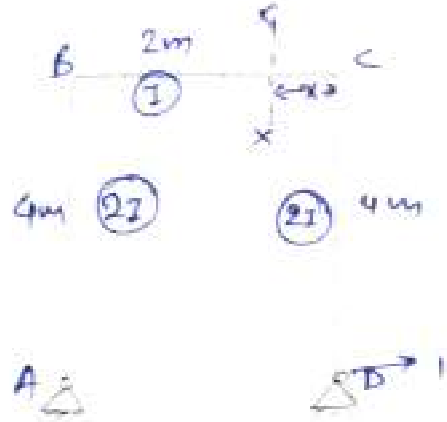
$$\delta_{HD} = \int \frac{M_x \cdot m \cdot dx}{EI}$$

M_x = B.M at xx due to applied load.

m = B.M at xx due to unit Horizontal load at 'D'.

Sol^n:-



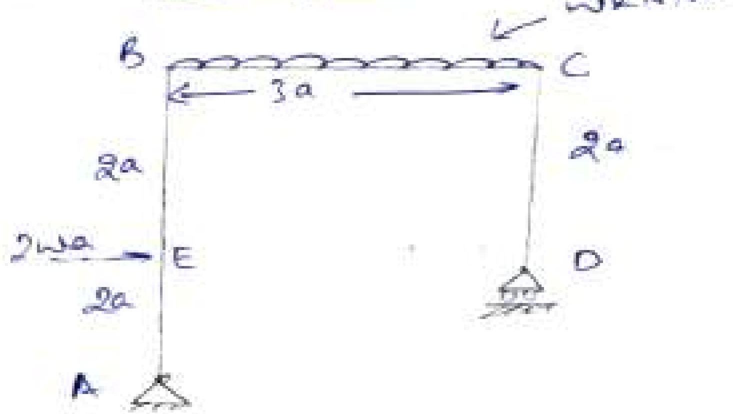


Member	Mx	M	$\delta_{HD} = \int \frac{M_x \cdot M\, dx}{EI}$
CD	0	$+1 \times x$	0
CB	$+5x$	$+1 \times 4$	$\int_0^2 \frac{5x \times 4}{EI} \cdot dx = \left[\frac{20x^2}{2EI}\right]_0^2 = \frac{80}{2EI} = \frac{40}{EI}$

$$\delta_{HD} = \frac{2 \times 40}{EI} = \frac{80}{EI}$$

{+ve sign indicates deflection is in the direction of unit load}.

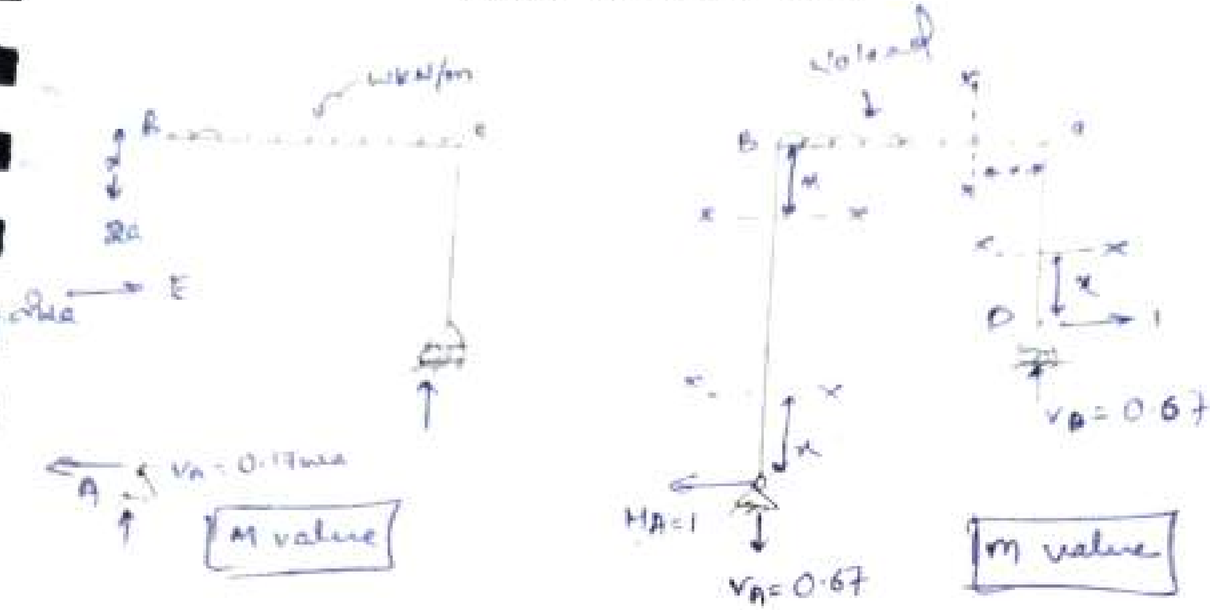
Q. Find Horizontal deflection at joint D shown in fig:-



Note :-

$$\delta_{HD} = \int_{x} \frac{M_x \cdot m \, dx}{EI}$$

where, m = BM at xx due to unit load at 'D'.



Step1 :- Find support Reaction :-

for M_x diagram.

$\Sigma x = 0 \Rightarrow -H_A + 2wa = 0$

$$\boxed{H_A = 2wa}$$

$\Sigma M_D = 0 \Rightarrow +V_A \times 3a + H_A \times 2a - (w \times 3a) \times \frac{3a}{2} = 0$

$$\boxed{V_A = 0.17wa}$$

$\Sigma y = 0 \Rightarrow +V_A + V_D - w \times (3a) = 0$

$$\boxed{V_D = 2.83\,wa}$$

for m diagram.

$\Sigma x = 0 \Rightarrow -H_A + 1 = 0 \Rightarrow H_A = 1$

$\Sigma M_D = 0$

$(+1 \times 2a) - V_A \times 3a = 0$

$V_A = 0.67 (\downarrow)$

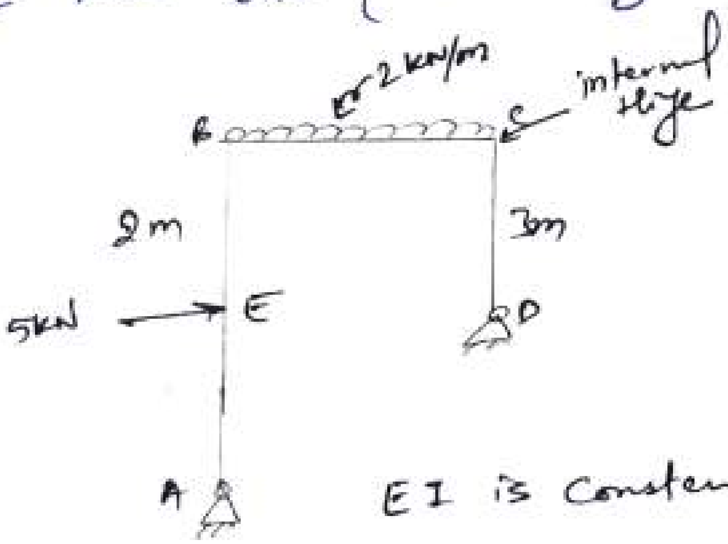
$V_D = 0.67 (\uparrow)$

→

Step 2 :-
$$\delta_{HD} = \int \frac{M_x \cdot m \cdot dx}{EI}$$

Member	M_x	M	$\delta_{HD} = \int \frac{M_x \cdot m\, dx}{EI}$
CD	0	$+(1\times x)$	0
BC	$+0.83\,wa\,\cancel{\mathrm{b}}\,x - \frac{w\cdot x\cdot x}{2}$ $= 2.83\,wax - \frac{wx^2}{2}$	$(0.67\times x) + (1\times 2a)$ $= (2a + 0.67x)$	$\int_0^{3a} \frac{\left(2.83\,wax - \frac{wx^2}{2}\right)(2a+0.67}{EI}$
BE	$(2.83\,wa)3a - \frac{w\times(3a)^2}{2}$ $= 4wa^2$	$(0.67\times 3a) + 1(2a - x)$ $= 2a + 2a - x$ $= (4a - x)$	$\int_0^{2a} \frac{(4wa^2)(4a - x)\,dx}{EI}$
AE	$+(2wa)x$	$+(1\times x) + V_A \times 0$	$\int_0^{2a} \frac{(2wax)\,x\,dx}{EI}$ $= \frac{2wa\cdot(2a)^3}{3EI} = \frac{16wa^4}{3EI}$

Q. Find δ_{HC} {20 Marks}.

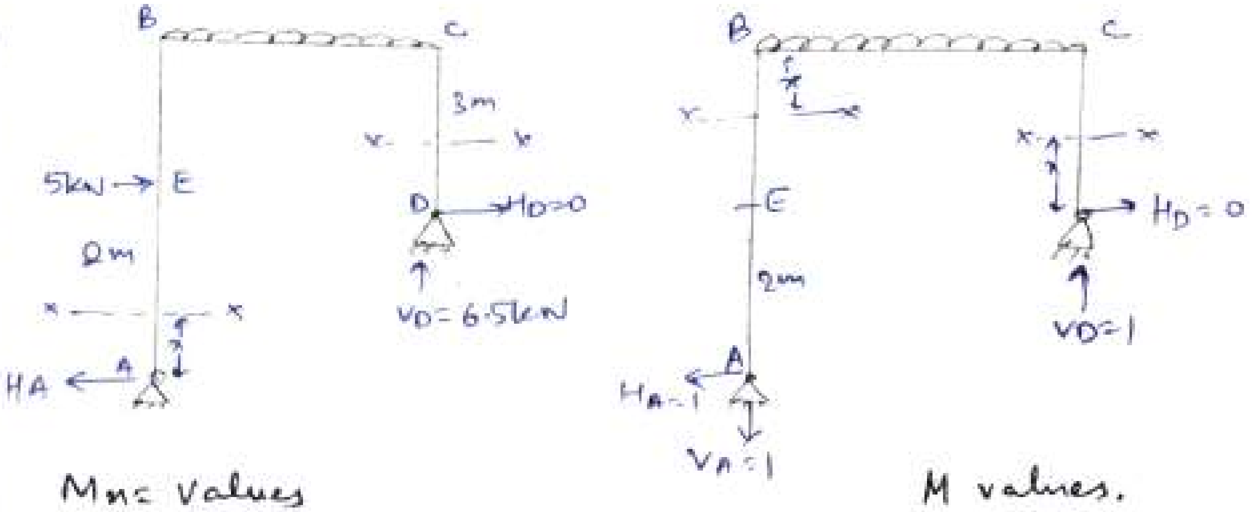


EI is Constant.

Note :-
$$\delta_{HC} = \int \frac{M_x \cdot m\, dx}{EI}$$

where, m = B.M at x-x due to unit load at 'C'.

Sol^n :-



Step 1 :- Support Reaction :-

for Mx diagram

$\Sigma M_D = 0 \Rightarrow +V_A \times 4 + 5 \times 1 + 5 \times 1 - (2 \times 4) \times 2 = 0$

$4V_A = 6$

$\boxed{V_A = 1.5kN}$

$\Sigma y = 0 \Rightarrow V_A + V_D - 2 \times 4 = 0$

$\boxed{V_D = 6.5kN}$

for m diagram :-

$\Sigma M_D = 0$

$\underbrace{1 \times 3}_{at\,C} + \underbrace{1 \times 1}_{at\,A} - V_A \times 4 = 0$

$V_A = 1 \; (\downarrow)$

$\Sigma y = 0 \Rightarrow -V_A + V_D = 0$

$\boxed{V_D = 1}$

→

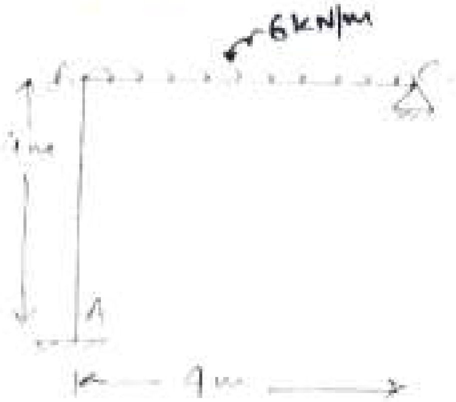
Step II $\quad \delta_{HC} = \int \frac{M_x \cdot m \cdot dx}{EI}$

Member	M_x	m	$\delta_{HC} = \int \frac{M_x \cdot m\,dx}{EI}$
CD	0	0	0
BC	$+(6.5x) - \frac{2x^2}{2}$ $= 6.5x - x^2$	$(1 \times x)$	$\int_0^4 \frac{(6.5x - x^2) \cdot x\,dx}{EI}$ $= \frac{24}{3EI}$
BE	$6.5 \times 4 - \frac{2 \times 4 \times 4}{2} = 10$	$+1 \times 4 - 1 \times x$ $= (4 - x)$	$\int_0^2 \frac{10(4-x)\,dx}{EI} = \frac{60}{EI}$
AE	$+(5 \times x)$	$+(1 \times x)$	$\int_0^2 \frac{(5x)x\,dx}{EI} = \frac{5 \times 2^3}{3EI}$ $= \frac{40}{3EI}$

$$\delta_{HC} = +\frac{148}{EI}$$

{+ve sign indicate that the deflection is in the direction is in the direction of unit load i.e; towards rigid.}.

Q: Analyse the portal frame shown in fig by strain energy method and draw BMD, EI is constant.



Solution: —

Analyse the portal frame by moment distribution Method. Distribution factors.

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution factor
B	BA	$\frac{I}{4} = \frac{4I}{16}$	$7I/16$	$4/7$
	BC	$\frac{3}{4} \times \frac{I}{4} = \frac{3I}{16}$		$3/7$

Find fixed end Moments:-

$\bar{M}_{AB} = 0$

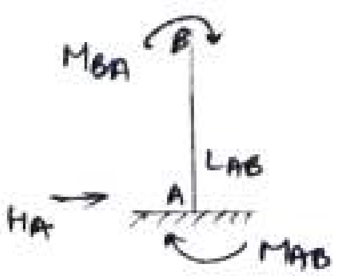
$\bar{M}_{AB} = 0$

$\bar{M}_{BC} = \frac{6 \times 4^2}{12} = -8\,kN\text{-}m$

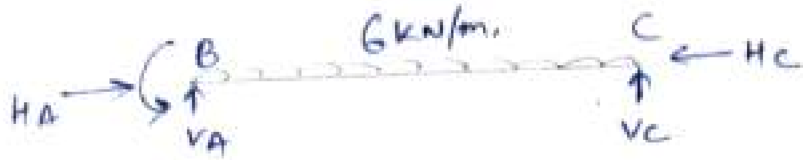
$\bar{M}_{CB} = +8\,kNm.$

A		4/7	3/7	C
0	0	-8	+8
		-4	-8
0	0	-12	0
	+6.857	+5.143	
+3.429			
+3.429	+6.857	-6.857	0

Reaction:-



$$H_A = \frac{M_{AB} + M_{BA}}{L_{AB}} = \frac{3.429 + 6.857}{4} = 2.572\,kN(\rightarrow)$$



$\Sigma M_c = 0$

$\Rightarrow V_A \times 4 - 6.857 - 6\times 4 \times 2 = 0$

$V_A = 13.71\,kN$

Also, $H_A - H_c = 0$

$H_A = H_c = 2.572\,kN\ (\leftarrow)$

$V_c = 6\times 4 - V_A = 24 - 13.71 = 10.29\,kN$

find Bending moment in $BC = \frac{6\times 4^2}{8} = 12\,kNm.$

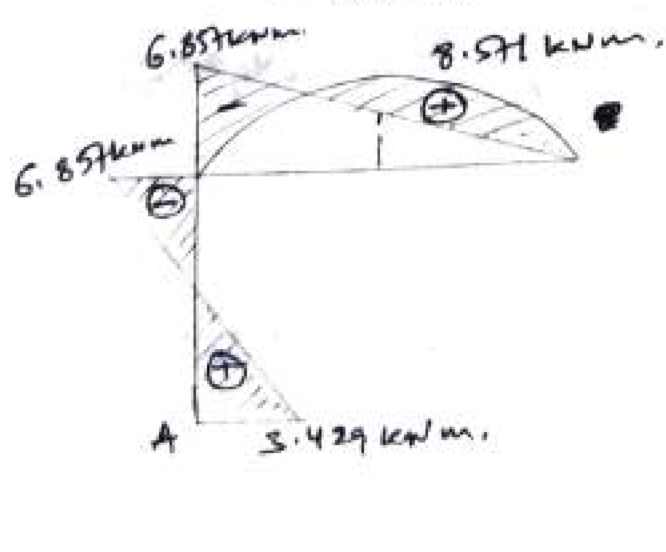
Taking outer face as reference face.

Net. Bending moment at the centre of BC.

$= 12 - \frac{6.857}{2}$

$= 12 - 3.429$

$= 8.571\,kN\text{-}m$



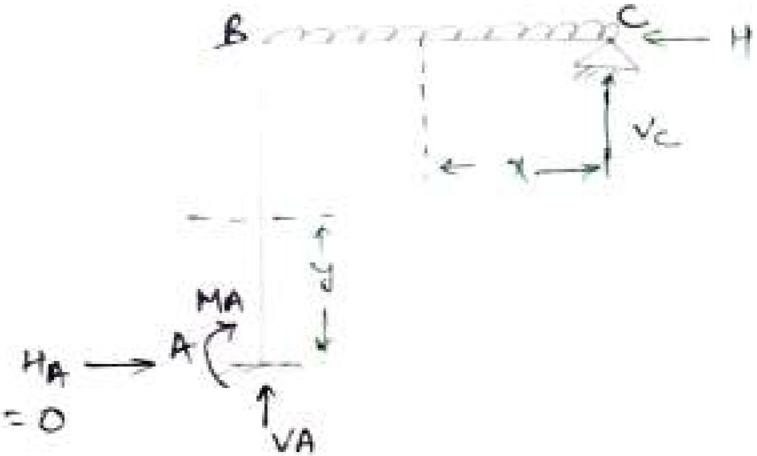
(11) Analyse by strain energy Method :-

$\Sigma F_y = 0$

$\Rightarrow V_A + V_C - 6 \times 4 = 0$

$V_A = 24 - V_C$

$V_C = 24 - V_A$



$\Sigma M_C = 0$

$\Rightarrow V_A \times 4 - H \times 4 + M_A - 6 \times 4 \times 2 = 0$

$\Rightarrow 4V_A - 4H + M_A - 48 = 0$

$\Rightarrow V_A = H + 12 - \frac{M_A}{4}$

$V_C = 24 - V_A = 24 - H - 12 + \frac{M_4}{4}$.

$V_C = 12 - H + \frac{M_A}{4}$

We have,

$$U = \int \frac{M^2 ds}{2EI}$$

The value of H will be such that the total strain energy is minimum i.e.,

$$\Rightarrow \frac{\partial U}{\partial H} = 0$$

$$\Rightarrow \frac{\partial U}{\partial H} = \int \frac{2M \cdot \frac{\partial M}{\partial H} \cdot \partial s}{2EI} = 0$$

$$\Rightarrow \frac{\partial U}{\partial H} = \int \frac{M \cdot \frac{\partial M}{\partial H} \cdot \partial s}{EI} = 0$$

Similarly,
$$\frac{\partial U}{\partial M_A} = 0$$

$$\frac{\partial U}{\partial M_A} = \int \frac{M \cdot \frac{\partial M}{\partial M_A} \cdot \partial s}{EI} = 0.$$

→

Member	M	$\frac{\partial M}{\partial H}$	$\frac{\partial M}{\partial M_A}$	limit of integration
AB	$M_A - Hy$	$-y$	1	0 to 4
BC	$12x - Hx + \frac{M_A x}{4} - 3x^2$	$-x$	$x/4$	0 to 4

from (1), we get

$$\Rightarrow \int_0^4 \frac{[(M_A - Hy)(-y)\,dy]}{EI} + \int_0^4 \frac{[(12x - Hx + \frac{M_A}{4}x - 3x^2)(-x)\,dx]}{EI} = 0$$

$$\Rightarrow \int_0^4 (-M_A y + Hy^2)\,dy + \int_0^4 (-12x^2 + Hx^2 - \frac{M_A}{4}x^2 + 3x^3]\,dx = 0$$

$$\Rightarrow \left[-\frac{M_A y^2}{2} + \frac{Hy^3}{3}\right]_0^4 + \left[\frac{12x^3}{3} + \frac{Hx^3}{3} - \frac{M_A x^3}{12} + \frac{3x^4}{4}\right]_0^4 = 0$$

$$\Rightarrow \left[-8M_A + \frac{64H}{3}\right] + \left[-256 + \frac{64H}{3} - \frac{16M_A}{3} + 192\right] = 0$$

$$\Rightarrow -8M_A + \frac{64H}{3} - 256 + \frac{64H}{3} - \frac{16M_A}{3} + 192 = 0$$

$$\Rightarrow -\frac{40M_A}{3} + \frac{128H}{3} - 64 = 0$$

$$\Rightarrow -40M_A + 128H - 192 = 0$$

$$16H - 5M_A = 24.$$

From (11), we get

$$\int_0^4 \frac{(M_A - Hy) \times 1 \times dy}{EI} + \int_0^4 \frac{(12x - Hx - \frac{M_A x}{4} - 3x^2) \times x/4 \times dx}{EI} = 0$$

$$\Rightarrow \left[M_A y - \frac{Hy^2}{2}\right]_0^4 + \frac{1}{4}\left[\frac{12x^3}{3} - \frac{Hx^3}{3} + \frac{M_A x^3}{12} - \frac{3x^4}{4}\right]_0^4 = 0$$

$$\Rightarrow 4M_A - 8H + \frac{1}{4}\left[4\times4^3 - \frac{4^3H}{12} + \frac{4^3\times M_A}{12} - \frac{3\times4^4}{4}\right] = 0$$

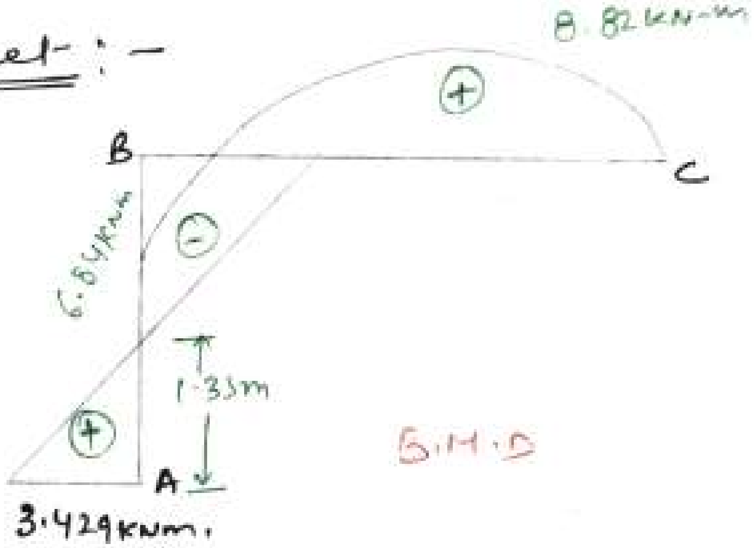
$$\Rightarrow 4M_A - 8H + 64 - \frac{16H}{3} + \frac{4M_A}{3} - 48 = 0$$

$$\frac{16}{3}M_A - \frac{40H}{3} + 16 = 0$$

$$-40H + 16M_A = -48$$

$$-5H + 2M_A = -6 \quad \longleftarrow (iv)$$

Solving (iii) and (iv) we get :-



$H = 2.571\text{ kN}$

$M_A = 3.429\text{ kN-m.}$

$$\therefore V_A = H + 12 - \frac{M_A}{4}$$

$$= 2.571 + 12 - \frac{3.429}{4}$$

$$= 13.71\text{ kN}$$

and $V_C = 24 - V_A$

$$= 24 - 13.71$$

$$= 10.29\text{ kN}$$

Assuming outer face as reference face.

Bending moment in BC :-

$$M_x (x \text{ from } C) = 10.29x - \frac{6x^2}{2} \quad [0 \le x \le 4]$$

$$M_x = 10.29x - 3x^2$$

$$M_C = 0$$

$$M_B = 10.29\times4 - 3\times(4)^2 = -6.84\text{ kNm.}$$

for, $M_{max} = \frac{dM_x}{dx} = 0$

$\Rightarrow \quad 10.29 - 6x = 0$

$x = \frac{10.29}{6} = 1.715\,m.$

$\therefore M_{max} = 10.29 \times 1.715 - 3 \times (1.715)^2 = 8.82\,kN\text{-}m.$

Bending moment in AB

$M_y (Y \text{ from } A) = M_A - Hy \; (0 \le y \le 4]$

$M_y = 3.429 - 2.571y$

$M_A = 3.429\,kNm.$

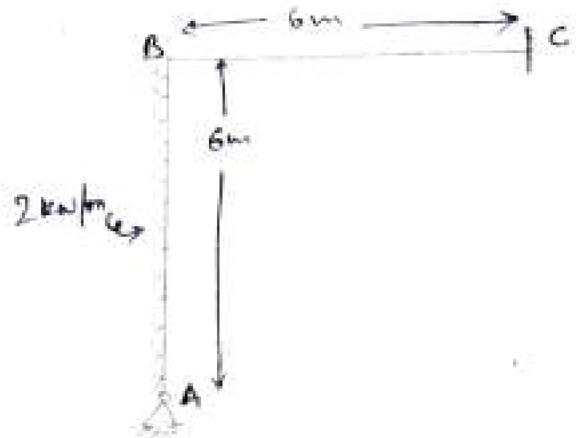
$M_B = 3.429 - 2.571 \times 4 = -6.85\,kNm.$

for. $M_y = 0$, we have.

$3.429 - 3.571y = 0$

$\Rightarrow y = 1.33\,m.$

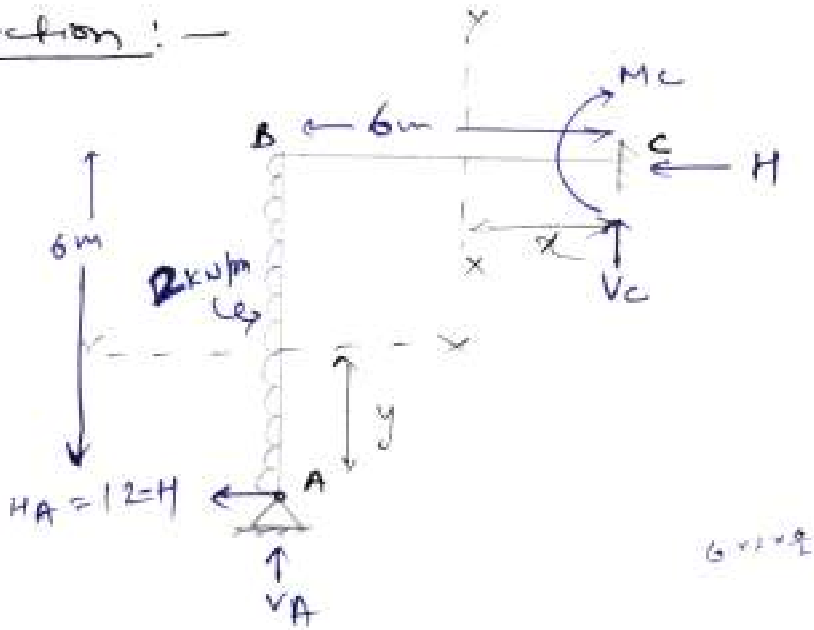
Q. Analyse the frame shown above by compatibility method. EI is constant. Draw B.M and SF diagram.



Solution: —

let verticle reaction at A and C be $V_A(\uparrow)$ and $V_C(\uparrow)$.

Calculate the reaction: —



$\Sigma F_y = 0$

$V_A + V_C = 0$

$V_A = -V_C$

$\Sigma F_x = 0$

let $H_C = H$

$H_A - 2\times6 + H = 0$

$H_A = 12 - H$

$\Sigma M_A = 0$

$\Rightarrow M_C - 6\times V_C - H\times6 + 2\times6\times3 = 0$

$\Rightarrow M_C - 6V_C - 6H + 36 = 0$

$\Rightarrow M_C + 36 - 6H = 6V_C$

$\Rightarrow V_C = 6 - H + \frac{M_C}{6}$

$V_A = -V_C = H - \frac{M_C}{6} - 6$

→

let Redundants are H and M_c

$$U = \int \frac{M^2 ds}{2EI}$$

The value of H and M_c will be such that the total strain energy is minimum i.e.,

$$\Rightarrow \quad \frac{\partial U}{\partial H} = 0$$

$$\Rightarrow \quad \frac{\partial U}{\partial H} = \Sigma \int \frac{M \frac{\partial M}{\partial H} \cdot ds}{EI} = 0 \quad \text{— (1)}$$

Similarly $\quad \frac{\partial U}{\partial M_c} = 0$

$$\Rightarrow \quad \Sigma \int \frac{M \cdot \frac{\partial M}{\partial c} \cdot ds}{EI} = 0 \quad \text{— (ii)}$$

The bending moment M at any section and the corresponding values of $\frac{\partial M}{\partial H}$ and $\frac{\partial M}{\partial M_c}$ of the frame are

Member	M	$\frac{\partial M}{\partial H}$	$\frac{\partial M}{\partial M_c}$	Limit of intergration
AB	$(12-H)y - 2y \cdot y/2$	$-y$	0	0 to 6
CB	$\left(6 - H + \frac{M_c}{6}\right)x - M_c$	$-x$	$x/6 - 1$	0 to 6

from eqⁿ (i) we get;

$$\Rightarrow \int_0^6 \frac{\left[(12-H)y - 2y\frac{y}{2}\right](-y)\,dy}{EI} + \int_0^6 \frac{\left\{\left(6-H+\frac{M_c}{6}\right)x - M_c\right\}(-x)\,dx}{EI} = 0$$

$$\Rightarrow \int_0^6 (12y - Hy - y^2)(-y)\,dy + \int_0^6 \left(6x - Hx + \frac{M_c x}{6} - M_c\right)(-x)\,dx = 0$$

$$\Rightarrow \int_0^6 (-12y^2 + Hy^2 + y^3)\,dy + \int_0^6 \left(-6x^2 + Hx^2 - \frac{M_c x^2}{6} + M_c x\right) dx = 0$$

$$\Rightarrow -12\times\frac{6^3}{3} + H\times\frac{6^3}{3} + \frac{6^4}{4} - \frac{6\times6^3}{3} + \frac{H\times6^3}{3} - \frac{M_c\times6^3}{18} + \frac{M_c\times6^2}{2} = 0$$

$$\Rightarrow -864 - 72H + 324 - 432 + 72H - 12M_c + 18M_c = 0$$

$$144H + 6M_c = 972$$

$$\Rightarrow \quad 24H + M_c = 162 \quad \text{--- (iii)}$$

From eq. (ii) we get,

$$\Rightarrow \int_0^6 \frac{\left[(12-H)y - 2y\frac{y}{2}\right](0)\,dy}{EI} + \left[\int_0^6 \left\{\left(6-H+\frac{M_c}{6}\right)x - M_c\right\}\left(\frac{x}{6}-1\right)dx\right] = 0$$

$$\Rightarrow \int_0^6 \frac{(6x - Hx + M_c x - M_c)\left(\frac{x}{6}-1\right)}{EI}\,dx = 0$$

$$\Rightarrow \int_0^6 \left(x^2 - \frac{Hx^2}{6} + \frac{M_c x^2}{36} - \frac{M_c x}{6} - 6x + Hx - \frac{M_c x}{6} + M_c\right) dx = 0$$

$$\Rightarrow \left[\frac{x^3}{3} - \frac{Hx^3}{18} + \frac{M_c x^3}{108} - \frac{M_c x^2}{12} - \frac{6x^2}{2} + \frac{Hx^2}{2} - \frac{M_c x^2}{12} + M_c x\right]_0^6 = 0$$

$$\Rightarrow \left[\frac{6^3}{3} - \frac{H\times6^3}{18} + \frac{M_c\times6^3}{108} - \frac{M_c\times6^2}{12} - \frac{6\times6^2}{2} + \frac{H\times6^2}{2} - \frac{M_c\times6^2}{12} + 6M_c\right] = 0$$

$$\Rightarrow 72 - 12H + 2M_c - 3M_c - 108 + 18H - 3M_c + 6M_c = 0$$

$$\Rightarrow -36 + 6H + 2M_c = 0$$

$$\Rightarrow M_c + 3H = 18 \quad \text{--- (iv)}$$

Solving (iii) and (iv), we get ;

$H = 6.8571 kN$

$M_c = -2.571 kN\text{-}m.$

$H_c = H = 6.8571 kN$

$H_A = 12 - H = 5.1429 kN$

$V_A = H - \frac{M_c}{6} - 6 = 6.8571 + \frac{2.571}{6} - 6$

$= 1.2856 kN$

$V_c = 6 - H + \frac{M_c}{6} = 6 - 6.8571 + \frac{(-2.571)}{6}$

$= -1.2856 kN$

Bending Moment diagram :-

Portion AB

$M_y (y \text{ from } A) = H_A y - 2y \frac{y}{2}$

$= 5.1429y - y^2$

At $y=0$ $\quad M_y = M_A = 0$

$M_B = 5.1429 \times 6 - 6^2$

$= -5.143 kNm.$

for M_{max} $\quad \frac{dM_y}{dy} = 0$

$5.1429 - 2y = 0$

$y = 2.571 m.$

$\therefore M_{max} = 5.1429 \times 2.571 - (2.571)^2$

$= 6.61 kNm.$

For $M_y = 0$, we have.

$$5.1429y - y^2 = 0$$
$$y(5.1429 - y) = 0$$
$$y = 0 \text{ and } y = 5.1429 m.$$

Portion CB

$$M_x = V_c x - M_c$$
$$= -1.2856x + 2.571$$

$M_c = 2.571$ kNm.

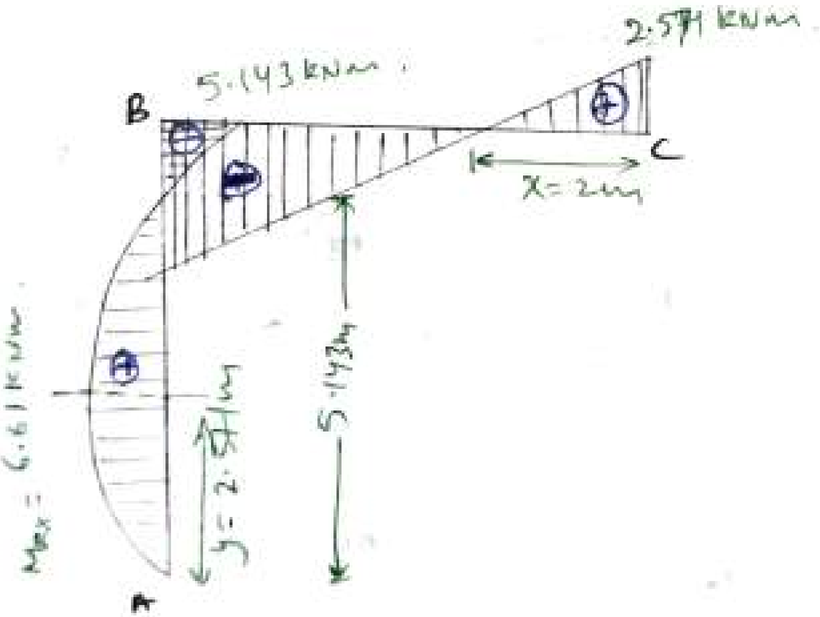
$M_B = -5.143$ kN-m.

$M_x = 0$, we have

$$-1.2856x + 2.571 = 0$$

$\Rightarrow$ $x = 1.999m \simeq 2m$.

$\Rightarrow$ Taking outer face as reference face.



BMD

→

Shear force diagram :-

Portion AB :-

Shear force in AB = $H_A - 2y$

$= 5.1429 - 2y$

At $y = 0$, $SF_A = 5.1429$ KN

At $y = 6m$, $SF_B = -6.8571$ KN

for SF to be zero

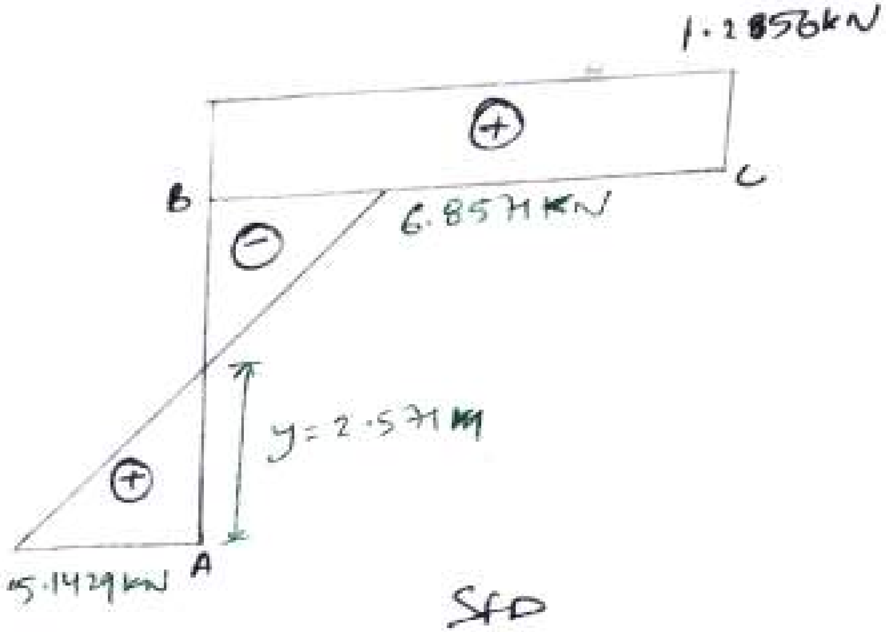
$\Rightarrow 5.1429 - 2y = 0$

$\Rightarrow y = 2.571m.$

Portion CB :

$SF = -V_C$

$= 1.2856$ KN



SFD

Q. Analyse the beam in fig. using the strain energy method and draw bending moment diagram and shear force diagram.

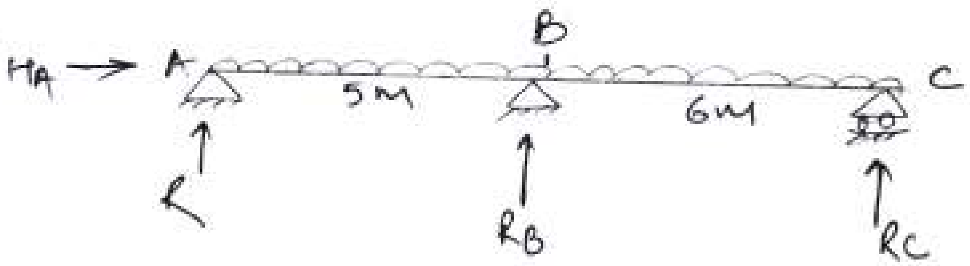
Sol^n:-

$$D_s = R_e - 3$$

$$R_e = 2+1+1 = 4$$

$$\therefore D_s = 4-3 = 1$$

Let reaction R_A is redundant, say $R_A = R$.



$\Sigma F_x = 0$

$\therefore H_A = 0$

$\Sigma F_y = 0$, $R + R_B + R_C = 30 \times 11$

$$R_B + R_C = 330 - R$$

$\Sigma M_C = 0$; $R \times 11 + R_B \times 6 - 30 \times 11 \times 5.5 = 0$

$$6R_B = 1815 - 11R$$

$$R_B = \frac{1815 - 11R}{6} (\uparrow)$$

from eq^n ①

$$R_C = 330 - R - R_B$$

$$R_C = \frac{165 + 5R}{6} (\uparrow)$$

→

The true value of redundant 'R' will be that for which total strain energy of the system is minimum

i.e.,
$$\frac{\partial U}{\partial R} = 0$$

$$U = U_{AB} + U_{BC}$$

$$= \int_0^5 \frac{M_x^2\,dx}{2EI} + \int_0^6 \frac{M_x^2\,dx}{2\times 2EI}$$

$$= \int_0^5 \frac{\left(Rx - \frac{wx^2}{2}\right)^2}{2EI}\,dx + \int_0^6 \frac{\left[\left(\frac{165+5R}{6}\right)x - \frac{wx^2}{2}\right]^2}{4EI}\,dx.$$

$$\frac{\partial U}{\partial R} = \int_0^5 \frac{2\left(Rx - \frac{wx^2}{2}\right)x}{2EI}\,dx + \int_0^6 \frac{2\left[\left(\frac{165+5R}{6}\right)x - \frac{wx^2}{2}\right]\cdot x\cdot\frac{5}{6}}{4EI}$$

$$\frac{\partial U}{\partial R} = \int_0^5 \frac{\left(Rx - \frac{wx^2}{2}\right)x}{EI}\,dx + \int_0^6 \frac{\frac{5}{12}\left[\left(\frac{165+5R}{6}\right)x - \frac{wx^2}{2}\right]x\,dx}{EI}.$$

$$\therefore \int_0^5 12\left(Rx^2 - \frac{wx^3}{2}\right)dx + \int_0^6 5\left[\left(\frac{165+5R}{6}\right)x^2 - \frac{wx^3}{2}\right]dx = 0$$

$$12\left[R\frac{x^3}{3} - \frac{wx^4}{2\times 4}\right]_0^5 + 5\left[\left(\frac{165+5R}{6}\right)\frac{x^3}{3} - \frac{wx^4}{2\times 4}\right]_0^6 = 0$$

$$12\left[\frac{125R}{3} - 2343.75\right] + 5\left[\left(\frac{165+5R}{6}\right)\times 72 - 4860\right] = 0$$

$$500R - 28125 + 9900 + 300R - 24300 = 0$$

$$800R = 42525$$

$$R = 53.15\,kN\,(\uparrow)$$

$$R_B = \frac{1815 - 11 \times 53.15}{6} = 20$$

Hence, $$R_C = \frac{165 + 5 \times 53.15}{6} = 71.80\,kN$$

Check:- $$\Sigma F_y = R_A + R_B + R_C - 30 \times 11$$

$$= 53.15 + 205.05 + 71.80 - 330$$

$$= 0$$

For shear force diagram:-

Portion AB:-

$$S_x (x \text{ from } A) = +R_A - wx$$

$$S_x = 53.15 - 30x$$

at $x = 0$ $\quad S_A = 53.15\,kN$

at $x = 5m$.

$$S_B (\text{Just left of } B) = 53.15 - 30 \times 5 = -96.85\,kN$$

$$M = Rx - Wx \times \frac{x}{2} = 53.15x - \frac{30x^2}{2} = 53.15x - 15x^2$$

For M_{max} $\quad \frac{dM}{dx} = 0$

$$53.15 - 15 \times 2x = 0$$

$$x = 1.77m \text{ (from A)}.$$

$\therefore M_{max} (\text{at } x = 1.77m) = 53.15 \times 1.77 - \frac{30 \times (1.77)^2}{2}$

$$= 47.08\,kN\text{-}m.$$

Portion BC:

$$S_x (x \text{ from } C) = -R_C + wx$$

$$S_x = -71.80 + 30x$$

at $x = 0$, $\quad S_C = -71.80\,kN$

$\rightarrow$

at $x = 6m$,

S_B (Just right of B) $= 108.2 kN$

for bending moment diagram :-

Portion AB

$$M_x (x \text{ from } A) = R_A x - \frac{wx^2}{2} \quad [0 \le x \le 5]$$

$$M_x = 53.15x - \frac{30x^2}{2} \quad \text{[Parabola]}$$

at $x = 0$,

at $x = 5m$, $M_B = 53.15 \times 5 - \frac{30 \times 5^2}{2} = -109.25 \text{ kN-m}$

Portion BC

$$M_x (x \text{ from } C) = R_C x - \frac{wx^2}{2} \quad [0 \le x < 6]$$

$$M_x = 71.80x - \frac{30x^2}{2}$$

at $x = 0$ $\quad M_C = 0$

at $x = 6m$, $M_B = 71.80 \times 6 - \frac{30 \times 6^2}{2} = -109.20 \text{ kNm}$.

from M_{max}.

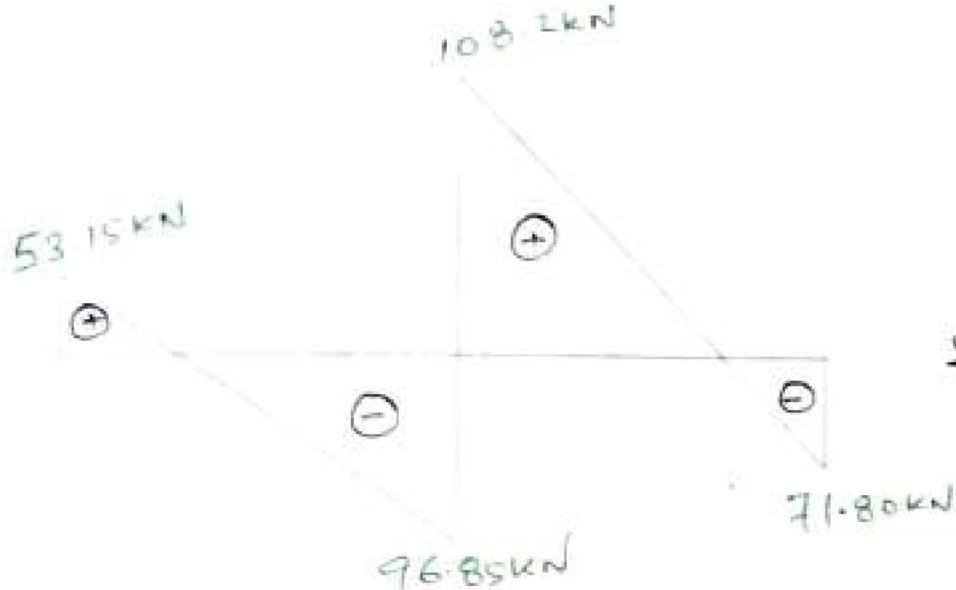
$$\frac{dM}{dx} = 0$$

$$71.80 - 15 \times 2x = 0$$

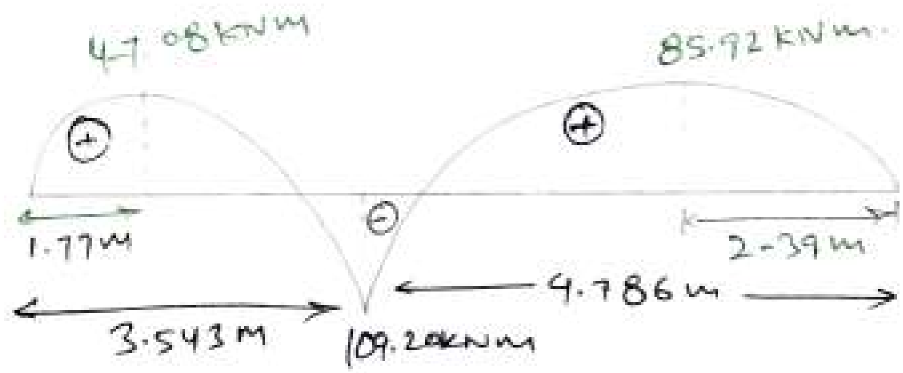
$$x = 2.30m \text{ (from C)}.$$

M_{max} (at $x = 2.39m$) $= 71.80 \times 2.39 - 15 \times (2.39)^2$

$= 85.92 \text{ kN-m}$.


108.2kN
53.15kN
96.85kN
71.80kN

SFD


47.08kNm
85.92kNm
1.77m
2.39m
3.543m
4.786m
109.20kNm

BMD