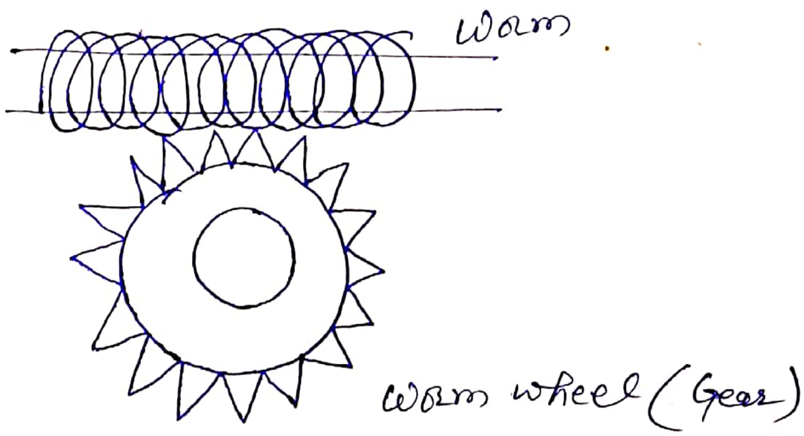
Module – IV

Worm Gears

Introduction :- The worm gears are widely used for transmitting power at high velocity ratio between non-intersecting shafts that are generally, but not necessary at right angles. It can give velocity ratio as high as 300:1. The worm gears are mostly used as a speed reducer, which consist of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an ~~cylindrical form~~ involute rack.

The worm are generally made of steel while the worm gear is made of bronze or cast iron for light service.



* Types of worms :-

The following are the two types of worms.

(i) cyllindrical or straight worm.

(ii) Cone or double enveloping worm.

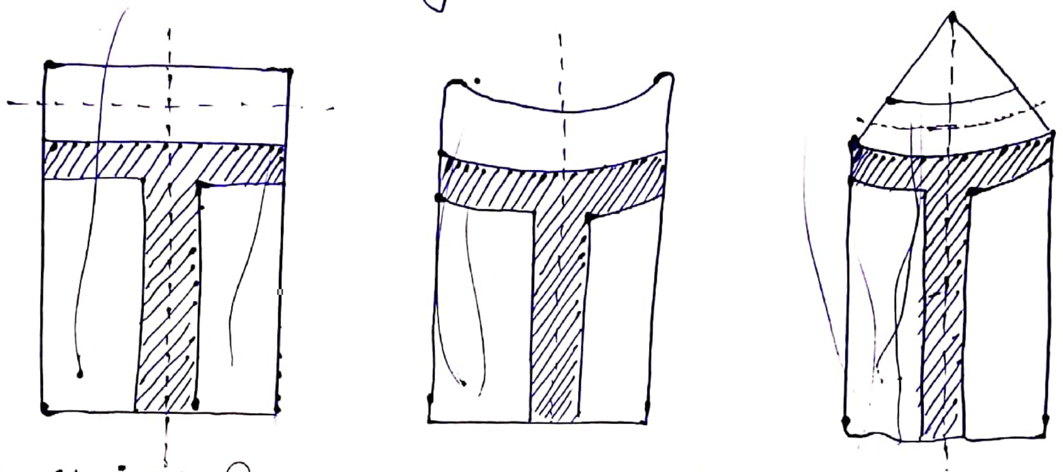
The cyllindrical or straight worm is most commonly used. The shape of the thread is involute helicoid of pressure angle $14\frac{1}{2}°$ for single and double threaded worm and $20°$ for triple threaded and quardruple threaded worm. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm.

The cone or double enveloping worm is used to some extent but it requires extremely accurate alignment.

* Types of worm Gears :-

There are following three types of worm gears.

1. Straight face worm gear.
2. Hobbed straight face worm gear.
3. Concave face worm gear.



(a) Straight face (b) Hobbed straight face (c) Concave face.

→ The straight face worm gear is like a helical gear in which the straight teeth are cut with a form cutter. since it has only point contact with the worm thread, therefore it is used for light service.

→ The hobbed straight face worm gear is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

→ The concave face worm gear is the accepted standard form and is used for all heavy service and general ~~industrial~~ industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the mating worm to increase the contact area.

* Terms used in Worm Gearing.

The worm and worm gear in mesh is shown in fig. The following terms, in connection with the worm gearing are important from the subject point of view.

(i) Axial pitch :- It is also known as linear pitch of a worm. it is the distance measured axially (i.e parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm. It may be noted that the axial pitch (P_a) of a worm is equal to the circular pitch (P_c) of the mating worm gear, when the shaft ~~and~~ are at right angles.

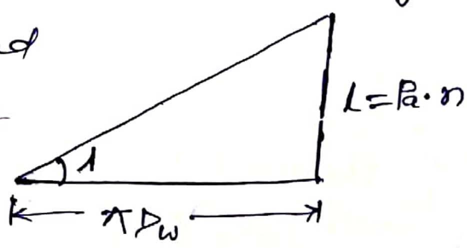
2. Lead :→ It is a linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

$$\text{Lead} \quad \boxed{l = P_a \cdot n}$$

where P_a = Axial pitch and n = Number of starts.

3. Lead angle :- It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

If one complete turn of a worm thread be imagined to be unwound from the body of the worm. it will form an inclined plane whose base is equal to the ~~lead~~ pitch circumference of the worm and altitude equal to the lead of the worm.



From geometry of the figure,

$$\tan\lambda = \frac{\text{Lead of the worm}}{\text{Pitch circumference of the worm}}$$

$$= \frac{l}{\pi D_w} = \frac{P_a \cdot n}{\pi D_w} \qquad \because l = P_a \cdot n.$$

$P_a = P_c$ and $P_c = \pi m$

$$= \frac{P_c \cdot n}{\pi D_w} = \frac{\pi m \cdot n}{\pi D_w} = \frac{mn}{D_w}$$

where m = module

D_w = Pitch circle diameter of worm.

The lead angle may vary from $9°$ to $45°$. By F. A. Halsey that a lead angle less than $9°$ results in rapid wear and the safe value of λ is $12\frac{1}{2}°$.

4. Tooth Pressure angle :- It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle.

- Recommended values of lead angle and Pressure angle :-

Lead Angle (λ) in degree	0 — 16	16 — 25	25 — 35	35 — 45
Pressure angle (ϕ) in degree	$14\frac{1}{2}$	20	25	30

For automotive application, the pressure angle of $30°$ is recommended to obtain a high efficiency and to permit overhauling.

5. Normal Pitch :→ It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm.

Mathematically,

Normal Pitch $P_N = P_a \cos\lambda$

6. Helix angle :→ It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by the ~~as~~ a_w. The worm helix angle is the complement of worm lead angle.

i.e $a_w + \lambda = 90$

It may be noted that the helix angle on the worm is generally quite large and on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. Velocity ratio :→ It is the ratio of the speed of worm (N_W) in r.p.m to the speed of the worm gear (N_G) in r.p.m.

Mathematically,

$$\boxed{V.R = \frac{N_W}{N_G}}$$

* Velocity ratio may also be defined as the ratio of number of teeth on the worm gear to the number of starts of the worm.

$$\boxed{V.R. = \frac{T_G}{n}}$$

* Efficiency of worm Gearing :-

- The efficiency of worm gearing may be defined as the ratio of work done by the worm gear to the work done by the worm.

Mathematically,

$$\eta = \frac{\tan\lambda(\cos\phi - \mu\tan\lambda)}{\cos\phi\tan\lambda + \mu} \quad \text{---} (1)$$

where ϕ = Normal pressure angle
μ = Coeff^n of friction.
λ = Lead angle.

- The efficiency is maximum, when

$$\tan\lambda = \sqrt{1+\mu^2} - \mu$$

In order to find the approximate value of the efficiency, assume square thread, the following relation may be used.

$$\text{Efficiency} \quad \eta = \frac{\tan\lambda(1-\mu\tan\lambda)}{\tan\lambda + \mu}$$

$$\eta = \frac{1-\mu\tan\lambda}{1+\mu/\tan\lambda}$$

$$\boxed{\eta = \frac{\tan\lambda}{\tan(\lambda+\phi_1)}}$$

$\because$ For square thread $\phi = 0$
& ϕ_1 = Angle of friction $= \tan\phi_1 = \mu$

Note :→ If the efficiency of worm gearing is less than 50%, then the worm gearing is said to be self locking. i.e it cannot be driven by applying a torque to the wheel. This property

Q: A triple threaded worm has teeth of 6 mm module and pitch circle diameter of 50 mm. If the worm gear has 30 teeth of $14\frac{1}{2}°$ and the coefficient of friction of the worm gearing is 0.05:

Find (i) The lead angle of the worm.

(ii) velocity ratio.

(iii) Centre distance

(iv) Efficiency of the worm gearing.

Solⁿ:- Given $n = 3$, $m = 6$, $D_w = 50$ mm, $T_G = 30$, $\phi = 14.5°$

$\mu = 0.05$

(i) Lead angle of the worm

$$\because \quad \tan\lambda = \frac{m.n}{D_w} = \frac{6\times3}{50} = 0.36$$

$$\lambda = \tan^{-1}(0.36) = 19.8° \text{ Ans}$$

(ii) velocity ratio

We know that velocity ratio

$$V.R = \frac{T_G}{n} = \frac{30}{3} = 10 \text{ Ans}$$

(iii) Centre distance

We know that pitch circle diameter of the worm gear

$$D_G = m.T_G = 6\times30 = 180 \text{ mm}$$

∴ Centre distance

$$x = \frac{D_w + D_G}{2} = \frac{50+180}{2} = 115 \text{ mm Ans}$$

(iv) Efficiency

$$\eta = \frac{\tan\lambda(\cos\phi - \mu\tan\lambda)}{\cos\phi\cdot\tan\lambda + \mu}$$

$$= \frac{\tan(19.8)[\cos 14.5 - 0.05\tan 19.8]}{\cos 14.5°\times\tan 19.8° + 0.05}$$

$$= \frac{0.36(0.9681 - 0.05\times0.36)}{0.9681\times0.36 + 0.05} = 0.858 = 85.8\% \text{ A}$$

* Strength of Worm Gear teeth :-

In finding the tooth size and strength, it is safe to assume that the teeth of worm gear are always weaker than the threads of the worm. In worm gearing two or more teeth are contact, but due to ~~load~~ uncertainity of load distribution among themselves, it is assumed that the load is transmitted by one tooth only.

A/c to Lewis equation.

$$W_T = (\sigma_o \cdot C_v)\, b \cdot \pi\, m y$$

where W_T = Permissible tangential tooth load or beam strength of gear tooth.

σ_o = Allowable static stress

C_v = velocity factor

b = face width

m = module

y = Tooth form factor or Lewis factor

Note: 1. The velocity factor is given by

$$C_v = \frac{6}{6+v}$$ where v is the peripheral velocity of the worm gear in m/sec.

2. The tooth form factor or Lewis factor (y) may be obtained in the similar manner as discussed in spur gear i.e.

$$y = 0.124 - \frac{0.684}{T_G}, \text{ for } 14\tfrac{1}{2}^\circ \text{ involute teeth.}$$

$$= 0.154 - \frac{0.912}{T_G} \text{ for } 20^\circ \text{ involute teeth}$$

3. The dynamic tooth load on the worm gear is given by -

$$W_D = \frac{W_T}{C_v} = W_T\left(\frac{6+v}{6}\right)$$

where W_T = Actual tangential load on the tooth.

4. The static tooth load or endurance strength of the tooth (W_S) may also be obtained in the similar manner as -

$$\boxed{W_S = \sigma_e \cdot b \cdot \pi m y}$$

where σ_e = flexural endurance limit. it value may be taken as 84 MPa for cast iron and 168 MPa for phosphor bronze gears.

* Wear tooth load for worm gear :-

The limiting or maximum load for wear (W_w) is given by -

$$\boxed{W_w = D_g \cdot b \cdot K}$$

where D_g = Pitch circle diameter of the worm gear.

b = face width of the worm gear

K = Load stress factor (Also known as material combination factor). which is taken from data book

* Thermal Rating of worm Gearing :-

In the worm gearing, the heat generated due to the work lost in friction must be dissipated in order to avoid over heating of the drive and lubricating oil.

The quantity of heat generated (Q_g) is given by -

Q_g = Power lost in friction in watts.

$$\boxed{Q_g = P(1-\eta)} \quad \text{--- (1)}$$

where P = Power transmitted in watts.

η = Efficiency of the worm gearing.

The heat generated must be dissipated through the lubricating oil to the gear box housing and then to the atmosphere.

The heat dissipating capacity depends upon the following factors.

(i) Area of the housing (A).

(ii) Temperature difference between the housing and surrounding air i.e (t_2-t_1).

(iii) Conductivity of the material (K).

Mathematically,

$$\boxed{Q_d = A(t_2-t_1)K} \quad \text{--- (2)}$$

From equation (1) & (2) we can find the temp. diffn. (t_2-t_1).

The average value of K may be taken as 378 $W/m^2 {}^\circ C$.

<u>Note</u>:-(i) The maximum temperature (t_2-t_1) should not exceed 27° to 38°C.

(ii) The maximum temperature of the lubricant should not exceed 60°C.

(iii) The limiting input power of a plain worm gear unit from the standpoint of heat dissipation, for worm gear speeds upto 2000 rpm may be checked from the following relation, i.e.

$$P = \frac{3650\, x^{1.7}}{V.R + 5}$$

where P = Permissible input power in KW.

x = Centre d/s in meters.

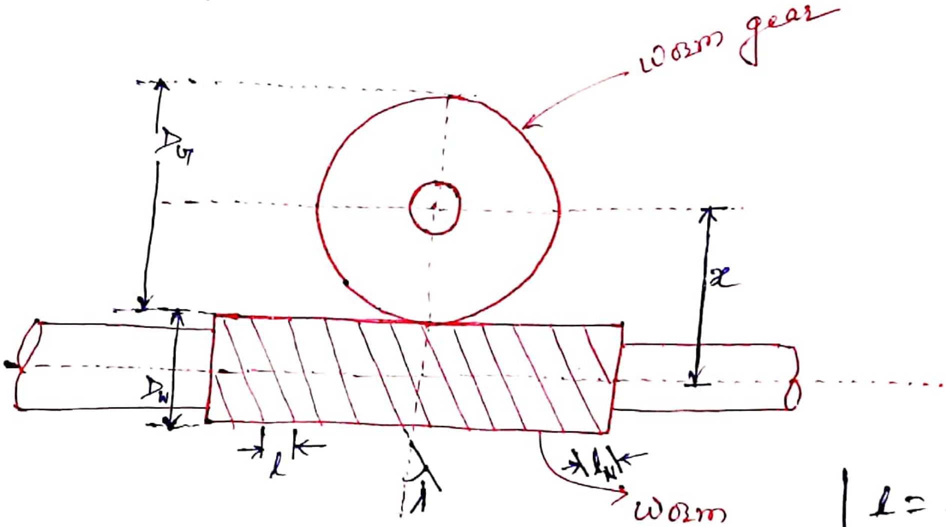
$V.R$ = velocity ratio or transmission ratio.

* Design of worm gearing :-

In ~~cn~~ designing a worm and worm gearing, the quantities like the power transmitted, speed, velocity ratio and the centre distance between the shafts are usually given and the quantities such as

lead angle, lead, and number of threads on the worm are to be determined.

In order to determine the satisfactory combination of lead angle, lead and centre distance, the following method may be used.



Worm and worm gear.

l = Axial lead
λ = Lead angle.
l_N = Normal lead = $l \cos\lambda$

From fig,

- Centre distance $\boxed{x = \dfrac{D_W + D_G}{2}}$

- The centre distance may be expressed in terms of axial lead (l), lead angle (λ) and velocity ratio (V.R) as follow-

$$x = \frac{l}{2\pi}\left(\cot\lambda + V.R\right)$$

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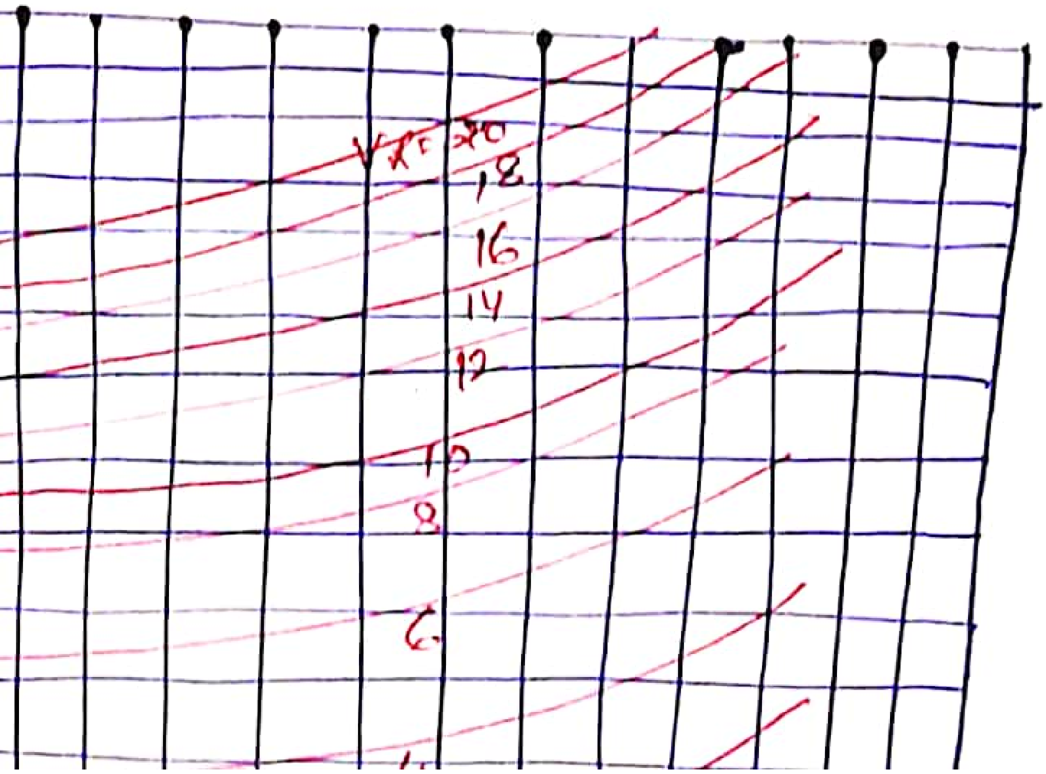
...mal lead ($l_N = l\cos\lambda$)

$$\left(\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right)$$

$$\boxed{\left[\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right]}$$



Q: Design 20° involute worm and gear to transmit 10 KW with worm rotating at 1400 r.p.m and to obtain a speed reduction of 12:1. The d/s b/w the shafts is 225 mm.

Given $\phi = 20°$, $P = 10\,KW$, $N_W = 1400\,rpm$, $V.R = 12$, $x = 225\,mm$

Soln – 1. Design of worm

Let l_N = Normal lead

λ = Lead angle

We know that $\frac{x}{l_N}$ will be minimum when

$V.R = \cot^3\lambda$

$\cot^3\lambda = 12$

$\therefore \lambda = 23.6°$

&

$$\frac{x}{l_N} = \frac{1}{2\pi}\left[\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right]$$

$$\frac{225}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin 23.6°} + \frac{12}{\cos 23.6°}\right)$$

$\therefore l_N = 90\,mm$

and axial lead $l = \frac{l_N}{\cos\lambda} = \frac{90}{\cos 23.6°} = 98.2\,mm$.

From table we find that,

For a velocity ratio of 12, the numbers of starts or threads on the worm

$n = T_W = 4$

$\therefore$ Axial pitch of the threads on the worm

$P_a = l/4 = \frac{98.2}{4} = 24.55\,mm$

module $m = P_a/\pi = \frac{24.55}{\pi} = 7.8\,mm$

$\simeq 8\,mm$.

(taking standard value)

∴ Axial pitch of the thread on the worm

$$P_a = \pi m = \pi \times 8 = 25.136 \text{ mm}$$ Ans

& Normal lead

$$l_N = l \cos\lambda = 100.544 \cos 23.6$$

$$= 92 \text{ mm}$$ Ans

We know that the centre distance

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R}{\cos\lambda}\right)$$

$$= \frac{92}{2\pi}\left(\frac{1}{\sin 23.6} + \frac{12}{\cos 23.6}\right)$$

$$= 230 \text{ mm}$$ Ans

Let D_W = Pitch circle diameter of the worm.

We know that,

$$\tan\lambda = \frac{l}{\pi D_W}$$

$$D_W = \frac{l}{\pi \tan\lambda} = \frac{l}{\pi \tan 23.6} = 73.24 \text{ mm}$$ Ans

Since the velocity ratio is 12 and the worm has quadruple threads (i.e $n = T_W = 4$), therefore number of teeth on the worm gear

$$T_G = 12 \times 4 = 48$$

From data book,

The face length of the worm or the length of threaded portion is,

$$L_W = P_c (4.5 + 0.02 T_W) \qquad (\because P_c = P_a)$$

$$= 25.136 (4.5 + 0.02 \times 4)$$

$$= 115 \text{ mm}$$

This length should be increased by 25 to 30 mm for the feed marks produced by the vibrating grinding wheel

∴ $L_W = 140$ mm (taken) Ans

We know that depth of tooth,

$$h = 0.623\,p_c = 0.623 \times 25.136 = 15.66 \text{ mm} \quad \text{Ans}$$

and addendum $a = 0.286\,p_c = 0.286 \times 25.136$

$$= 7.2 \text{ mm} \quad \text{Ans}$$

∴ outside diameter of worm,

$$D_{ow} = D_w + 2a = 73.24 + 2 \times 7.2 = 87.64 \text{ mm} \quad \text{Ans}$$

2. Design of worm gear.

We know that pitch circle diameter of the worm gear

$$D_G = m \cdot T_G = 8 \times 48$$

$$= 384 \text{ mm} = 0.384 \text{ m} \quad \text{Ans}$$

outside diameter of the worm gear (from table)

$$D_{OG} = D_G + 0.8903\,p_c$$

$$= 384 + 0.8903 \times 25.136 = 406.4 \text{ mm} \quad \text{Ans}$$

Throat diameter,

$$D_T = D_G + 0.572\,p_c = 384 + 0.572 \times 25.136$$

$$= 398.4 \text{ mm} \quad \text{Ans}$$

and face width

$$b = 2.15 \times p_c + 5 \text{ mm}$$

$$= 2.15 \times 25.136 + 5 = 59 \text{ mm}.$$

Let us now check the designed worm gearing from the stand point of tangential load, static load or endurance strength, dynamic load, wear load and heat dissipation.

(a) Check for the tangential load,

Let N_G = speed of the worm gearing in r.p.m.

We know that velocity ratio of the drive,

$$V.R = \frac{N_w}{N_G} \Rightarrow N_G = \frac{N_w}{V.R} = \frac{1400}{12} = 116.7 \text{ rpm}.$$

∴ Torque transmitted $T = \dfrac{P \times 60}{2\pi N_G} = \dfrac{10{,}000 \times 60}{2\pi \times 116.7} = 818.2 \text{ N-m}$

Tangential load acting on the gear,

$$W_T = \frac{2 \times Torque}{D_4} = \frac{2 \times 818.2}{0.384}$$

$$= 4260 \text{ N.}$$

We know that pitch line or pheripheral velocity of the worm gear,

$$v = \frac{\pi \times D_4 \times N_4}{60} = \frac{\pi \times 0.384 \times 116.7}{60}$$

$$= 2.35 \text{ m/sec.}$$

∴ velocity factor

$$C_v = \frac{6}{6+v} = \frac{6}{6+2.35} = 0.72$$

Tooth form factor for 20° involute teeth,

$$y = 0.154 - \frac{0.912}{T_4} = 0.154 - \frac{0.912}{48}$$

$$= 0.135$$

Since the worm gear are generally made of phosphorous bronze, therefore taking the allowable static stress for phosphor bronze $\sigma_0 = 84$ MPa or 84 N/mm²

We know that the designed tangential load,

$$W_T = (\sigma_0 \cdot C_v) b . \pi . m . y$$

$$= (84 \times 0.72) 59 . \pi \times 8 \times 0.135$$

$$= 12110 \text{ N.}$$

Since this is more than the tangential load acting on the gear (i.e 4260 N), therefor the design is safe from the stand point of tangential load.

(b) check for dynamic load,

We know that the dynamic load;

$$W_D = \frac{W_T}{C_v} = \frac{12110}{0.72} = 16820 \text{ N.}$$

Since this is more than $W_T = 4260$ N, therefore the design is safe from the stand point of dynamic load.

(c) check for static load or endurance strength :→

We know that the flexural endurance limit for phosphor bronze is $\sigma_e = 168$ MPa.

∴ static load or endurance strength,

$$W_S = \sigma_e \cdot b \cdot \pi \cdot m \cdot y$$
$$= 168 \times 59 \times \pi \times 8 \times 0.135$$
$$= 33635 \text{ N}.$$

Since this is much more than $W_T = 4260$ N, therefore the design is safe from the stand point of static load or endurance strength.

(d). Check for wear :→

From table, the value of load stress factor,

$$K = 0.55 \text{ N/mm}^2$$

∴ Limiting or maximum load for wear

$$W_w = D_g \cdot b \cdot K = 384 \times 59 \times 0.55$$
$$= 12461 \text{ N}.$$

Since this is more than $W_T = 4260$ N, therefore the design is safe.

(e) check for heat dissipation :-

We know that rubbing velocity,

$$V_r = \frac{\pi D_w \cdot N_w}{\cos \lambda} = \frac{\pi \times 0.07324 \times 1400}{\cos 23.6^\circ} = 351.6 \text{ m/min}$$

∴ Coefficient of friction

$$\mu = 0.025 + \frac{V_r}{18000} = 0.025 + \frac{351.6}{18000} = 0.0445$$

Angle of friction

$$\phi_1 = \tan^{-1}(\mu) = \tan^{-1}(0.0445) = 2.548^\circ$$

We know that efficiency

$$\eta = \frac{\tan\lambda}{\tan(\lambda+\phi_1)} = \frac{\tan 23.6^\circ}{\tan(23.6^\circ+2.548)}$$

$$= 0.89 = 89\%$$

Assume 25 % overload, heat generation.

$$\therefore Q_g = 1.25\, P(1-\eta)$$

$$= 1.25 \times 10,000\,(1-0.89) = 1375\,W.$$

We know that projected area of the worm

$$A_w = \frac{\pi}{4}(D_w)^2 = \frac{\pi}{4}(73.24)^2 = 4214\ mm^2$$

& Projected area of the gear,

$$A_g = \frac{\pi}{4}(D_g)^2 = \frac{\pi}{4}(384)^2 = 115827\ mm^2$$

$\therefore$ Total Projected area $A = A_w + A_g = 120041\ mm^2$

$\therefore$ Heat dissipating capacity

$$Q_d = A(t_2-t_1)K = 120041\times10^{-6}(t_2-t_1)\,378 = 45.4(t_2-t_1)$$

The heat generated must be dissipated in order to avoid over heating of the drive, therefore equating $Q_g = Q_d$

$$\therefore Q_g = Q_d$$

$$\therefore t_2 - t_1 = \frac{1375}{45.4} = 30.3^\circ c$$

Since this temperature diffⁿ (t_2-t_1) is within 27 to 38°c therefore the design is safe.

Q: A speed reducer unit is to be designed for an input of 1.1 KW with a transmission ratio 27. The speed of the hardened steel worm is 1440 r.p.m. The worm wheel is to be made of phosphor bronze. The tooth form is to be 20° involute.

Sol^n. Given $P = 1.1$ KW, $V.R = 27$, $N_W = 1440$ rpm, $\phi = 20°$

A speed reducer unit (i.e worm and worm ~~wheel~~ gear) may be designed as discussed below.

Since the centre distance b/w the shafts is not given therefore let us assume that for this size unit, the centre distance $x = 100$ mm.

We know that pitch circle diameter of the worm

$$D_W = \frac{x^{0.875}}{1.416} = 39.7 \simeq 40 \text{ mm.}$$

∴ Pitch circle diameter of the worm gear,

$$D_G = 2x - D_W = 2 \times 100 - 40 = 160 \text{ mm.}$$

For the transmission ratio of 27, we shall use double start worm,

∴ Number of teeth on the worm gear

$$T_G = 2 \times 27 = 54$$

∵ Axial pitch of the thread = circular pitch of the teeth on the worm gear

$$\therefore P_a = P_c = \frac{\pi D_G}{T_G} = \frac{\pi \times 160}{54} = 9.3 \text{ mm}$$

module $$m = \frac{P_c}{\pi} = \frac{9.3}{\pi} = 2.963 \simeq 3 \text{ mm.}$$

∴ Actual circular pitch

$$P_c = \pi m = \pi \times 3 = 9.4 \text{ mm.}$$

∴ Actual pitch circle diameter of the worm gear $$D_G = \frac{P_c T_G}{\pi} = \frac{9.4 \times 54}{\pi} = 162 \text{ mm}$$

Actual pitch circle diameter of the worm

$$D_W = 2x - D_G$$
$$= 2\times100 - 162 = 38\ mm \quad \underline{Ans}$$

Face width $b = 0.73 D_W = 0.73\times38$
$$= 27.7 \simeq 28\ mm.$$

Now check the designed,

(i) <u>check for tangential load,</u>

velocity ratio for the drive,

$$V.R = \frac{N_W}{N_G}.$$

or $N_G = \frac{N_W}{V.R} = \frac{1440}{27} = 53.3$ rpm.

Peripheral velocity of the worm gear,

$$V = \frac{\pi D_G \cdot N_G}{60} = \frac{\pi\times0.162\times53.3}{60} = \cancel{0.452\ m}/sec.$$
$$= 0.452\ m/sec.$$

and velocity factor

$$C_V = \frac{6}{6+V} = \frac{6}{6+0.452} = 0.93$$

We know that for 20° involute teeth, the tooth form factor

$$y = 0.154 - \frac{0.912}{T_G} = 0.137$$

for phosphor bronze

$$\sigma_o = 84\ MPa$$

∴ Tangential load transmitted

$$W_T = (\sigma_o\cdot C_V)b.\pi m y = (84\times0.93)\times28\times\pi\times3\times0.137$$
$$= 2825\ N.$$

∴ Power transmitted due to tangential load

$$P = W_T\times V = 2825\times0.452$$
$$= 1.277\ KW$$

Since this power is more than the given power (1.1 KW), therefore the design is safe.

2. Check for the dynamic load.

We know for the dynamic load

$$W_D = \frac{W_T}{C_V} = \frac{2825}{0.93} = 3038 \text{ N.}$$

and power transmitted due to the dynamic load,

$$P = W_D \times V = 3038 \times 0.452 = 1373 \text{ W}$$
$$= 1.373 \text{ KW}$$

Since this power is more than the given power to be transmitted, therefore the design is safe. ~~from abt~~

3. Check the static load or endurance strength :-

for phosphor bronze

$$\sigma_e = 168 \text{ mPa.}$$

∴ Static load or endurance strength.

$$W_S = \sigma_e \cdot b \cdot \pi m \cdot y = 168 \times 28 \times \pi \times 3 \times 0.137 = 6075 \text{ N.}$$

& Power transmitted

$$P = W_S \times V = 6075 \times 0.452 = 2746 \text{ W} = 2.746 \text{ KW.}$$

Since this power is more than the power to be transmitted (1.1 therefore design is safe.

4. Check for heat dissipation,

We know that permissible input power,

$$P = \frac{3650 (x)^{1.7}}{V.R + 5} = \frac{3650 (0.1)^{1.7}}{27 + 5} = 2.27 \text{ KW.}$$

Since this power is more than the given power to be transmitted (1.1 KW), Therefore design is safe.

Clutch.

* Introduction:-

A clutch is a machine member used to connect a driving shaft to a driven shaft so, that the driven shaft may be started or stopped at will, without stopping the driving shaft. The uses of a clutch is mostly found in automobile. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. it is therefe necessary that the ~~the driven driving~~ driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

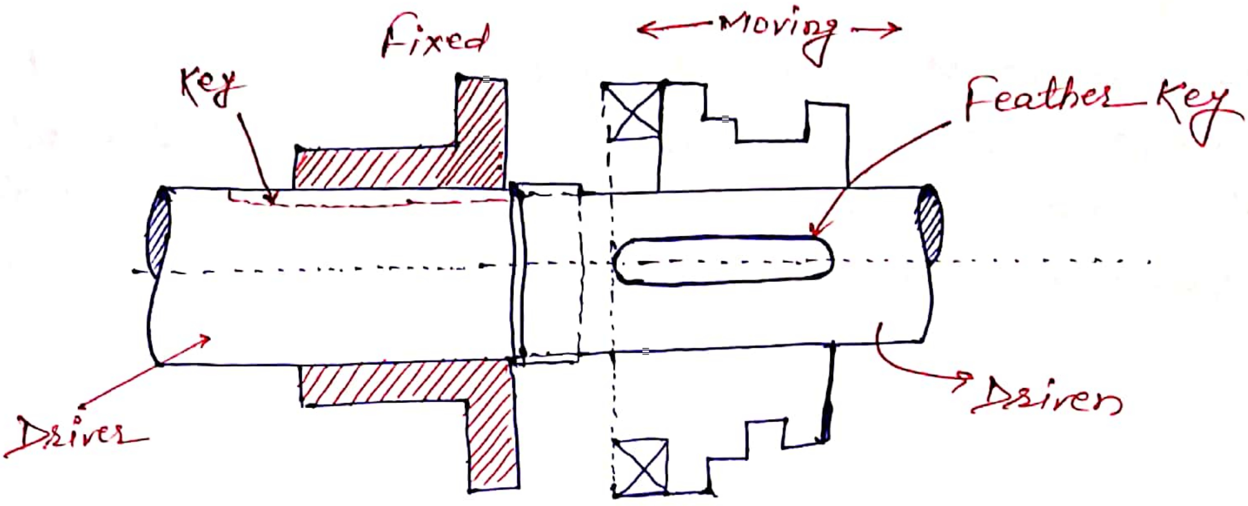
* Types of clutches.

There are following two types of clutches are generally used. -

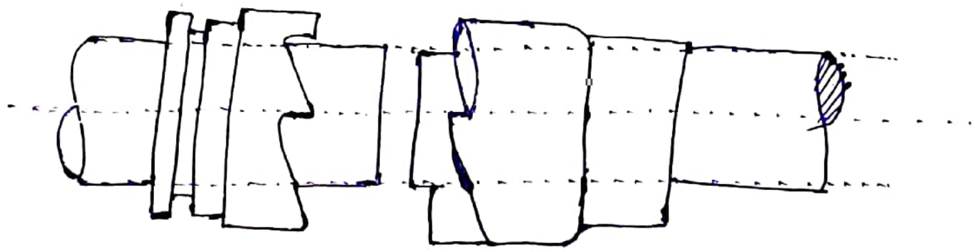
(i) Positive clutch.

(ii) Friction clutch.

Positive clutch:- The positive clutches are used when a positive drive is required. The simplest types of a positive clutch is a jaw or claw clutch. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws.



(a) Square Jaw clutch



(b) Spiral Jaw clutch.

A square Jaw clutch is used where engagement and disengagement in motion and under load is not necessary. This types of clutch will transmit power in either direction of rotation. The spiral Jaw may be left hand or right hand, because power transmit by them is in one direction only. This type of clutch is occationally used where the clutch must be engaged and disengaged while in motion. The use of Jaw clutch is are frequently applied to spocket wheel, gears and pulley. in such a case the non-sliding part is made integral with the hub.

(ii) Friction clutches :-

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. its application is also found in case in which power is to be delivered to machines partially or fully loaded.

* Material for friction surfaces:→

The material used for lining of friction surface of a clutch should have the following characteristics -

(i) It should have a high and uniform coefficient of friction.

(ii) It should not be affected by moisture and oil.

(iii) It should have the ability to withstand high temperature caused by slippage.

(iv) it should have high heat conductivity.

(v) It should have high resistance to wear.

The materials commonly used for lining of friction are as follows -

- Cast iron on cast iron or steel.
- ~~cast iron on cast iron or~~
- Hardened steel on hardened steel.
- Bronze on cast iron or steel.
- Pressed asbestos on cast iron or steel.

* Considerations in designing a friction clutch:-

The following consideration must be kept in mind while designing a friction clutch.

(i) The suitable material forming the contact surface should be selected.

(ii) The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed load.

(iii) The clutch should not require any external force to maintain contact of the friction surfaces.

(iv) The clutch should have provision for carrying away the heat generated at the contact surfaces.

* Types of friction clutches:-

There are following types of friction clutches -

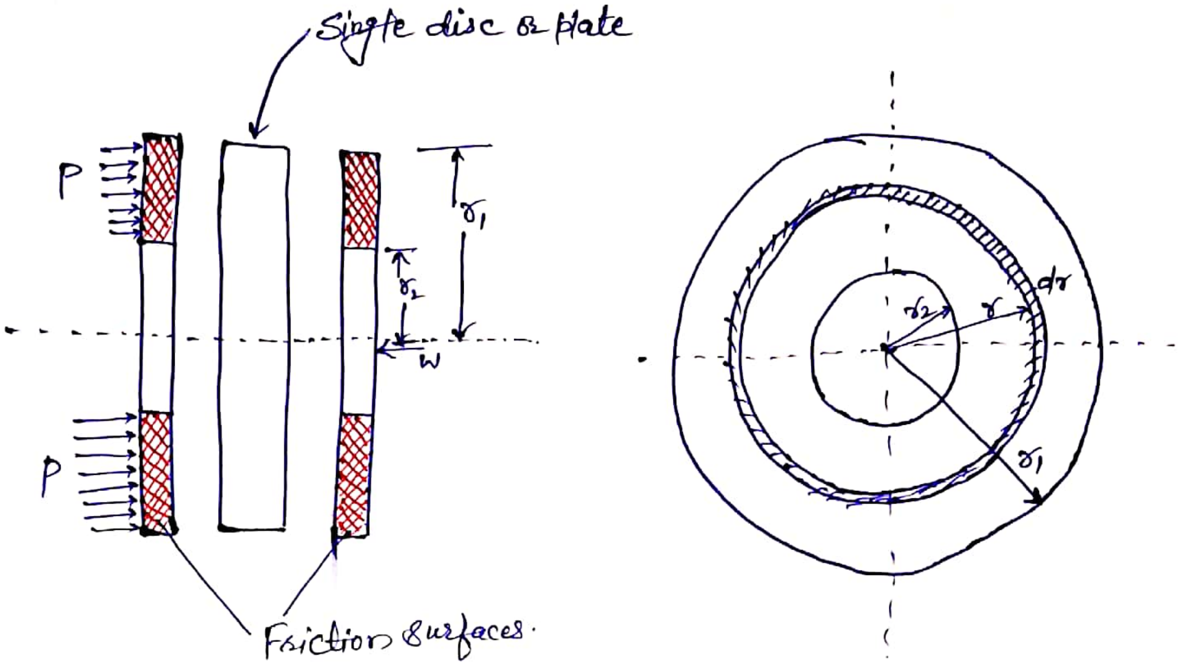
(i) Disc or plate clutches (single disc or multiple disc clutches)

(ii) Cone clutches.

(iii) Centrifugal clutches.

The disc and cone clutches are known as axial friction clutches, while the centrifugal clutch is called radial friction clutch.

* Design of a Disc or plate clutch :-



Let T = Torque transmitted by the clutch

p = Intensity of axial pressure with which the contact surfaces are held together.

r_1 and r_2 = External and internal radii of friction faces.

r = Mean radius of the friction face.

μ = coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in fig.

We know that area of the contact surface or friction surface

$$= 2\pi r\, dr$$

$\therefore$ Normal or axial force on the ring

$$\delta W = \text{Pressure} \times \text{Area}$$

$$= p \times 2\pi r\, dr = 2\pi p r\, dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu p \cdot 2\pi r\, dr$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu p \times 2\pi r\, dr \times r = 2\pi \mu p r^2\, dr$$

We shall now consider the following two cases.

(i) when there is a uniform pressure

(ii) when there is a uniform wear

* **Uniform pressure theory :-**

When the pressure is uniformly distributed over the entire area of the friction face as ~~ab~~, then the intensity of pressure,

$$P = \frac{W}{\pi[r_1^2 - r_2^2]}$$

where W = Axial thrust with which the friction surfaces are held together.

We know that the frictional torque on the elementary ring of radius r and thickness dr is -

$$T_r = 2\pi \mu p r^2 dr$$

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi \mu p r^2 dr = 2\pi \mu P \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$T = 2\pi \mu P \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$T = 2\pi \mu \cdot \frac{W}{\pi[r_1^2 - r_2^2]} \cdot \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$\boxed{T = \mu W R}$$ where $R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$

Considering uniform wearing:-

The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The ~~for~~ work of friction is proportional to the product of normal pressure (p) and the sliding velocity (v). Therefore,

$$\text{Normal wear} \propto \text{work of friction} \propto p.v$$

$$\therefore \quad pv = K$$

$$\text{or,} \quad p = \frac{K}{v}$$

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing ~~&~~ in process continues until the product pv is constant over the entire surface.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore,

$$pr = C$$

$$\text{or} \quad p = \frac{C}{r}$$

and the normal force on the ring,

$$\delta w = p.2\pi r\,dr = \frac{C}{r}.2\pi r\,dr = 2\pi C\,dr$$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\,dr = 2\pi C(r_1 - r_2)$$

Or, $C = \frac{W}{2\pi(r_1 - r_2)}$

Frictional torque acting on the ring

$$T_r = 2\pi \mu p r^2 dr = 2\pi \mu \frac{C}{r} \times r^2 dr$$

∴ Total torque acting on the friction surface

$$T = \int_{r_2}^{r_1} 2\pi \mu C r \, dr = 2\pi \mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$T = \cancel{\pi} \frac{\mu \cdot W}{2 \cdot \cancel{\pi} \cancel{(r_1 - r_2)}} (r_1 + r_2) \cancel{(r_1 - r_2)}$$

$$T = \mu W \frac{(r_1 + r_2)}{2} \Rightarrow \boxed{T = \mu W R}$$ where $R = \frac{r_1 + r_2}{2}$

Note: 1. In general, total frictional torque acting on the friction surfaces is given by:-

$$\boxed{T = n \mu W R}$$

where n = Numbers of pairs of friction surfaces

R = Mean radius of friction surfaces.

$= \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$ → For uniform pressure

$= \frac{r_1 + r_2}{2}$ → For uniform wear

Note-2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (i.e $n = 2$)

Note-3. Since the intensity of pressure is maximum at the ~~outer~~ inner radius (r_2) of the friction or contact surface therefore,

$$\boxed{P_{max} \times r_2 = C} \quad \text{or} \quad \cancel{\boxed{P_{min} \times r_1 = C}}$$

or $$P_{max} = \frac{C}{r_2}$$

Note-4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore,

$$\boxed{P_{min} \times r_1 = C}$$

Note-5 The average pressure (P_{av}) on the friction or contact surface is given by -

$$P_{av} = \frac{\text{Total force on friction surface.}}{\text{Cross-sectional area of friction surface}}$$

$$\boxed{P_{av} = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}}$$

Note-6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch, the uniform wear theory is more approximate.

Note-7: The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

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Q: Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 KN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

Sol^n Given $W = 4\ KN = 4 \times 10^3\ N$, $r_1 = 100\ mm$, $r_2 = 50\ mm$

$\because$ $P_{max} \cdot r_2 = C$

$\therefore$ $C = 50\ P_{max}$

We also know that total force on the contact surface (W).

$$W = 2\pi C (r_1 - r_2)$$

$$4000 = 2\pi \times 50 P_{max} (100 - 50)$$

$$\therefore P_{max} = 0.2546\ N/mm^2 \quad \text{Ans.}$$

Also $P_{min} \times r_1 = C$

$$\therefore C = 100\ P_{min}$$

We also know that,

$$W = 2\pi C (r_1 - r_2)$$

$$4000 = 2\pi \times 100 \cdot P_{min} (100 - 50)$$

$$\therefore P_{min} = 0.1273\ N/mm^2 \quad \text{Ans}$$

Average Pressure,

$$P_{av} = \frac{W}{\pi [r_1^2 - r_2^2]} = \frac{4000}{\pi [100^2 - 50^2]} = 0.17\ N/mm^2$$

Ans

Q: A single plate clutch, effective on both sides, is required to transmit 25 KW at 3000 rpm. Determine the outer and inner diameters of frictional surfaces if the coefficient of friction is 0.255, ratio of dia. is 1.25 and the max^m pressure is not to exceed 0.1 N/mm^2. Also determine the axial thrust to be provided by spring. Assume the theory of uniform wear.

Sol^n: Given $n = 2$, $P = 25$ KW $= 25 \times 10^3$ W, $N = 3000$ rpm

$\mu = 0.255$, $\frac{r_1}{r_2} = 1.25$, $P_{max} = 0.1$ N/mm^2

We know that, the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^3 \times 60}{2\pi \times 3000} = 79600 \text{ N-mm.}$$

For uniform wear conditions,

$$Pr = C$$

$$P_{max} \cdot r_2 = C \quad \Rightarrow C = 0.1\, r_2 \text{ N/mm}$$

and,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 0.1\, r_2 (1.25 r_2 - r_2)$$
$$= 0.157\, r_2^2$$

Also for uniform wear theory,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2} = 1.125\, r_2$$

~~Also~~ Torque transmitted T

$$T = n \mu W R = 2 \times 0.255 \times 0.157\, r_2^2 \times 1.125 r_2$$

$$r_2^3 = \frac{79.6 \times 10^3}{0.09} = 884 \times 10^3$$

$\therefore r_2 = 96$ mm. Ans $\Rightarrow d_2 = 192$ mm Ans

$\therefore r_1 = 1.25 r_2 = 1.25 \times 96 = 120$ mm Ans

$\therefore d_1 = 240$ mm

Axial thrust,

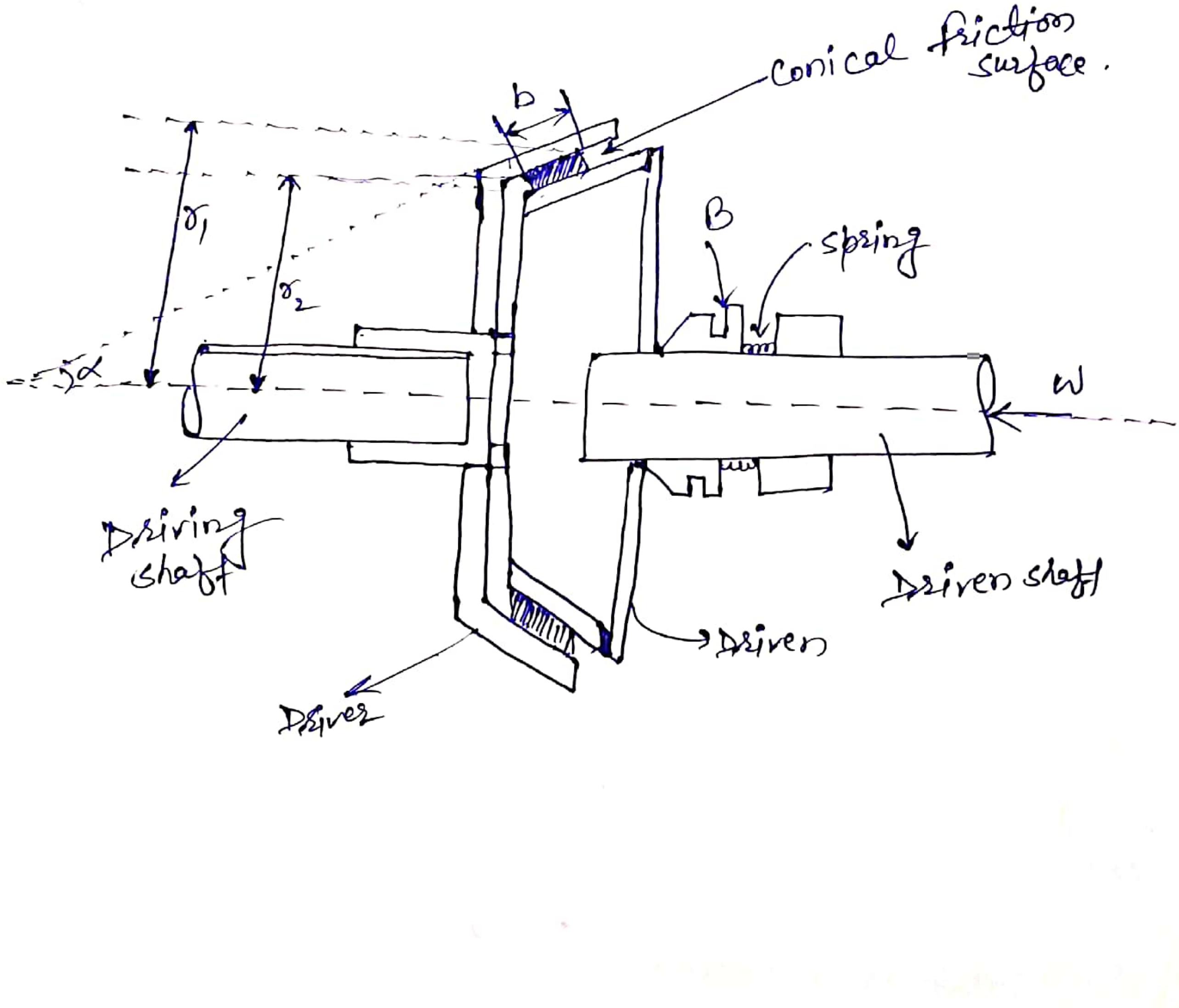
$$W = 2\pi C(r_1 - r_2) = 2\pi \times 0.1\, r_2(1.25 r_2 - r_2)$$

$$W = 2\pi \times 0.1 \times 96 \times 0.25 \times 96$$

$$= 1447\ N.$$ Ans

Cone clutch :-

A Cone clutch was extensively used in automobile, but now a day it ~~is~~ has been replaced completely by the disc clutch. it Consist of one pair of friction surface only . . . .



Design of a Cone clutch :-

* According to uniform pressure theory, Total torque transmitted by the clutch can be given as -

$$\boxed{T = \frac{2}{3} \mu W \operatorname{cosec}\alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

* According to uniform wear theory, Total torque transmitted can be given by -

$$\boxed{T = \mu W \operatorname{cosec}\alpha \left[\frac{r_1 + r_2}{2}\right]}$$

$T = \mu W R \operatorname{cosec}\alpha$ where $R = \frac{r_1 + r_2}{2}$

Note-1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

Note-2. If the clutch is engaged when one member is stationary and the other rotating (i.e during engagement of the clutch) then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu W_n \cos\alpha$) act on the clutch which resist the engagement and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$W_e = W + \mu W_n \cos\alpha = W_n \sin\alpha + \mu W_n \cos\alpha$

$$\boxed{W_e = W_n(\sin\alpha + \mu\cos\alpha)}$$

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Q: An engine developing 45 kW at 1000 rpm is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm². Determine

(i) The face width required

(ii) The axial spring force necessary to engage the clutch.

Sol^n

$P = 45\,kW = 45\times10^3\,W$, $N = 1000\,rpm$, $\alpha = 12.5°$, $D = 500\,mm$

$\mu = 0.2$, $P_n = 0.1\,N/mm^2$, $R = 250\,mm$

$$\because \quad T = \frac{P\times60}{2\pi N} = \frac{45\times10^3\times60}{2\times\pi\times1000} = 430\,N\text{-}m$$

We know that torque developed by the clutch (T),

$$T = 2\pi\mu P_n R^2 b$$

$$430\times10^3 = 2\pi\times0.2\times0.1\times250^2\times b$$

$$\therefore b = 54.7 \simeq 55\,mm \quad \text{Ans}$$

Normal force acting on the contact surface,

$$W_n = P_n\times2\pi R\cdot b$$

$$= 0.1\times2\pi\times250\times55 = 8640\,N.$$

$\therefore$ Axial spring force necessary to engage the clutch,

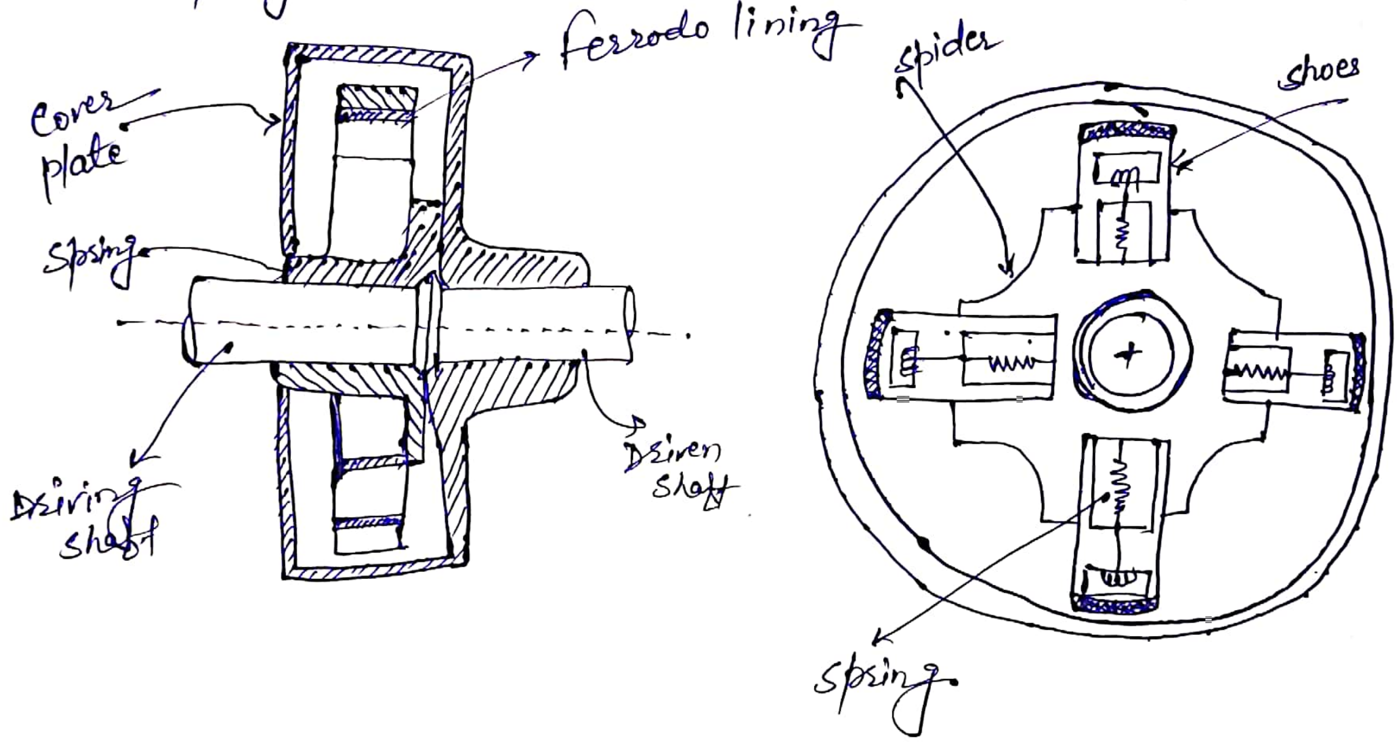
$$W_e = W_n(\sin\alpha + 0.25\mu\cos\alpha)$$

$$= 8640(\sin12.5 + 0.25\times0.2\times\cos12.5)$$

$$= 2290\,N. \quad \text{Ans.}$$

Centrifugal clutch :-

- The centrifugal clutches are usually incorporated into the motor pulleys. it consist of a number of shoes on the inside of a rim of the pulley as shown in fig. the outer surface of the shoes are covered ~~by~~ with a friction material. these shoes, which can move radially in guides are held against a boss. on the driving shaft by means of springs.



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* Design of a Centrifugal clutch :-

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for design a Centrifugal clutch.

(i) Mass of the shoes :-

Let m = mass of each shoe.

n = number of shoes.

r = Distance of centre of gravity of the shoe from the Centre of the spider.

R = inside radius of the pulley rim

N = Running speed of the pulley in r.p.m.

ω = Angular running speed of the pulley in rad/sec

$= \frac{2\pi N}{60}$

ω_1 = Angular speed at which the engagement begins to take place

μ = Coefficient of friction b/w shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed

$$P_c = m\omega^2 r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore,

$$P_s = m(\omega_1)^2 r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16} m\omega^2 r$$

∴ Net outward radial force (i.e centrifugal force) with which the shoe pressing against the rim at the running speed.

$$= P_c - P_s = \frac{7}{16} m\omega^2 r$$

and the frictional force acting tangentially on each shoe

$$F = \mu(P_c - P_s)$$

∴ frictional torque $= F \times R = \mu(P_c - P_s)R$

and total frictional torque transmitted

$$T = \mu(P_c - P_s)R \times n = nFR$$

2. <u>Size of the shoes:-</u>

Let l = Contact length of the shoes
b = width of the shoes,
R = Contact radius of the shoes,
θ = Angle subtended by the shoes at the centre of the spider
P = intensity of the pressure

We know that $\theta = \frac{l}{R}$ or $l = \theta R = \frac{\pi}{3} R$ [Assume $\theta = 60° = \pi/3$]

∴ Area of Contact of the shoe

$= l \times b$

and the force with which the shoe presses against the rim,

$= A \times p = l b p$.

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$ therefore,

$$\boxed{l b p = P_c - P_s}$$

$b = ?$

(3). <u>Dimensions of the spring:-</u>

We know that the load on the spring is given by -

$$\boxed{P_s = \frac{9}{16} \times m \omega^2 r}$$