COURSE FILE

SUBJECT : DESIGN OF TRANSMISSION SYSTEM

SUBJECT CODE : ME-621

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Mechanical Engineering (ME621)

Design of Transmission System

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Objectives: To learn about the design procedures for mechanical power transmission components
Contents:

Module-I Flexible transmission elements- design of flat belts & pulleys, selection of V-belts and pulleys, selection of hoisting wire ropes and pulleys, design of chains and sprockets. (6)

Module-II Gear transmission- speed ratios and number of teeth, force analysis, tooth stresses, dynamic effects, fatigue strength, factor safety, gear materials; Design of straight tooth spur gear and parallel axis helical gears based on strength and wear considerations, pressure angle in the normal and transverse plane; equivalent number of teeth and forces for helical gears. (6)

Module-III Straight bevel gear- tooth terminology, tooth forces and stresses, equivalent number of teeth. Estimating the dimensions of a pair of straight bevel gears. (4)

Module-IV Worm gear, merits & demerits, terminology, thermal capacity, materials, forces & stresses, efficiency, estimating the size of worm gear pair. Cross helical gears, terminology, helix angles, sizing of a pair of helical gears. (4)

Module-V Gear box- geometric progression, standard step ratio, Ray diagram, kinematics layout, Design of sliding mesh gear box- Design of multi-seed gear box for machine too application ; constant mesh gear box, speed reducer unit; Variable speed gear box; Fluid couplings, Torque converters for automotive applications. (10)

Module-VI Cam design, types; pressure angle and undercutting base circle determination, forces and surface stresses; Design of plate clutches, axial clutches, cone clutches, internal expandingrim clutches; Electromagnetic clutches; Band and Block brakes. (6)

Module-VII External shoe brakes, internal expanding shoe brake. (4) Course Outcomes: Upon completing this course the students will be able to design transmission systems for engines and machines.

Text Books:

(i) Shigley J., Mischke C., Budynas R. and Nisbett K., Mechanical Engineering Design, 8thed., Tata McGraw Hill, 2010

(ii) Jindal U.C., Machine Design: Design of Transmission System, Dorling Kindersley, 2010.

(iii) Maitra G. and Prasad L., Handbook of Mechanical Design, 2nd ed., Tata McGraw Hill,2001

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Flat Belt drives.

- **Introduction:-** The belt or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors.

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The condition under which the belt is used.

- **Selection of belt drives:-**

Following factors for selection of belt drives.

1. Speed of the driving and driven shafts.
2. Speed reduction ratio.
3. Power to be transmitted.
4. Centre d/s b/w the shaft
5. Positive drive requirements.
6. Shaft layout.
7. Space available
8. Service Conditions.

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* Types of belt drives:-

The belt drives usually classified into the following three groups.

1. Light drives – These are used to transmit small power at belt speeds upto 10 m/sec as in agricultural machines and small machine tools.
2. Medium drives — These are used to transmit medium power at belt speed over 10 m/sec upto 22 m/sec, as in m/c tools.
3. Heavy drives — These are used to transmit large power at belt speed above 22 m/sec in compressor and generators.

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* Coefficient of friction b/w belt and Pulley.

The coefficient of friction b/w belt and pulley depends upon the following factors.

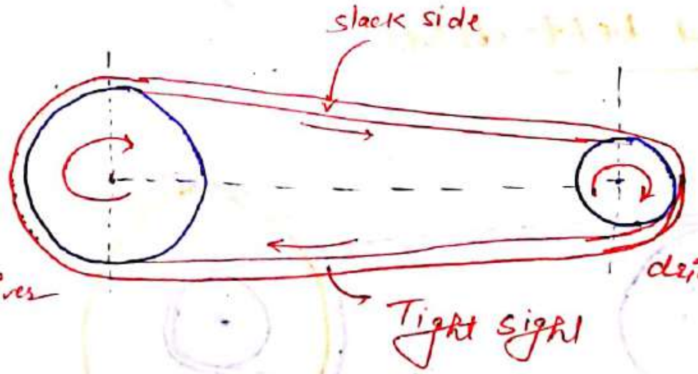
1. The material of belt.
2. the material of pulley.
3. the slip of the belt.
4. the speed of belt.

A/c to C.G. Barth the Coeff^n of friction (μ) can be given by

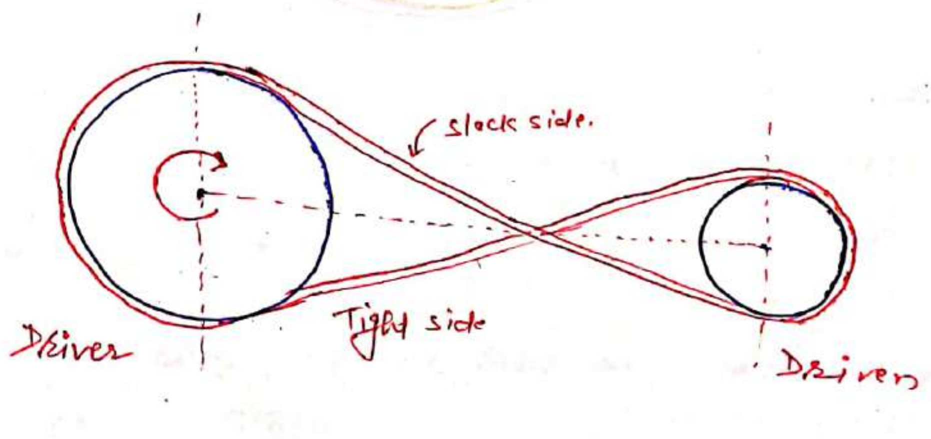
$$\boxed{\mu = 0.54 - \frac{42.6}{152.6 + V}}$$

* Types of belt drives:-

(1) open belt drives.



2. Crossed or twist belt drive:-



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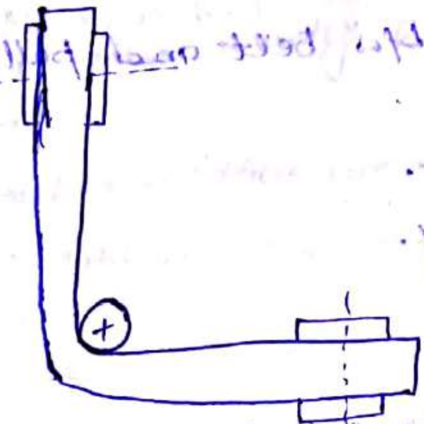
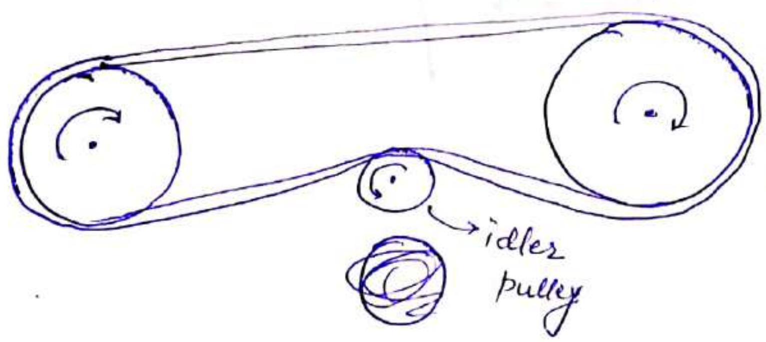
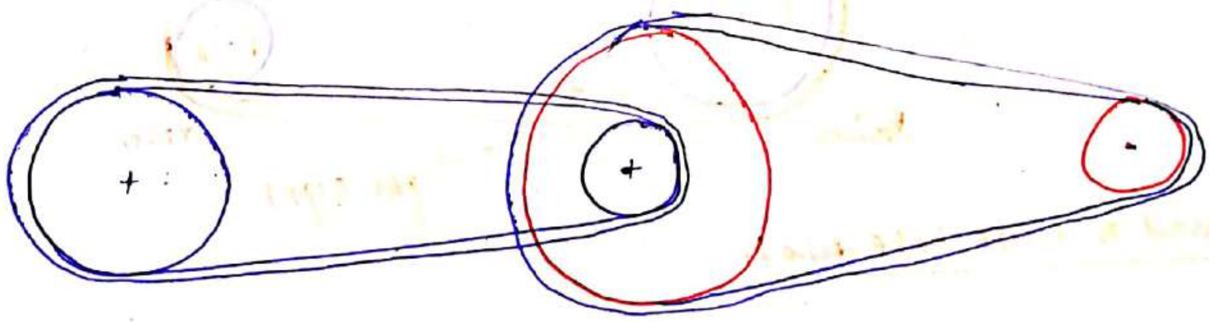
3. Quarter turn belt drive.



4. Belt drive with idler pulley



5. Compound belt drives.



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* velocity ratio of belt drives

It is the ratio ~~of~~ between the velocities of the driver and the follower or driven.

Let d_1 = dia of the driver
d_2 = " " follower or driven
N_1 = speed of driver in rpm.
N_2 = " " driven

∴ length of the belt that passes over the driver, in one minutes
$= \pi d_1 N_1$.

Similarly length of the belt that passes over the driven or follower in one minute
$= \pi d_2 N_2$.

Since the length of the belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minutes. So

therefore,

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

when thickness of belt is considered then,

$$\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$$

Note :- the velocity ratio of a belt drive may be also be obtained as -

- We know that the periferal velocity of the belt on the driving pulley
$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/sec.}$$

- Peripheral velocity of the belt on the driven pulley
$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/sec.}$$

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when there is no slip.

then $v_1 = v_2$

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

2. In Case of Compound belt drives,

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} = \frac{\text{Product of dia. of drivers.}}{\text{" " " " driven.}}$$

* Slip of the belt :-

when slip of the belt is also consider then,

Let $s_1\%$ = slip b/w the driver and the belt

$s_2\%$ = " " belt " " follower

$$\boxed{\therefore \frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1-\frac{s}{100}\right)}$$

where $s = s_1 + s_2$
= Total % of slip.

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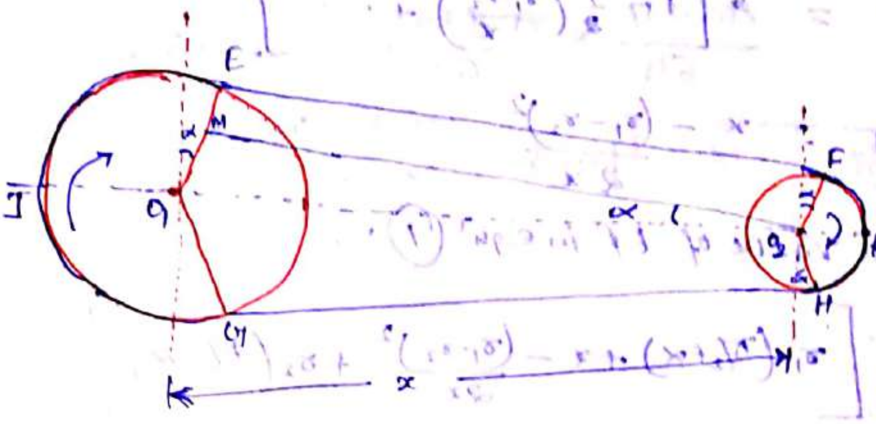
* Length of an open belt drives:-



Let r_1 and r_2 = Radii of the larger and smaller pulleys.

x = D/s b/w centres of pulley

L = Total length of ~~pulley~~ Belt.

We know that the length of the belt,

$$L = Arc\ GJE + EF + Arc\ FKH + HG$$

$$= 2(Arc\ JE + EF + Arc\ FK) \quad \text{———} \quad (1)$$

From geometry,

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

$\theta = \frac{Arc}{Radius}$

$Arc = Radius \times \theta$

$= r_1 \times (\pi/2 + \alpha)$

Since α is very less

$$\therefore \sin\alpha = \alpha = \frac{r_1 - r_2}{x}$$

$$Arc\ JE = r_1\left(\pi/2 + \alpha\right)$$

Similarly $Arc.\ FK = r_2\left(\pi/2 - \alpha\right)$

$$EF = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

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Expanding through bionomial distribution theorem.

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\right]$$

$$EF = x - \frac{(r_1 - r_2)^2}{2x}$$

Putting the value of EF in eqn (1).

$$L = 2\left[r_1\left(\pi/2 + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\pi/2 - \alpha\right)\right]$$

$$= 2\left[r_1 \times \pi/2 + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \pi/2 - r_2\alpha\right]$$

$$= 2\left[\pi/2\,(r_1 + r_2) + \alpha\,(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Put $\alpha = \frac{r_1 - r_2}{x}$

$$\therefore L = \pi(r_1 + r_2) + 2 \times \left(\frac{r_1 - r_2}{x}\right)(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

$$\boxed{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}}$$

* Length of Cross belt

$$\boxed{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}}$$

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Chain drive.

A chain can be defined as a series of links connected by pin ~~pin~~ joints. Chain drive is a positive drive with constant velocity ratio, i.e ratio of angular velocity of driving sprocket wheel and angular velocity of driven sprocket wheel, with no slip and no creep which occurs in belt drive. - The chain drive is intermediate between belt and ~~ten~~ gear drive.

* Advantage of chain drives compared with belt and gear drive

(i) Chain drives can be used for long as well as short centre distance. They are particularly suitable for medium centre distance, where gear drives will require additional idler gears. Thus chain drive can be used over a wide range of centre distances.

(ii) A single chain can be drive a number of shafts, from a single source of power.

(iii) Chain drives have small over all dimensions than belt drives, resulting in compact unit.

(iv) A chain does not slip and to that extent chain drive is a positive drive.

(v) The efficiency of chain drives is high. For properly lubricated chain the efficiency of chain drive is 96% to 98%.

(vi) Chains are easy to replace.

(vii) Atmospheric conditions and temperatures do not affect the performance of chain drives. They do not present any fire hazard.

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* Disadvantages of Chain drive:-

(i) Chain drives are not suitable where precise motion is required due to polygonal effect. The velocity of the chain is not Constant resulting in non-uniform speed of the driven shaft.

~~(ii) Chain drive is unsuitable where precise motion is required due to polygonal effect. The velocity of chain is not Constant.~~

(ii) chain drives are not suitable for non-parallel shafts. Bevel and Worm gears and quarter-turn belt drives can be used for non-parallel shaft.

(iii) chain drives require housing.

(iv) Compared with belt drives, chain drives requires precise alignment of shafts. However the centre distance is not as critical as in case of gear drive.

(v) Chain drives require housing

(vi) Chain drives generate noise.

* Types of chains.

There are different types of chains with respect to their purpose, chains are classified into the following three groups.

(i) Load lifting chains.

(ii) Hauling chains.

(iii) Power transmission chains.

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* Velocity Ratio of chain drives:-

The velocity ratio of chain drive is given by -

$$V.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

where N_1 = speed of rotation of smaller sprocket in r.p.m.

N_2 = " " " larger

T_1 = Numbers of teeth on the smaller sprocket.

T_2 = " " " larger "

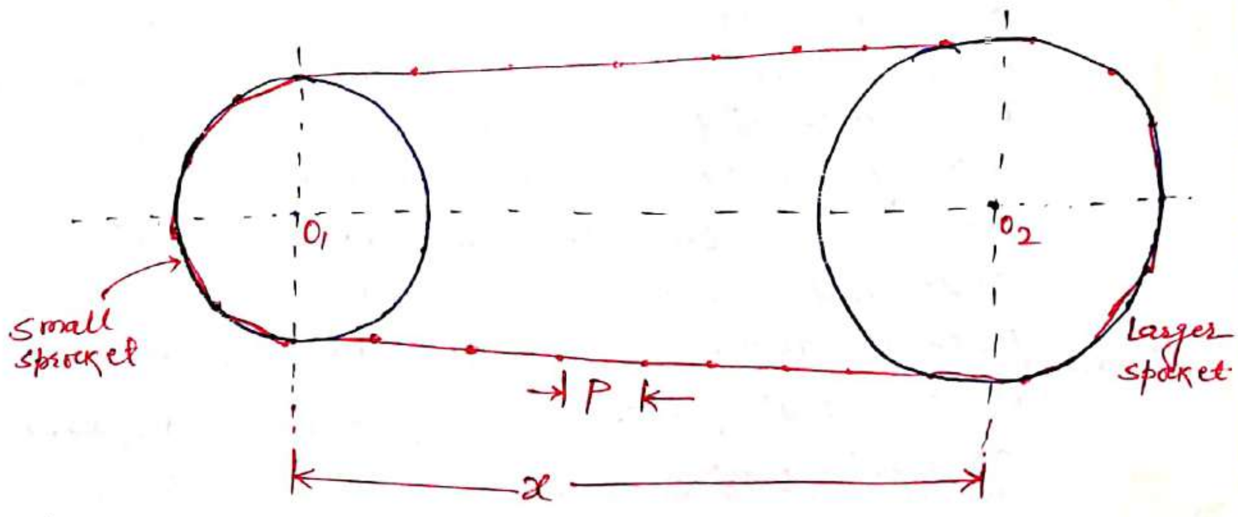
The average velocity of the chain is given by -

$$v = \frac{\pi D N}{60}$$ where D = Pitch circle diameter of the sprocket in m.

$$= \frac{T.P.N}{60}$$ P = Pitch of the chain in m

* Length of chain and centre distance:-

An open chain drive system connecting the two sprockets as shown in fig.



Let T_1 = Numbers of teeth on the smaller sprocket.

T_2 = " " " larger "

P = Pitch of the chain

x = Centre distance.

The length of chain (L) must be equal to the Product of the number of chain links (K) and the pitch of the chain (P). Mathematically,

$$\boxed{L = K \cdot P}$$

where K = no's of links.
P = pitch of the chain.

The number of chain links may be obtained from the following expression. i.e-

$$\boxed{K = \frac{T_1 + T_2}{2} + \frac{2x}{P} + \left[\frac{T_2 - T_1}{2\pi}\right]^2 \frac{P}{x}}$$

The value of K must be approximately even number.

The centre distance is given by-

$$\boxed{x = \frac{P}{4}\left[K - \frac{T_1+T_2}{2} + \sqrt{\left(K - \frac{T_1+T_2}{2}\right)^2 - 8\left(\frac{T_2 - T_1}{2\pi}\right)^2}\right]}$$

* Material for chain drives:-

Small sprockets are made of steel and sometimes they are made integral with the shaft. In large sizes, sprockets are made of cast iron which is strong enough, because pull is taken by all the teeth in contact with the chain. 55C8 steel with C 0.5 – 0.6% and Mn 0.6 – 0.9% is used for cycle and industrial chains. For higher strength, chain parts are also made from 35Ni1Cr60 alloy steel which contains C 0.3 – 0.4%, Si 0.1 – 0.35%, Mn 0.6 – 0.9%, Ni 1 – 1.5% & Cr 0.45% – 0.75%.

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* Selection of chain:-

A chain has to be selected from a catalogue of a manufacturer and selection depends upon the numbers of factors.

1. Numbers of teeth on smaller sprockets wheel.
2. Power to be transmitted.
3. Pitch of the chain i.e distance b/w centres of rollers.
4. speed of smaller sprocket.
5. Tanjential force in chain during power transmission.

* Design of Procedure of chain drive :-

The chain drive is designed as discuss below-

1. First of all, determine the velocity ratio of the chain drive.
2. Select the minimum number of teeth on the smaller sprocket or pinion from table.

	Number of teeth at velocity ratio					
	1	2	3	4	5	6
Roller	31	27	25	23	21	17
silent	40	35	31	27	23	19

3. Find the number of teeth on the larger sprocket.
4. Determine the design power by using the service factor such as -

   Designed Power = Rated Power x service factor

5. Choose the type of chain, numbers of strands for the design Power and r.p.m of the smaller sprocket.
6. Note down the parameter of the chain, such as pitch, roller diameter, minimum width of roller etc.
7. Find Pitch circle diameter and pitch line velocity of the smaller sprocket.

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8. Determine the load (W) on the chain by using the following rel^n:

$$W = \frac{\text{Rated Power}}{\text{Pitch line velocity}}$$

9. Calculate the factor of safety by dividing the breaking load (W_B) to the load on the chain (W).

10. Find the centre d/s b/w the sprockets.

11. Determine the length of chain.

Q: Design a chain drive to actuate a compressor from 15 KW electric motor running at 1000 rpm, the compressor speed being 350 r.p.m. The minimum centre d/s is 500 mm. The compressor operates 16 hours per day. The chain tension may be adjusted by shifting the motor on slides.

Sol^n Given Rated Power = 15 KW, $N_1 = 1000$ rpm, $N_2 = 350$ rpm

Centre distance = 500 mm.

We know

Step-1. velocity ratio $V.R = \frac{N_1}{N_2} = \frac{1000}{350} = 2.86 \simeq 3$

Step 2. We find that for the roller chain, the number of teeth on the smaller sprocket or pinion (T_1) for a velocity ratio of 3 is 25.

$$\therefore \frac{T_2}{T_1} = \frac{N_1}{N_2}$$

$$\therefore T_2 = T_1 \times \frac{N_1}{N_2} = 25 \times \frac{1000}{350}$$

$$T_2 = 71.5 \simeq 72$$

Step 3. Designed power = Rated Power × service factor (K_s)

~~& K_s = 1.5 for vari~~

$K_s = K_1 \times K_2 \times K_3$

where K_1 = Load factor = 1, for constant load.
= 1.5 for heavy shock load
= 1.25 for variable load with mild shock.

K_2 = Lubrication factor = 0.8 for continuous lubrication.
= 1 for drop lubrication
= 1.5 for periodic lubrication

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K_3 = Rating factor = 1, for 8 hours/day
= 1.25 for 16 hours/day
= 1.5 for continuous service.

$\therefore K_s = K_1 \times K_2 \times K_3$
$= 1.5 \times 1 \times 1.25$
$= 1.875$

$\therefore$ Design Power $= 15 \times 1.875$
$= 28.125$ KW. Ans

step-4. From table we find that corresponding to a pinion speed of 1000 rpm, the power transmitted for chain No 12B is 15.65 KW per strand. Therefore a chain no 12B width two strands can be used to transmit the required power. from table we find that

step-5: Pitch, $P = 19.05$ mm.

Roller diameter $d = 12.07$ mm

Minimum width of roller $w = 11.68$ mm

Breaking load $W_B = 59$ KN

Pitch can also be given as
$P = \frac{x}{(30-60)}$
30 to 60

$\therefore$ Pitch circle diameter of the smaller sprocket or pinion

$$d_1 = P \operatorname{cosec}\left(\frac{180}{T_1}\right) = 19.05 \operatorname{cosec}\left(\frac{180}{25}\right)$$
$$= 152 \text{ mm.}$$

Pitch circle diameter of the larger sprocket or gear

$$d_2 = P \operatorname{cosec}\left(\frac{180}{T_2}\right) = 19.05 \operatorname{cosec}\left(\frac{180}{72}\right)$$
$$= 436 \text{ mm.}$$

Pitch line velocity of the smaller sprocket,

$$v_1 = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.152 \times 1000}{60} = 7.96 \text{ m/sec.}$$

step-6. Load on the chain

$$W = \frac{\text{Rated Power}}{\text{Pitch line velocity}} = \frac{15}{7.96} = 1844 \text{ N.}$$

step-7. Factor of safety $= \frac{W_B}{W} = \frac{59 \times 10^3}{1844} = 32$

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This value is more than the given value in the table.

$\therefore$ Factor of safety = 11

The minimum centre d/s b/w the smaller and larger sprockets should be 30 to 50 times the pitch. Let us take it as 30 times the pitch.

step 8. $\therefore$ Centre d/s b/w the sprockets,

$$= 30P = 30 \times 19.05 = 572 \text{ mm}$$

Lit ← sag | In order to accomodate initial sag in the chain, the value of centre d/s is reduced by 2 to 5 mm.

$\therefore$ Correct centre distance $x = 572 - 4$

$$x = 568 \text{ mm.}$$

We know that the number of chain links

$$K = \frac{T_1 + T_2}{2} + \frac{2x}{P} + \left[\frac{T_2 - T_1}{2\pi}\right]^2 \frac{P}{x}$$

$$= \frac{25+72}{2} + \frac{2 \times 568}{19.05} + \left[\frac{72-25}{2\pi}\right]^2 \times \frac{19.05}{568}$$

$$= 110$$

Step 9. $\therefore$ Length of chain

$$L = KP = 110 \times 19.05$$

$$= 2.096 \text{ m} \quad \underline{\text{Ans}}$$

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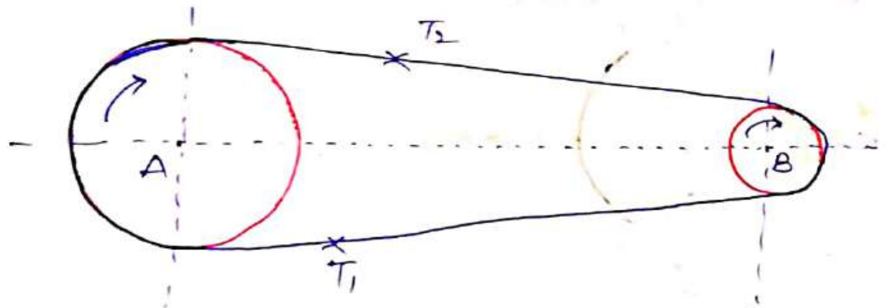
Power transmitted by a Belt :-



Let T_1 and T_2 = Tension in the tight side and slack side of the belt respectively.

r_1 and r_2 = Radius of the driving and driven pulley.
v = velocity of the belt in m/sec.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference b/w the two tensions (i.e $T_1 - T_2$).

∴ Work done per second = $(T_1 - T_2)\,v$ N m/sec

∴ $\boxed{\text{Power transmitted} = (T_1 - T_2)\,v}$ Watt. ∵ $1\ \frac{N\cdot m}{s} = W$.

& Torque exerted on the driving Pulley is $(T_1 - T_2)\,r_1$.

& Torque exerted on the driven pulley is $(T_1 - T_2)\,r_2$.

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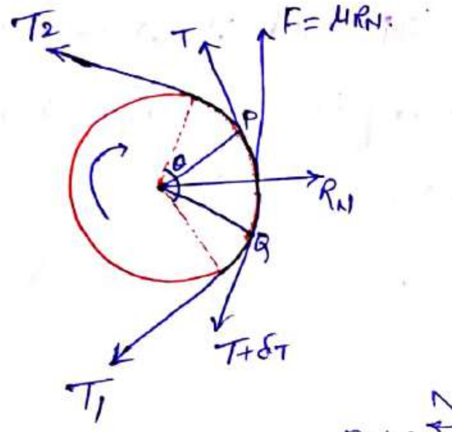
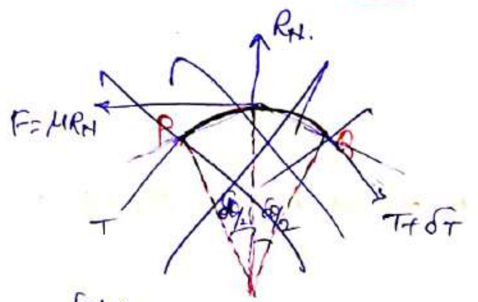
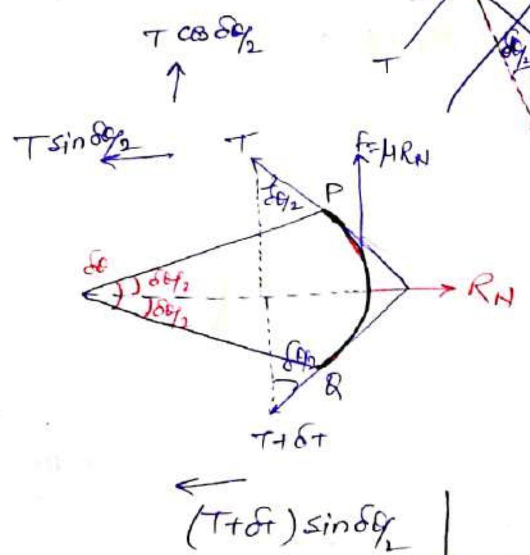
* Ratio of tension for flat belt drive.







Resolving the forces horizontally and vertically we get,

$$R_N = (T+\delta T)\sin\delta\theta/_2 + T\sin\delta\theta/_2 \quad \text{———} \quad (1)$$

$$\Rightarrow R_N = (T+\delta T)\,\delta\theta/_2 + T\cdot\delta\theta/_2 \Rightarrow R_N = T\cdot\delta\theta + \frac{\delta T\cdot\delta\theta}{2} \xrightarrow{\text{neglecting}} \therefore \boxed{R_N = T\,\delta\theta}$$

&

$$\mu R_N + T\cos\delta\theta/_2 = (T+\delta T)\,\cancel{\sin\delta\theta/_2}\,\cos\delta\theta/_2 \quad \text{———} \quad (2)$$

$$\Rightarrow \mu R_N + T = (T+\delta T)$$

$$\cancel{\Rightarrow \mu R_N + T = T\cdot\frac{\delta\theta}{2} + \frac{\delta T\cdot\delta\theta}{2}}$$

$$\cancel{\Rightarrow \mu\cdot T\delta\theta + T = T\frac{\delta\theta}{2}}$$

neglecting

$$\Rightarrow \mu R_N + \cancel{T} = \cancel{T} + \delta T$$

$$\mu R_N = \delta T$$

$$\Rightarrow \mu R_N = \delta T$$

$$R_N = \frac{\delta T}{\mu}$$

2. ~~$T\delta\theta = \mu\delta$~~

Now
$$T\delta\theta = \frac{\delta T}{\mu}$$

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu\int_0^{\theta} \delta\theta$$

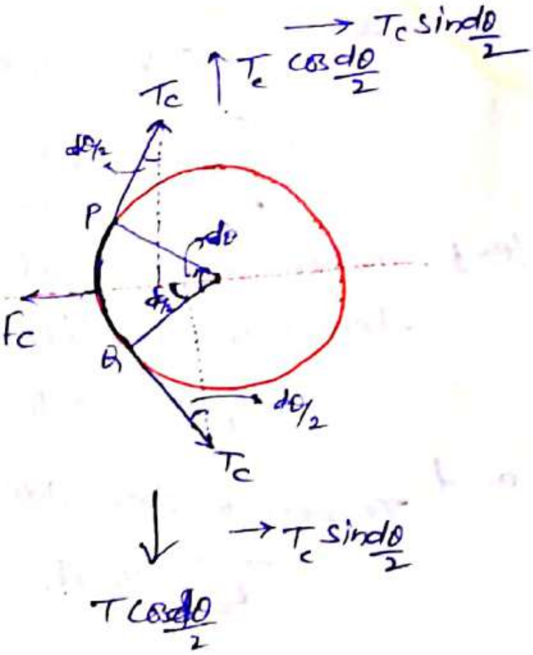
$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

In terms of Corresponding logarithm to the base 10 i.e.

$$\boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta}$$

Centrifugal Tension.



Since the belt run continuously over the pulley, therefore, some centrifugal force is caused whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension.

Let m = mass of belt per unit length in kg.

v = Linear velocity of belt in m/s.

r = Radius of pulley over which the belt runs.

T_c = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r\,d\theta$$

and mass of the belt PQ $= m r\,d\theta$

$\therefore$ Centrifugal force acting on the belt PQ,

$$F_c = m r\,d\theta \times \frac{v^2}{r}$$

$$F_c = m\,d\theta \cdot v^2$$

The centrifugal tension T_c acting tangentially at P and Q keeps the belt in equilibrium. Now

$$F_c = T_c \cdot \sin\frac{d\theta}{2} + T_c \sin\frac{d\theta}{2}$$

$$F_c = T_c \cdot \frac{d\theta}{2} + T_c \cdot \frac{d\theta}{2}$$

$$F_c = T_c \cdot d\theta$$

$$\Rightarrow \quad m\,d\theta \cdot v^2 = T_c\,d\theta.$$

$$\therefore \boxed{T_c = m v^2}$$

Note-1. When centrifugal tension is taken into account, then total tension in the tight side

$$T_{t_1} = T_1 + T_c$$

and Tension in slack side

$$T_{t_2} = T_2 + T_c$$

Note-2. Power transmitted

$$P = (T_{t_1} - T_{t_2})v$$

$$\boxed{P = (T_1 - T_2)v}$$

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* Condition for the transmission of Maximum Power :-

We know that

$$P = (T_1 - T_2)v \quad \text{——— (1)}$$

Also

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta}} \quad \text{——— (2)}$$

Putting the value of T_2 in eqn (1).

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)v$$

$$P = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v = T_1 v C \quad \text{where } C = 1 - \frac{1}{e^{\mu\theta}} \quad \text{——— (3)}$$

We also know that

$$T_1 = T - T_c$$

where T = maximum tension to which the belt can be subjected in newtons.

T_c = Centrifugal tension.

Putting the value of T_1 in eqn (3)

$$\therefore P = (T - T_c)vC \qquad \Big|\ \because T_c = mv^2$$

$$P = (Tv - mv^3)C$$

For max^m power

$$\frac{dP}{dv} = 0$$

$$T - 3mv^2 = 0$$

$$T = 3mv^2$$

$$T = 3T_c$$

It shows that when the power transmitted is maximum, $\frac{1}{3}$rd of the maximum tension is absorbed as centrifugal tension.

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Note 1. We know that $T_1 = T - T_c$ and

and for max^m power $T_c = \frac{T}{3}$

$\therefore T_1 = T - \frac{T}{3}$

$$\boxed{T_1 = \frac{2T}{3}}$$

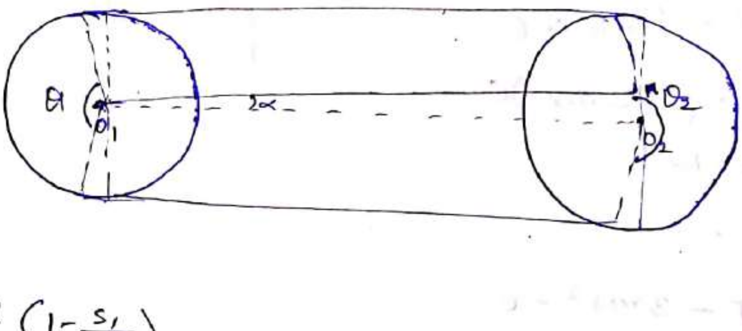
Note-2. The velocity of the belt for maximum power

$$\boxed{v = \sqrt{\frac{T}{3m}}}$$

Q: Design a belt drive to transmit 110 KW for a system consisting of two pulleys of diameters 0.9 m and 1.2 m, centre d/s of 3.6 m, a belt speed 20 m/sec, coeff^t of friction 0.3, a slip of 1.2% at each pulley and 5% friction loss at each shaft, 20% over load.

Sol^n Given $P = 110\ KW = 110 \times 10^3 W$, $d_1 = 0.9m \Rightarrow r_1 = 0.45m$, $d_2 = 1.2m$, $r_2 = 0.6m$.
$x = 3.6m$, $v = 20m/sec$, $\mu = 0.3$, $S_1 = S_2 = 1.2\%$

Let N_1 = speed of the driving pulley
N_2 = " " driven pulley



We know that

$$v = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

$$20 = \frac{\pi \times 0.9 \times N_1}{60}\left(1 - \frac{1.2}{100}\right)$$

$\therefore N_1$ = 430 r.p.m.

Peripheral velocity of the driven pulley

$$\frac{\pi d_2 N_2}{60} = v\left(1 - \frac{S_2}{100}\right)$$

$$\frac{\pi \times 1.2 \times N_2}{60} = 20\left(1 - \frac{1.2}{100}\right)$$

$\therefore N_2 = 315$ rpm.

We know that torque acting on the driven shaft

$$\because P = \frac{2\pi N_2 T}{60}$$

$$\Rightarrow T = \frac{P \times 60}{2\pi N_2} = \frac{110 \times 10^3 \times 60}{2\pi \times 315} = 3334 \text{ N-m}$$

Since there is a losses of 5% due to friction, therefore torque acting on the belt

$$= 1.05 \times 3334 = 3500 \text{ N-m}.$$

Since the belt is to be designed for 20% over load, therefore designed torque $= 1.2 \times 3500 = 4200$ N-m.

We Also know that, torque exerted on the driven pulley

$$T = (T_1 - T_2) r_2$$

$$4200 = (T_1 - T_2) \times 0.6$$

$$T_1 - T_2 = 7000 \text{ N-m} \quad \text{——— (1)}$$

Also from diagram,

$$\sin\alpha = \frac{O_2 M}{O_1 O_2} = \frac{r_2 - r_1}{x} = \frac{0.6 - 0.45}{3.6} = 0.0417$$

$\therefore \alpha = 2.4°$

Also $\frac{T_1}{T_2} = e^{\mu\theta_1}$

$$\frac{T_1}{T_2} = 2.51 \quad \text{——— (2)}$$

From (1) & (2) $T_1 = 11636$ N-m

$T_2 = 4636$ N-m

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Since the belt speed is more than 10m/sec. therefor Centrifugal tension must be taken into Consideration. Assuming a leather belt for which the density may be taken as 1000 kg/m3.

$\therefore$ Mass of the belt per meter length

$$m = \text{Area} \times \text{length} \times \text{density}$$
$$= b \times t \times l \times \rho$$
$$= b \times 0.015 \times 1 \times 1000 = 15b \text{ kg/m}.$$

and Centrifugal tension

$$T_c = mv^2$$
$$= 15b \times 20^2 = 6000\, b \text{ N}.$$

We know that maximum tension in the belt.-

Let
$\sigma = 2.5$ mPa $= 2.5 \times 10^6$ N/mm^2
t = thickness of belt = 15mm
b = width of the belt in m.

$$T = T_1 + T_c = \sigma bt$$
$$\Rightarrow 11636 + 6000b = 2.5 \times 10^6 \times b \times 0.015$$
$$\therefore b = 0.37\text{m} = 370\text{mm}.$$

and length of the belt

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_2 - r_1)^2}{x}$$
$$= \pi(0.6 + 0.45) + 2 \times 3.6 + \frac{(0.6 - 0.45)^2}{3.6}$$
$$= 10.506\text{m}.$$

Ans

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Module-2.

Gear drive.

Gear:- Gear is defined as a machine element used to transmit power between rotating shafts by means of progressive engagement of projection called teeth.

Gears are toothed members which transmit power/motion between two shafts by meshing without any slip. Hence, gear drives are also called positive drives.

→ Gears operates in pairs, the smaller of the pair called the pinion and the larger is gear.

Types of gear drives

Relative position of shafts.

- Parallel
  - → Spur gear
  - → Helical gear
  - → Herringbone gear
- Intersecting
  - → Straight bevel gear
  - → Spiral bevel gear
- Non-intersecting
  - → Hypoid gear
  - → Worm gear

* Spur gear :-

→ spur gears have their teeth parallel to the axis and are used for transmitting power between two parallel shafts.

→ They are simple in construction, easy to manufacture and cost is less.

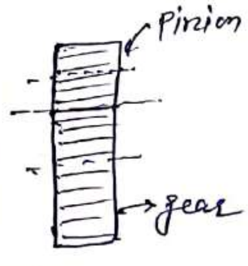
→ They have highest efficiency and excellent precision rating.

Depending of size and type.

Maximum power = 18 KW , speed = 10,000 rpm

circumferential velocity = 200 m/sec.

Over all efficiency = 96 – 99%



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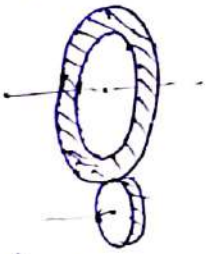
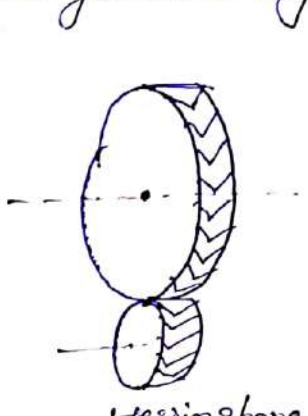
* Helical gear :-

Have teeth inclined to the axis of rotation. Helical gears are not so noisy because more gradual engagement during meshing



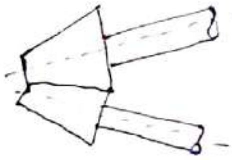
(Helical gear)



Herringbone gear
(Both side helix).

* Bevel gear :-

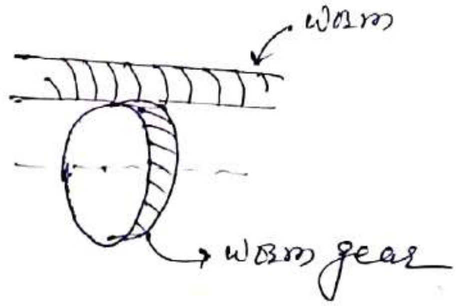
Have teeth formed on conical surfaces and are mainly used for transmitting power between intersecting shafts.



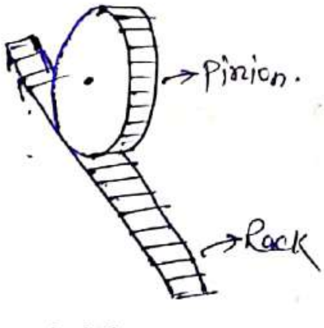
(used in Automobile).

* Worm and worm gear :-

worm gear resembles a screw.



* Rack and pinion



used in drilling machine.

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International Organization for ~~Standards~~ Standardization (ISO) recommendations.

ISO No. 888

international vocabulary of gears.

ISO No. R701

International Gear Notations

Symbols for Geometrical data.

* Law of gearing :-

The fundamental law of gearing states that the angular velocity ratio between the gears of a gear set must remain constant throughout the mesh.

$$\boxed{\frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}}$$

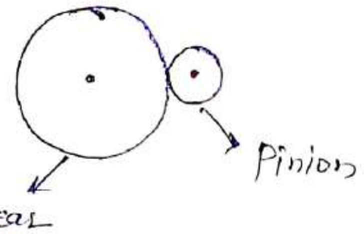
where

n = speed

d = diameter

z = number of teeth

ω = angular speed



* Gear Profile.

Profiles which can satisfy the law of gearing are -

1. Involute
2. cycloidal
3. Circular arc or Novikov.

For producing constant velocity ratio, the curved profiles of the mating teeth must be such that the law of gearing is satisfied.

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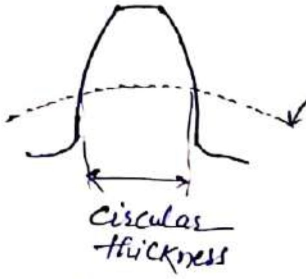
Involute Profile.



* Advantages of involute profile

(i) variation in centre distance does not affect the velocity ratio.

(ii) Pressure angle remains constant throughout the engagement which result in smooth running.

(iii) straight teeth of basic rack for involute admit simple tools. Hence manufacturing becomes simple and cheap.

* Advantages of cycloidal gears:-

(i) cycloidal gears do not have interference.

(ii) cycloidal teeth is generally stronger than an involute tooth owing to spreading flanks in contrast to the radial flanks of an involute tooth.

(iii) Because of the spreading flanks, they have high strength and compact drives are achievable.

(iv) cycloidal teeth have longer life since the contact is mostly rolling which results in low wear.

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- Spur gear.

$$\text{Gear ratio} := \frac{\text{No's of teeth on driven shaft.}}{\text{No's of teeth on driver shaft.}}$$

[For spur gear, Gear ratio lies 1:1 to 6:1].

- Advantages of spur gear :-

1. Simple in construction.
2. Easy to manufacture.
3. Low cost.
4. Excellent precision rating.

- Disadvantages of spur gear :-

1. We can transmits power when centre d/s is limited.
2. Noise at high speed
3. Teeth are straight so large stress occurs.
4.

- Application of spur gear :-

(i) used in watches.
(ii) washing machine.
(iii) It is used for increase or decrease the torque. in gear pump.
(iv) used in motor cycle gear boxes.

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Helical gear

→ The gear whose teeth are inclined to axis are Known as Helical gear.

Advantages of Helical gear

1. Higher load carrying capacity.
2. Smoother and quiter.
3. Less wear and tear.
4. Constant output.

Disadvantages:-

1. Costlier than spur gears.
2. Required thrust bearing
3. Generation of heat.

Applications of helical gear

1. Automobile Gear box.
2. Fertiliser industry.
3. Printing Industry.
4. Plastic Industry
5. Food Industry.

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3. Herringbone gear (Double helical gear)

Double helical gear

Herring.bone gear is a double helical gear. This gear pair is also used to transmit power and motion b/w two parallel shaft. Herringbone gear has opposite helical teeth.

- Gear ratio → 10:1

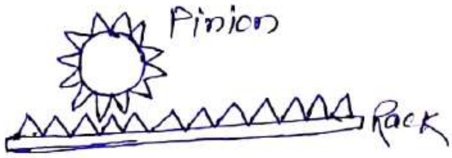
Advantages:-

1. Shaft is free from axial force.
2. High power transmission capacity. → Due to double helical.
3.

Disadvantages

1. Manufacturing cost is high:- Due to double ~~helical~~ helical
2. Load must be equally distributed.
3.

4. Rack and Pinion.



Rack is a segment of gear with infinite diameter. The teeth are spur or helical i.e teeth are parallel or inclined to the pinion. This gear pair are used to convert rotary motion of pinion into linear motion of Rack.

Advantages :-

1. Easiest way ~~to~~ of conversion. → Rotary motion into Reciprocating motion or vice versa.
2. This gear pair is compact in size.
3. Robust in construction.
4. It is cheaper.

Disadvantages:-

1. Noise in the operation.
2. Always work with friction.
3.

Application.

1. Lathe carriage
2. Drilling m/c.
3. Steering system.
4. Stair lift.

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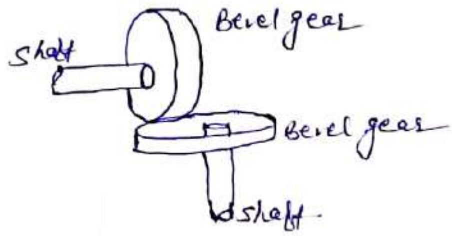
5. Bevel gear :-



Bevel gears are used to transmit power and motion b/w two intersecting shaft. Here shaft are not exactly 90°.

Gear ratio 3:2 to 10:1

Advantages :-

1. High speed and load.
2. It is suitable for 1:1 gear velocity ratio.

Disadvantages:-

1. Complicated in design and manufacture.
2. It requires more precision.

Applications:-

1. Automobile differential.
2. Material handling equipments.
3. Food canning industry
4. Food packaging industry

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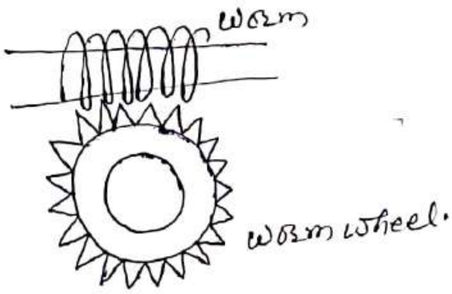
6. Worm and Worm wheel:-



This gear pair consist of worm and similar to screw.
This gear pair transmit power and motion where shafts are non parallel or non intersecting.

Gear ratio — 20:1

Advantages:-

1. Higher reduction ratio.
2. Compact in size
3. self locking system
4. Effective meshing
5. Reduce speed and torque.

Disadvantages

1. Heat at contact point.
2. Low efficiency.
3. manufacturing cost is more.
4. High power losses.

Applications.

1. Steering mechanism in Automobile.
2. Conveyers.
3. Presses.
4. Lifts.

* Design Procedure for spur gears.

For design spur gears, the following procedure may be followed: -

Step 1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relations -

$$\boxed{W_T = \frac{P}{V} \times C_s}$$

where W_T = Permissible tangential tooth load in N.

P = Power transmitted.

V = Pitch line velocity $= \frac{\pi D N}{60}$ m/sec.

D = Pitch circle diameter in m.

C_s = Service factor

- circular pitch $P_c = \frac{\pi D}{T} = \pi m$ where m = module

Values of service factor (C_s)

Types of load	Types of services		
	Intermittent or 3 hrs per day	8 10 hrs/day	Continuous 24 hrs/day.
Steady	0.8	1.00	1.25
Light shock	1.00	1.25	1.54
Medium shock	1.25	1.54	1.80
Heavy shock	1.54	1.80	2.00

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(iii) When the pinion and the gear are made of different material, then the ~~load~~ product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.

(iv) The product $(\sigma_w \times y)$ is known as strength factor of the gear.

(v) The face width (b) may be taken as $3p_c$ to $4p_c$ or $9.5m$ to $12.5m$ } for cut teeth

and $2p_c$ to $3p_c$ or $6.5m$ to $9.5m$ } for cast teeth.

Step-3. Calculate the dynamic load (W_D), on the tooth by using Buckingham equation i.e.

$$W_D = W_T + W_I$$
$$= W_T + \frac{21V(bc + W_T)}{21V + \sqrt{bc + W_T}}$$

& $W_T = \frac{P}{V}$ where P = watt & V = m/sec.

Step 4. Find the static tooth load (i.e beam strength or the endurance strength of the tooth) by using the relⁿ.

$$W_S = \sigma_e \cdot b \cdot p_c \cdot y = \sigma_e \cdot b \cdot \pi \cdot m \cdot y$$

For safety against breakage, W_S should be greater than W_D.

Step 5. Find the wear tooth load by using the relⁿ

$$W_W = D_p \cdot b \cdot Q \cdot K$$

The wear load (W_W) should not be less than the dynamic load (W_D).

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* Causes of Gear Tooth failure :-

The different modes of failure of gear teeth and their possible remedies to avoid the failure are as follows -

(i) Bending failure → Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear will fail in bending. In order to avoid such failure the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

(ii) Pitting :- It is the surface failure which occurs due to many repitition of contact stresses.

In order to avoid the pitting, the dynamic load b/w the gear tooth should be less than the wear strength of the gear tooth.

(iii) scoring :- The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubrication fails.

This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricants. so that the temperature at the rubbing faces is with in the permissible limits.

(iv) Abrasive wear :- The foreign particles in the lubricants such as dirt, dust or burr enter b/w the tooth and damage the form of tooth.

This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil.

(v) Corrosive wear :-

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* Design Considerations for a Gear drive.

There are following Consideration for designing a gear drive -

(i) The Power to be transmitted.

(ii) The speed of the driving gear.

(iii) velocity ratio.

(iv) The Centre distance.

The following requirement must be met in the design of a gear drive -

(a) The gear teeth should have sufficient strength so that they will not ~~fail~~ fail under static loading or dynamic loading during normal running conditions.

(b) The gear teeth should have wear characteristics so that their life is satisfactory.

(c) The use of space and material should be economical.

(d) The lubrication of the gears must be satisfactory.

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Module - III

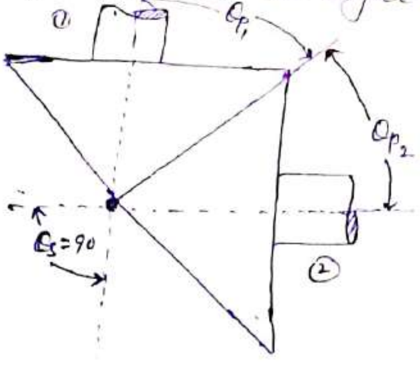
Bevel Gear :-

Introduction:- The Bevel gears are used to transmitting Power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surface for the bevel gear are frustums of cones.

* Classification of Bevel gears:-

The bevel gears may be classified into the following types depending upon the angles between the shafts and the pitch surface.

1. Mitre gears:- when equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle as shown in fig.



2. Angular bevel gears:- When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as angular bevel gears.

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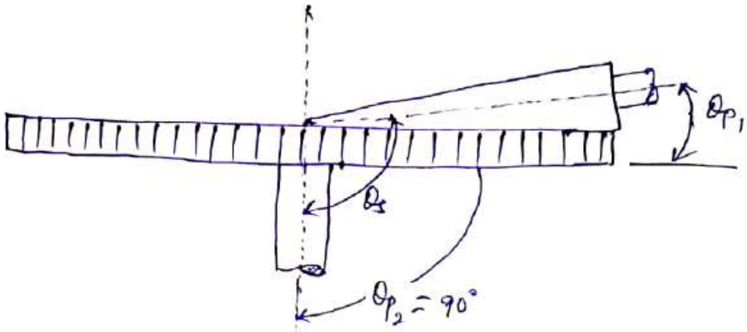
3. Crown bevel gears :-

when the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90°, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing.



Crown bevel gear

4. Internal bevel gears :- when the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as internal bevel gears.

Note: → The bevel gears may have straight or spiral teeth, it may be assumed, unless other wise stated that the bevel gear has straight teeth and the axes of the shafts intersect at right angle.

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Forms of teeth :-

In actual Practice, there are following two types of teeth are Commonly used.

(i) cycloidal teeth,

(ii) Involute teeth.

Cycloidal teeth :- A cycloidal is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle the curved traced by a point on the circumference of a circle is known as epicycloid. On the other hand if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called hypo cycloid.

Involute teeth :- An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel.

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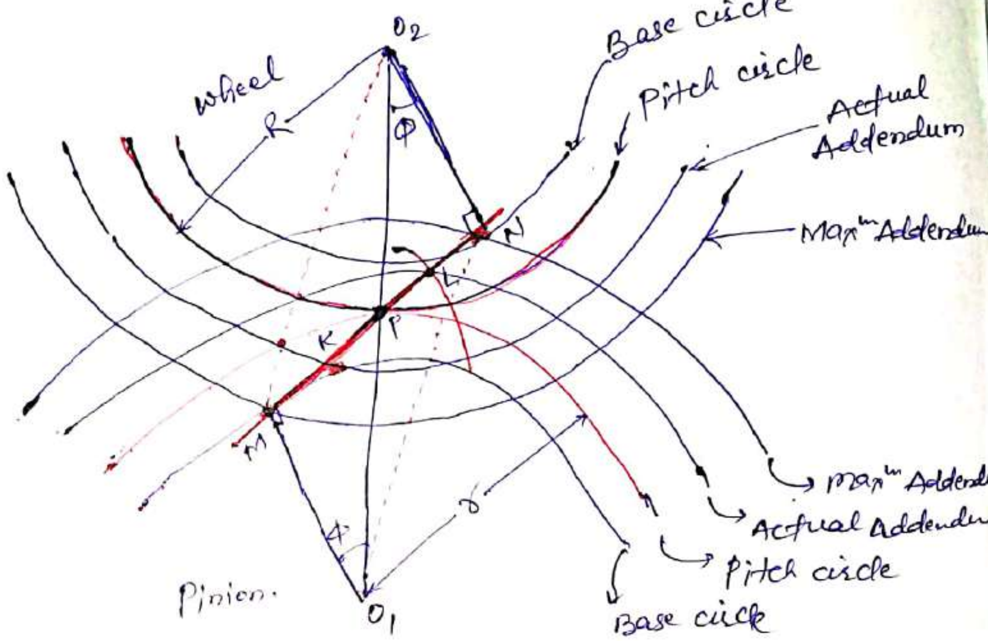
Interference in involute Gears :→

The phenomenon when the tip of a tooth undercuts the root on its mating gear is known as interference.



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Gear materials:

The material used for the manufacture of gears depends upon the strength and service conditions. like wear, noise etc. The gear may be manufactured from metallic or non metallic materials. The metallic gears with cut teeth are commercially obtained in cast iron, steel and bronze. The non metallic material like

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* Strength of a bevel gears:-

The strength of a bevel gear tooth is obtained by using the Lewis equation (Modified).
The modified form of the Lewis equation for the tangential tooth load is given by -

$$W_T = (\sigma_0 \times C_v)\, b\, m \pi y' \left(\frac{L-b}{L}\right)$$

where σ_0 = Allowable static stress.

C_v = velocity factor

$= \frac{3}{3+v}$, for teeth cut by form cutters.

$= \frac{6}{6+v}$, for teeth generated with precision machine

v = Peripheral speed in m/sec.

b = face width

m = module

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth

L = Slant height of pitch cone (or cone distance)

$$= \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2}$$

where D_G = Pitch diameter of the gear.

D_P = Pitch diameter of the Pinion.

Notes:

(i) The factor $\left(\frac{L-b}{L}\right)$ may be called as bevel factor

(ii) For satisfactory operation of the bevel gears, the face width should be from 6.3 m to 9.5 m, where m is the module. Also $\frac{L}{b}$ should not exceed 3.

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(iii) The dynamic load for bevel gears may be obtained in the similar manner as discussed for spur gears.

(iv) The static load or endurance strength of the tooth for bevel gears is given by -

$$W_s = \sigma_e \cdot b \cdot \pi m y' \left(\frac{L-b}{L}\right)$$

The value of flexural endurance limit (σ_e) may be taken from data book in spur gear.

(v) The maximum or limiting load for wear for bevel gear is given by

$$W_w = \frac{D_p \cdot b \cdot Q \cdot K}{\cos\theta_{P_1}}$$

where Q = Equivalent number of teeth such that

$$Q = \frac{2\,T_{EG}}{T_{EG} + T_{EP}}$$

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* Design of a shaft for Bevel gears:-

In designing a pinion shaft, the following Procedure may be adopted.

(i) First of all, find the torque acting on the pinion. it is given by - $T = \frac{P \times 60}{2\pi N_p}$ where P = Power transmitted. N_p = Speed of the pinion in r.p.m.

(2.) Find the tangential force (W_T) acting at the mean radius (R_m) of the pinion we know that,

$$\boxed{W_T = \frac{T}{R_m}}$$

(3) Find the axial and Radial forces (i.e W_{RH} and W_{RV}) acting on the pinion shaft as discussed above.

(4). Find resultant bending moment on the pinion shaft as follows.

The bending moment due to W_{RH} and W_{RV} is given by

$$M_1 = M_{RV} \times \text{Overhang} - W_{RH} \times R_m$$

and bending moment due to W_T

$$M_2 = W_T \times \text{overhang}$$

∴ Resultant bending moment

$$M = \sqrt{M_1^2 + M_2^2}$$

5. Since the shaft is subjected to twisting moment (T) and resultant bending moment (M). Therefore equivalent twisting moment

$$T_e = \sqrt{M^2 + T^2}$$

6. Now the diameter of the pinion shaft may be obtained by using the torsional equation.

$$T_s = \frac{\pi}{16} \times \tau \, d_p^3$$

where d_p = diameter of the pinion shaft
τ = Shear stress for the material of the pinion shaft.

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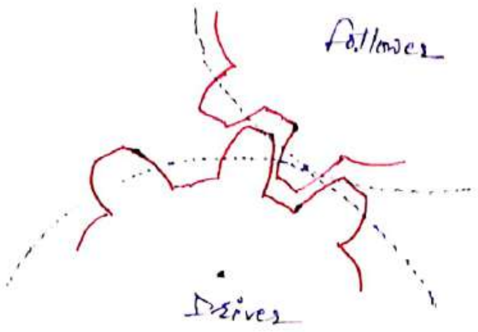
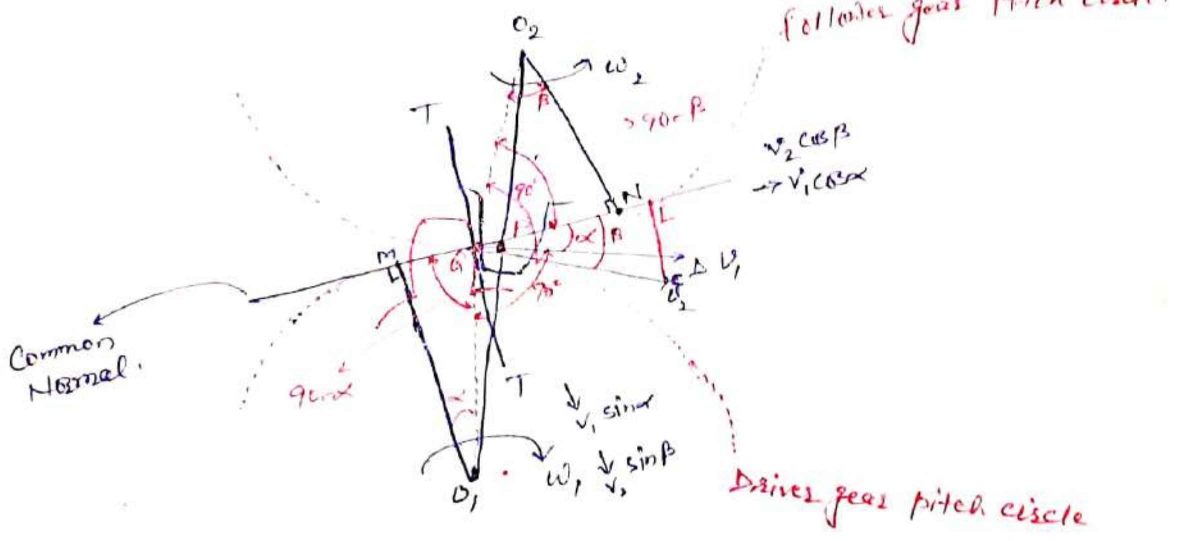
transmission if the both teeth are contact to

Law of gearing [Condition for constant velocity ratio of gear].





P → Pitch Point → It is the point at which both the pitch circle touches each other.

TT → common tangent

v_1 = Linear velocity of point Q along QD. if point Q lies on driver gear pitch circle.

v_2 = Linear velocity of point Q if point Q lies on follower gear

At the time of power transmission if the both teeth are contact to each other then,

Now $V_1 \cos\alpha = V_2 \cos\beta$ $\quad \because V = \omega r$

$\omega_1 (O_1 Q) \cos\alpha = \omega_2 (O_2 Q) \cos\beta$

V → Linear velocity
ω → Angular velocity
r → Radius of the circular path.

In $\triangle O_1 M Q$ & In $\triangle O_2 Q N$

$$\cos\alpha = \frac{O_1 M}{O_1 Q} \qquad \cos\beta = \frac{O_2 N}{O_2 Q}$$

From above equation,

$$\omega_1 \cancel{(O_1 Q)} \cdot \frac{O_1 M}{\cancel{O_1 Q}} = \omega_2 \cancel{(O_2 Q)} \cdot \frac{O_2 N}{\cancel{O_2 Q}}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} \quad \text{—— (1)}$$

From diagram

$\triangle O_2 P N$ and $\triangle O_1 P M$ are similar triangle.

~~Now~~ then $\dfrac{O_2 N}{O_1 M} = \dfrac{O_2 P}{O_1 P}$

$$\therefore \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}}$$

From above equation we see that the ratio of Angular velocity is inversely proportional to the ratio of $O_2 P$ and $O_1 P$.

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Q: A pair of Cast iron bevel gears connect two shafts at right angles. The pitch diameters of the pinion and gears are 80 mm and 100 mm respectively. The tooth Profiles of the gears are $14\frac{1}{2}°$ Composite form. The allowable static stress for both the gears is 55 MPa, if the pinion transmits 2.75 KW at 1100 rpm, find the module and number of teeth on each gears from the stand-point of strength and check the design from the stand point of wear. Take the surface endurance limit as 630 MPa and modulus of elasticity for cast iron as 84 KN/mm².

Sol: Given $\theta_s = 90°$, $D_P = 80\,mm = 0.08\,m$, $D_G = 100\,mm = 0.1\,m$

$\phi = 14\frac{1}{2}°$, $\sigma_{OP} = \sigma_{OG} = 55\,MPa = 55\,N/mm^2$, $P = 2.75 \times 10^3\,W$.

$N_P = 1100\,rpm$ $\quad \sigma_{es} = 630\,MPa = 630\,N/mm^2$, $E_P = E_G = 84 \times 10^3\,N/mm^2$

Let m = module in mm.

We know that pitch angle for the pinion,

$$\theta_{P_1} = \tan^{-1}\left(\frac{1}{V.R}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{80}{100}\right) = 38.66°$$

Pitch angle for gears,

$$\theta_{P_2} = 90 - 38.66° = 51.34°$$

We know that formative number of teeth on the pinion,

$$T_{EP} = T_P \cdot \sec\theta_{P_1} = \frac{80}{m} \times \sec 38.66 = \frac{102.4}{m} \qquad \left[\because T_P = \frac{D_P}{m}\right]$$

and formative number of teeth on the gear,

$$T_{EG} = T_G \cdot \sec\theta_{P_2} = \frac{100}{m} \sec 51.34 = \frac{160}{m} \qquad \left[\because T_G = \frac{D_G}{m}\right]$$

Since the both gears are made of the same material therefore Pinion is the weaker. Thus the design should be based on the pinion.

∵ Tooth form factor for the pinion having $14\frac{1}{2}°$ Composite teeth

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684}{\frac{102.4}{m}}$$

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$$y'_p = 0.124 - 0.00668\, m$$

and pitch line velocity

$$V = \frac{\pi D_p \cdot N_p}{60} = \frac{\pi \times 0.08 \times 1000}{60} = 4.6\ m/sec.$$

Taking velocity factor,

$$C_v = \frac{6}{6+V} = \frac{6}{6+4.6} = 0.566$$

and length of the pitch cone element or slant height of the pitch cone,

$$L = \sqrt{\left(\frac{D_g}{2}\right)^2 + \left(\frac{D_p}{2}\right)^2} = \sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{80}{2}\right)^2} = 64\ mm$$

Assume face width $b = \frac{1}{3} \times L = \frac{64}{3} = 21.3 \simeq 22\ mm.$

And torque

$$T = \frac{P \times 60}{2\pi N_p} = \frac{2750 \times 60}{2\pi \times 1000} = 23870\ N\text{-}mm.$$

and tangential load on the pinion

$$W_T = \frac{T}{D_p/2} = \frac{23870}{80/2} = 597\ N.$$

We also know tangential load on the pinion,

$$W_T = (\sigma_{op} \times C_v)\, b \times \pi m \times y'_p \left(\frac{L-b}{L}\right)$$

$$\Rightarrow 597 = (55 \times 0.566)\, 22 \times \pi \times m \times (0.124 - 0.00668\, m)\left(\frac{64-22}{64}\right)$$

$$\Rightarrow 597 = 1412\, m\,(0.124 - 0.00668\, m)$$

$$\Rightarrow 597 = 175 m - 9.43 m^2$$

$$\therefore m = 4.5 \simeq 5\ mm \quad \underline{Ans}$$

And Number of teeth on Pinion.

$$T_p = \frac{D_p}{m} = \frac{80}{5} = 16 \quad \underline{Ans}$$

and Number of teeth on Gear,

$$T_g = \frac{D_g}{m} = \frac{100}{m} = 20 \quad \underline{Ans}$$

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* Determination of pitch angle for Bevel Gears :-

Consider a pair of bevel gear in mesh as shown in fig.

Let θ_{p_1} = Pitch angle for the pinion.

θ_{p_2} = Pitch angle for the gears.

θ_s = Angle b/w the two shaft axes.

D_p = Pitch diameter of the pinion

D_g = Pitch diameter of the gears.

$\because$ $V.R = \cancel{\frac{D_p}{D_g}} \frac{D_g}{D_p} = \frac{T_g}{T_p} = \frac{N_p}{N_g}$

From fig. we find that,

$\theta_s = \theta_{p_1} + \theta_{p_2}$ or $\theta_{p_2} = \theta_s - \theta_{p_1}$

or, $\sin\theta_{p_2} = \sin(\theta_s - \theta_{p_1})$

$\sin\theta_{p_2} = \sin\theta_s \cdot \cos\theta_{p_1} - \cos\theta_s \cdot \sin\theta_{p_1}$ ——— (1)

We know that cone distance,

$$OP = \frac{D_p/2}{\sin\theta_{p_1}} = \frac{D_g/2}{\sin\theta_{p_2}} = \frac{\sin\theta_{p_2}}{\sin\theta_{p_1}} = \frac{D_g}{D_p} = V.R.$$

$\therefore$ $\boxed{\sin\theta_{p_2} = V.R \times \sin\theta_{p_1}}$ ——— (2)

From eqn (1) and (2),

$V.R \times \sin\theta_{p_1} = \sin\theta_s \cdot \cos\theta_{p_1} - \cos\theta_s \cdot \sin\theta_{p_1}$

Dividing by $\cos\theta_{p_1}$

$V.R \cdot \tan\theta_{p_1} = \sin\theta_s - \cos\theta_s \cdot \tan\theta_{p_1}$

$$\therefore \tan\theta_{p_1} = \frac{\sin\theta_s}{V.R + \cos\theta_s}$$

$$\therefore \boxed{\theta_{p_1} = \tan^{-1}\frac{\sin\theta_s}{V.R + \cos\theta_s}}$$

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Similarly, ~~for~~ we can find that,

$$\tan\theta_{P_2} = \frac{\sin\theta_s}{\frac{1}{V.R} + \cos\theta_s}$$

$$\boxed{\theta_{P_2} = \tan^{-1}\left[\frac{\sin\theta_s}{\frac{1}{V.R} + \cos\theta_s}\right]}$$

We also conclude that, when the angle b/w the shaft axes is 90°, then,

$$\boxed{\theta_{P_1} = \tan^{-1}\left[\frac{1}{V.R}\right] = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{T_P}{T_G}\right) = \tan^{-1}\left(\frac{N_G}{N_P}\right)}$$

and $$\boxed{\theta_{P_2} = \tan^{-1}(V.R) = \tan^{-1}\left(\frac{D_G}{D_P}\right) = \tan^{-1}\left(\frac{T_G}{T_P}\right) = \tan^{-1}\left(\frac{N_P}{N_G}\right)}$$

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