K. K. College of Engineering and Management

At:-Nairo, Bagsuma, Govindpur, Dhanbad

Sub:- Operations Research

Subject Code:-

Course Objectives:-

1 .To impart knowledge in concepts and tools of Operations Research
2. To understand mathematical models used in Operations Research
3. To apply these techniques constructively to make effective business decisions.

Course Outcomes:-

Identify and develop operational research models from the verbal description of the real system. Understand the mathematical tools that are needed to solve optimisation problems. Use mathematical software to solve the proposed models. Develop a report that describes the model and the solving technique, analyse the results and propose recommendations in language understandable to the decision-making processes in Management Engineering.

Syllabus:--

1.3 OPERATION RESEARCH

UNIT-I: Introduction: Definition and scope of operations research (OR), OR model, solving the OR model, art of modeling, phases of OR study. Linear Programming: Two variable Linear Programming model and Graphical method of solution, Simplex method, Dual Simplex method, special cases of Linear Programming, duality, sensitivity analysis. Total Lectures required =9

UNIT-II : Transportation Problems: Types of transportation problems, mathematical models, transportation algorithms, Assignment: Allocation and assignment problems and models, processing of job through machines. Total Lectures required =8

UNIT-III : Network Teachniques: Shortest path model, minimum spanning Tree Problem, Max-Flow problem and Min-cost problem. Project Management: Phases of project management, guidelines for network construction, CPM and PERT. Total Lectures required =8

UNIT-IV: Theory of Games: ; Rectangular games, Minima theorem, graphical solution of 2 x n or m x 2 games, game with mixed strategies, reduction to linear programming model. Quality Systems: Elements of Queuing model, generalized poisson queuing model, single server models. Total Lectures required =9

UNIT-V: Inventory Control Models of inventory, operation of inventory system, quantity discount., Replacement, Replacement models: Equipments that deteriorate with time, equipments that fail with time. Total Lectures required =8 Grand Grand Total Lectures required =42 Text / Reference Books: 1. Wayne L. Winston,"Operations Research" Thomson Learning,2003. 2. Hamdy H. Taha, "Operations Research-An Introduction" Pearson Education,2003. 3. R. Panneer Seevam, "Operations Research" PHI Learning, 2008. 4. V.K.Khanna, "Total Quality Management" New Age International, 2008.

Importance and Scope of Operations Research

Introduction

Operation is simply defined as the research of operations. Operations may be called a set of acts required for the achievement of a desired outcome. Operation research is used to maximize the utility of limited resources. The objective is to select the best alternative that is the one leading to the best result. O.R is scientific approach to problem solving for executive management

Scope of operations research in Management

The technique used in operations research has very wide application in various fields of business/industrial/government/social sector. Few areas of applications are mentioned below:

Marketing and sales:

1. Product selection and competitive strategies
2. Utilization of salesmen, their time and territory control, frequency of visits in sales force analysis.
3. Marketing advertising decision for cost and time effectiveness.
4. Forecasting and decision trends

Production Management

1. Product mix and product proportioning
2. Facility and production planning and scheduling sequencing
3. Physical distribution, warehousing and retail outlets planning-nature and localities
4. Project planning, scheduling , allocation of resources, monitoring and control systems
5. Queuing system design.
6. Quality control decision

Purchasing, procurement and inventory controls

1. Buying policies and prices
2. Negotiation and bidding policies
3. Time and quantity of procurement

Finance, Investments and Budgeting

1. Profit Planning

2. Cash flow analysis
3. Investment policy for maximum return
4. Dividend policies
5. Portfolio analysis

Personnel Management
1. Determination of optimum organizational level
2. Job evaluation and assignment analysis
3. Mixes of age and skills
4. Recruitment policies and job description

Research and Development
1. Determination of areas of thrust for research and development
2. Selection criteria for specific project
3. ‘what if’ analysis for alternative design and reliability
4. Trade –off analysis for time-cost relationship and control of development project.

Importance of Operations Research

Business Operations
OR could be very effective in handling issues of inventory planning and scheduling, production planning, transportation, financial and revenue management and risk management. Basically, OR could be used in any situation where improvements in the productivity of the business are of paramount importance.

Control
With OR, organizations are greatly relieved from the burden of supervision of all the routine and mundane tasks. The problem areas are identified analytically and quantitatively. Tasks such as scheduling and replenishment of inventories benefit immensely from OR.

Decision Making

OR is used for analyzing problems of decision making in a superior fashion. The organization can decide on factors such as sequencing of jobs, production scheduling and replacements. Also the organization can take a call on whether or not to introduce new products or open new factories on the basis of a good OR plan.

Coordination

Various departments in the organization can be coordinated well with suitable OR. This facilitates smooth functioning for the entire organization.

Systems

With OR, any organization follows a systematic approach for the conduct of its business.

OR essentially emphasizes the use of computers in decision making; hence the chances of errors are minimum.

Classification of OR Models

The first thing one has to do to use O.R. technique after formulating a practical problem is to construct a suitable model to represent the a practical problem. A model is a reasonably simplified representation of a real-world situration. It is a abstraction of reality. The model can broadly be classified as
Iconic(Physical) Models
Analog Models
Mathematical Models
Also as
Static Models
Dynamic Models
Deterministic Models
Stochastic Models
Models can further be subdivided as
Descriptive Models
Prescriptive Models
Predictive Models
Analytic Models
Simulation Models

Iconic Model:

This is physical or pictorial representation of various aspects of a system.
Example: Toy, Miniature Model of a building, scaled up model of a cell in biology etc.
Analogue or Schematic Model:
This uses one set of properties to represent another set of properties which a system under study has
Example: A network of water pipes to represent the flow of current in an electrical network or graph, organizational charts etc
.
Mathematical model or Symbolic model:

This uses a set of mathematical symbols (letters, numbers etc) to represent the decision variables of a system consideration. These variables related by mathematical equations or in equations which describes the properties of the system.
Example: A linear programming model, A system of equations representing an electrical network or differential equations representing dynamic systems etc.

Static Model:

This is a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.

Example: A linear programming problem, an assignment problem, transportation problem etc.

Dynamic Model:

This is a model which considers time as one of the important variables.
Example: A dynamic programming problem. A replacement problem.

Deterministic model:

This is a model which does not take uncertainty into account.
Example: A linear programming problem, an assignment problem etc.

Stochastic model:

This is a model which considers uncertainty as an important aspect of the problem
Example: Any stochastic programming problem, stochastic inventory models etc,

Descriptive model:

This is one which just describes a situation or system.
Example: An opinion poll, any survey.
Predictive Model is one which predicts something based on some data. Predicting election results before actually the counting is completed.
Prescriptive model is one which prescribes or suggests a course of action for a problem
Example: Any programming (linear, non linear, dynamic, geometric etc.) problem

Analytic model:

This is a model in which exact solution is obtained by mathematical methods in closed form.
Simulation model: This is a representation of reality through the use of a model or device which will react in the same manner as reality under a given set of conditions. Once a simulation model is designed, if takes only a little time, in general , to run a simulation on a computers.

Phases of O.R

1) Formulation of the problems: Identifying the objective, the decision variables involved and the constraints that arise involving the decision variables.
2) Construction of a Mathematical Model: Expressing the measure of effectiveness which may be total profit, total cost, utility etc., to be optimized by a Mathematical function called objective function. Representing the constraints like budget constraints, raw materials constraints, resource mathematical equations or inequalities.
3) Solving the Model constructed: Determining the solution by analytic or iterative or

Monte-Carlo method depending upon the structure of the mathematical model.
4) Controlling and Updating
5) Testing the model and its solution i.e., validating the model: Checking as far as possible either from the past available data or by expertise and experience whether the model gives a solution which can be used in practice.
6) Implementation: Implement using the solution to achieve the desired goal.

Linear Programming

Linear programming is a mathematical programming technique to optimize the performance (profit or cost), under a set of resource constraints (Machine hours, money, material, time men hours etc). As a special by an organization the usefulness of this technique is maximize even though the competitions in a linear programming model are two elaborate a sample list of applications of the linear programming problem is presented below.

1. Product mix problem
2. Diet planning problem
3. Manpower planning problem
4. Capital budgeting problem

Concept of Linear programming problem

The model of any linear programming problem will contain objective fuction, set of constraints and non – ve restrictions. Each of the company may consist of one of more of the following.

1. Decision variables
2. Objective function coefficients
3. Technological co efficient
4. Resource availabilities

Properties of Linear Programming model

Optimum solution

If there is no other superior solution to the solution obtained for a given linear programming model then the solution is obtained a treated as the optimum solution.

Feasible solution

If all the constraints of the linear programming model are satisfied by the solution of the model then the solution is called feasible solution.

Infeasible solution

When the constraints are not satisfied simultaneously, the LPP has no feasible solution. That solution is called infeasible solution. This means that there is no solution for the given model which can be implements.

Unbounded solution

When the values of the decision variable may be increased indefinitely or infinitely without violating any of constraints, the solution space (feasible region) is unbounded. The value of objective function in such cases, may increase (for maximization) or decrease (for minimization) indefinitely. Thus both solution space and objective function value are unbounded.

Bounded solution

When the values of the decision variable may by definite or finite with violating any of constraints, the solution space (feasible region) is bounded.

Linear Programming Problem Methods

Algorithm for Graphical Method:

The major steps in the solution of a linear programming problem by graphical method are summarized as follows:

Step 1: identify the problem – the decision variables, the objective and restrictions.

Step 2: Set up the mathematical formulation of the problem.

Step 3: Plot a graph representing all the constraints of the problem and identify the feasible region (solution space). The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

Step 4: The feasible region obtained in "step – 3" may be bounded or unbounded. Compute the co-ordinates of all the corner points of the feasible region.

Step 5: Find out the value of the objective function at each corner (solution) point determined in "step -4".

Step 6: Select the corner point that optimizes (maximizes or minimizes) the value of the objective function. It gives the "optimum feasible solution".

Slack Variables:

If the constraints has ≤ type in order to make it in equation we have to add something +ve quantity to left hand side. +ve quantities are called slack variables

Algorithm for Simplex Method:

Step 1: Check whether the given objective function is maximization type

Case a: If it is maximization type then go to next step.

Case b: If is minimization problem then we converting them into maximization type by using relation.

Min Z = -Max Z*

Step 2: check whether the right hand side constants are non – negative (positive)

Case a: if is non – negative then go to next step.

Case b: if at least one of the constant is negative (-ve) then we converting into +ve value by multiplying the corresponding constraint by "-1".

Step 3: Introduce slack and surplus variables to the constraints and the cost of objective function becomes zero.

Step 4: Express the given objective functions and constraints in matrix form.

Step 5: Check whether there exist initial basic feasible solution (IBFS).

Step 6: prepare initial simplex table by using the following columns.

C_j C_1 C_2 C_3 ... ...C_n

C_B S_B X_B X_1 X_2 X_3........................ X_n S_1 S_2 S_3 .... S_n

$Z_j - C_j$

Step 7: Find z-value i.e., $z = C_B X_B$

Step 8: Find net evaluations i.e., $Z_j - C_j$ where j = 1, 2, 3, ...n

Step 9: Check whether all the net evaluations are non –ve.

Case a: if it is so then there exist optimum solution.

Case b: If at least one net evaluation is –ve then go to next step

Step 10: Find entering variable which is most –ve net evaluation

Step 11: Find leaving variable which is

$_B$ X

Min

Entering column elements

. .

. .

. .

, Entering column elements > 0.

Step 12: Find pivotal or key element which is intersection of both entering and leaving variables.

Step 13: Do the simplex operations, new values of

Old values of the key row

Key row =

Key element

New values of the Non key row = Old values of the Non key row - (Non key element) · (New values of the key row)

Step 14: Repeat the procedure. Step 5 to step 12. Until optimum solution are unbounded solution has been obtained.

Big – M or Mixed or Penalty Method or Alternate Method:

Step 1: Check whether the given objective function is maximization type.

Case a: If it is maximization then go to next step

Case b: If it is minimization type then we convert them into maximization type by using relation.

Min Z = -Max Z*

Step 2: Check whether the right hand side constants are non –ve

Case a: If it is non –ve or positive then go to next step.

Case b: If it is –ve or negative then we convert them into positive (+ve) by multiplying the corresponding constraint by "-1" Automatically the signs will also change.

Step 3: Express the given objective function and constraints in matrix form.

Step 4: Check whether there exist IBFS

Case a: If it is exit so then go to next step

Case b: If it is not so, then introduce artificial variable to the constraints ≥ type and the cost co-efficient objective function becomes "-M".

Step 5: Prepare Initial Simplex Table

C_j

C_1 C_2 C_3.........Cn

C_B S_B X_B X_1 X_2 X_3...... X_n S_1 S_2 S_3 .....S_n A_1,A_2,............A_n

Z_j –C_j

Step 7: Find Z-value $Z = \sum C_B X_B$.

Step 8: Find Net evaluations i.e., $Z_j - C_j$ (j = 1, 2, 3 ....n)

Step 9: Check whether the net evaluations are non –ve.

Case a: If it is so and verifies the following conditions.

i) If all net evaluations are non –ve. If at least one artificial variable.
Still in the basis not at zero level then there exist infeasible solution.

ii) If all net evaluations are non –ve and if at least one artificial variable still in the basis at zero level then there exist optimum solution.

iii) If all net evaluations are non –ve there is no artificial variable in the basis then there exist optimum solution.

Case b: If it is so then go to next step.

Step 10: Find entering variable which is most –ve net evaluation i.e., Min $\{ (Z_j - C_j) \}$

Step 11: Find leaving variable which is

$$\text{Min}\left\{\frac{X_B}{\text{Entering column elements}}\right\}$$

, Entering column elements > 0.

Step 12: Find pivotal or key element which is intersection of both entering and leaving variables.

Step 13: Do the simplex operations, new values of

$$\text{Key row} = \frac{\text{Old values of the key row}}{\text{Key element}}$$

New values of the Non key row = Old values of the Non key row - (Non key element) · (New values of the key row)

Step 14: Repeat the procedure. Step 7 to step 13. Until optimum solution are unbounded solution has been obtained.

Duality in Linear programming problem

Every LPP there always exist another LPP which is basing on the same data and having same solution. The original problem is called “Primal problem” the associated one is called

“Dual Problem”.

Rules for Converting Primal problem Into Dual:

If the system of constraints in a given LPP consist of mixed equations (or) in equations non –ve variables or unrestricted variables then the dual of the given problem can be obtained by reducing to standard primal problem.

Step 1: first convert the objective function to maximization form, if not.

Step 2: If the constraints as ≥ type then we converts them into ≤ type by multiplying -1.

Step 3: If the constraints as an equal sign then it is replaced by two constraints involving the in equalities is in opposite direction simultaneously.

Step 4: Every unrestricted variables is replaced by difference of “2” Non –ve variable.

Step 5: We get the standard primal form of the given LPP in which

Case a: All the constraints have ≤ type the objective function is maximization.

Case b: All the constraints have ≥ type when the objective function is minimization.

Step 6: Identify the variables to be used in the dual problem. The no. of these variables equals to the no. of constraints in the primal problem.

Step7: Write the objective function. Objective function coefficient is the Right hand side constraints of primal constraints. If the primal problem is of maximization type the dual will be minimization problem and vice versa.

Step 8: Making use of “Step 6” write the constraints for the dual problem

Case a: If the primal is a maximization problem the dual constraints must be all are ≥ type, If the primal is minimization problem the dual constraints must be all are ≤ type.

Case b: The column co-efficient of the primal constraints become the row coefficients of dual constraints.

Case c: The co-efficient the primal objective function become the right hand side constants of the dual constraints.

Applications of Linear Programming

Linear programming is the most widely used techniques of decision –making in business and industry and in various other fields. In this section, we will discuss a few of the broad application areas of linear programming.

Agricultural Application:

This application fall into categories of farm economics and farm management. The farm deals with agricultural economy of a nation or region.

Linear programming study of farm economics deals with interregional competition and optimum allocation of crop production.

Linear programming can be applied in agricultural planning e.g. allocation of limited resources such as acreage, labor, water supply and working capital, etc. in a way so as to maximize net revenue.

Production Management:

Product Mix: A company can produce several different products each of which requires the use of limited production resources. In such cases, it is essential to determine the quantity of each product to be produced knowing its marginal contribution and amount of available resources used by it. The objective is to maximize the total contribution, subject to all constraints.

Production planning: This deals with determination of minimum cost6 production plan over planning period of an item with a fluctuating demand, considering the initial number of units in inventory, production, man power and all relevant cost factors.

Financial Management

Portfolio Selection: This deals with the selection of specific investment activity among several other activities. The objective is to find the allocation which maximizes the total expected return or minimize risk. Under certain limitations.

Profit planning: This deals with the maximization of the profit margin form investment in plant facilities and equipment, cash in hand and inventory.

Marketing management:

Medial selection: Linear programming techniques help in determining the advertising media mix. So as to maximize the effective exposure, subject to limitation of budget, specified exposure rates. To different market segments, specified minimum and maximum no. of advertisements in various media.

Transportation Problem

Introduction:

The transportation problem is one of the subclasses of LPPs in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized.

Examples of Transportation Problem

Source
Destination
Commodity
Objective
Plants Markets Finished goods Minimizing total cost
of shipping
Plants Finished goods
warehouses
Finished goods Minimizing total cost
of shipping
Suppliers Raw material
warehouses
Raw material Minimizing total cost
of shipping

The solution of transportation problem can be obtained in two stages, namely initial solution or initial basic feasible solution and optimum solution.

Initial solution

Initial solution cam be obtained by using any one of the three methods viz.,

I. North West corner rule (NWCR).

II. Row minimum rule
III. Column minimum rule.
IV. Least cost method or Matrix minima method.

2

V. Vogel’s approximation method (VAM).
VAM is preferred over the other two methods, since the initial basic feasible solution obtained by this method is either optimal of very close to the optimal solution.

Optimum solution

The improved solution of the initial basic feasible solution is called optimal solution which is the second stage of solution that can be obtained by MODI (modified distribution method) or U-V method.

Algorithm for Northwest-Corner rule

Step 1 : Find the minimum of the supply and demand values with respect to the current northwest corner cell of the cost matrix.

Step 2 : Allocate this minimum value to the current northwest-corner cell and subtract this minimum from the supply and demand values with respect to the current northwest-corner cell.

Step3 : Check whether exactly one of the row/column corresponding to the northwest-corner cell has zero supply/demand, respectively. If so, go to step 4.

Step4 : Delete that row/column with respect to the current northwest-corner cell which has the zero supply/demand and go to next step.

Step5 : Repeat above steps moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

Algorithm for Least Cost or Matrix Minima Method

Step 1 : Find the minimum of the values in the cost matrix (i.e. find the matrix minimum).

Step 2 : Find the minimum of the supply and demand values(X) with respect to the cell corresponding to the matrix minimum.

Step 3 : Allocate X units to the cell with the matrix minimum. Also, subtract X units from the supply and the demand values corresponding to the cell with the matrix minimum.

Step 4 : Check whether exactly one of the row/column corresponding to the cell with the matrix minimum has zero supply/demand, respectively. If yes, go to step 5.

Step 5 : Delete that row/column with respect to the cell with the matrix minimum which has the zero supply/zero demand.

Step 6 : Repeat above steps for the resulting reduced transportation table until all the rim requirements are satisfied.

3

Algorithm for Vogel’s Approximation Method (VAM)

Step 1 : Find row penalties, i.e. the difference between the first minimum and the second minimum in each row . If the two minimum values are equal, then the row penalty is zero.

Step 2 : Find column penalties, i.e. the difference between the first minimum and the second minimum in each column. If the two minimum values are equal, then the column penalty is zero.

Step 3 : Find the maximum amongst the row penalties and column penalties and identify whether it occurs in a row or in a column. If the maximum penalty is in a row, go to step 4 otherwise go to step 7.

Step 4 : Identify the cell for allocation which has the least cost in that row.

Step 5 : Find the minimum of the supply and demand values with respect to the selected cell.

Step 6 : Allocate this minimum value to that cell and subtract this minimum from the supply and demand values with respect to the selected cell and go to step 10.

Step 7 : Identify the cell for allocation which has the least cost in that column.
Step 8 : Find the minimum of the supply and demand values with respect to the selected cell.
Step 9 : Allocate this minimum value to the selected cell and subtract this minimum from the supply and demand values with respect to the selected cell.
Step 10 : Check whether exactly one of the rows and the column corresponding to the selected cell has zero supply/zero demand, respectively. If yes go to step 11.
Step 11 : Delete the row/column which has the zero supply/zero demand and revise the corresponding row/column penalties.
Step 12 : Repeat the procedure until all the rim requirements are satisfied.

Algorithm for MODI (modified distribution method) or U-V method.

Step 1 : Find the initial basic feasible solution of a Transportation Problem by using any one of the above methods.
Step 2 : Check whether the number of basic cells (occupied cells) in the set of initial basic feasible solution is equal to m+n-1. If yes go to next step.
Step 3 : Compute the values for U_1, U_2, U_3, ………. U_m and V_1, V_2, V_3, V_4, ………… V_n by applying the following formula to all the basic cells only.
4
$U_i + V_j = c_{ij}$
Step 5 : Compute penalties P_{ij} or the non-basic cells by using the formula.
$P_{ij} = U_i + V_j - c_{ij}$
Step 6 : Check whether all P_{ij} values are less than or equal to zero . If yes, go to step 11, otherwise, go to step 7.
Step 7 : Identify the non-basic cell which has the maximum positive penalty, and term that cell as the new basic cell.
Step 8 : Starting from the new cell, draw a closed loop consisting of only horizontal and vertical lines passing through some basic cells.(Note : Change of direction of the loop should be with 90 degrees only at some basic cell.)
Step 9 : Starting from the new basic cell, alternatively assign positive(+) and negative (-) signs at the corners of the closed loop.
Step 10: Find the minimum of the allocations made amongst the negatively signed cells.
Step 11 : The optimality is reached. Treat the present allocations to the set of basic cells as the optimum allocations.

Q→ Solve the following T. P. by N-W-C-R.

Factories	W_1	W_2	W_3	W_4	Supply
F_1	6	4	1	5	14
F_2	8	9	2	7	16
F_3	4	3	6	2	05
Demand	6	10	15	4	35

Sol^n:→ The problem is balanced
i.e. Total supply = Total demand.
For cell x_{11} the supply is 14 & demand is 6
Hence 6 is smaller than 14, allocate 6 into
cell $C_{11}x_{11}$.
Proceed in the same way to complete the
allocations.

	W_1	W_2	W_3	W_4	Supply
F_1	6 [6]	4 [8]	1	5	[illegible]
F_2	8	9 [2]	2 [14]	7	[illegible]
F_3	4	3	6 [1]	2 [4]	[illegible]
Demand	6 0	10 2 0	15 1 0	4 0	

Final allocations are as below

	W_1	W_2	W_3	W_4
F_1	6 [6]	4 [8]		
F_2		9 [2]	2 [14]	
F_3			6 [1]	2 [4]

$\therefore$ Transportation cost $= 6\times6 + 4\times8 + 9\times2 + 2\times14 + 6\times1 + 2\times4$

$= 36+32+18+28+6+8$

$= 36+32+18+28+14$

$=$ ₹ 128 Ans

Q→ Find the transportation cost by Row minima method.

	1	2	3	4	Supply
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
Req.	7	5	3	2	

Solⁿ:→ A/c to the question

Total requirement $= 7+5+3+2 = 17$

& Total supply $= 6+1+10 = 17$

Hence problem is balanced T.P.

Now, selecting 1st row with minimum cost matrix & proceed to next step.

	1	2	3	4	Supply
1	2 [6]	3	11	7	~~6~~ 0
2	1	0 [1]	6	1	~~1~~ 0
3	5 [1]	8 [4]	15 [3]	9 [2]	~~10~~ ~~9~~ ~~5~~ ~~2~~
Req:-	~~7~~ ~~1~~ 0	~~5~~ ~~4~~ 0	~~3~~ 0	~~2~~ 0	

∴ Total transportation cost =

$2\times6+0\times1+5\times1+8\times4+15\times3+9\times2$

$= 12+5+32+45+18$

$= 30+37+45$

$= 67+45$

$= ₹\,112$ Ans.

Q → Solve the T.P by column minima method

	1	2	3	4	Supply
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
Req	7	5	3	2	

Sol^n: → A/c to the question
Total Supply = 17
Total requirement = 17
Hence T.P. is balanced & it gives basic feasible sol^n.

First of all Select 1st column with minimum cost matrix & proceed to allocations.

	1	2	3	4	supply
1	2 (6)	3	11	7	6 → 0
2	1 (1)	0	6	1	1 → 0
3	5	8 (5)	15 (3)	9 (2)	10 → 5 → 3
Req.	7 → 1 → 0	5 → 0	3	2 → 0	

Now,

Total transportation cost =

$$2\times6+1\times1+8\times5+15\times3+9\times2$$

$$=12+1+40+45+18$$

$$=13+40+45+18$$

$$= ₹\ 116$$

Q→ Solve the following T.P. by least cost method or Matrix minima method.

	1	2	3	4	supply
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
Req.	7	5	3	2	

Solⁿ→ A/c to the question

Total supply = 17

Total requirement = 17

Hence, T.P. is balanced & its gives basic feasible solution.

In this method, select lowest cost cell & then further allocations is done in the cell with second lowest cost & so on.

→ In case of tie among the cost, select the cell where allocations of more no. of units can be made.

	1	2	3	4	Supply
1	2 [6]	3	11	7	~~6~~ 0
2	1	0 [1]	6	1	~~1~~ 0
3	5 [1]	8 [4]	15 [3]	9 [2]	~~10~~ ~~9~~ ~~5~~ ~~3~~
Req.	~~7~~ ~~1~~ 0	~~5~~ ~~4~~ 0	~~3~~	~~2~~ 0	

∴ Total transportion cost =

$2\times6+0\times1+5\times1+8\times4+15\times3+9\times2$

$= 12+5+32+45+18$

$= 30+37+45$

$= 67+45$

$=$ ₹ 112 Ans.

Q → Find the optimum sol^n of following T.P.

(1)

	W_1	W_2	W_3	W_4	W_5	Available
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Required	30	30	15	20	5	

Sol^n :→

Total supply = 40 + 30 + 20 + 10 = 100
(Available)

& Total required = 30 + 30 + 15 + 20 + 5 = 100

Here Total Available = Total required

∴ Transportation problem is balanced.

Now, its basic feasible sol^n by VAM as below

	W_1	W_2	W_3	W_4	W_5	Available	P_1	P_2	P_3	P_4	P_5	P_6
F_1	[5] 7	6	[15] 4	[20] 5	9	~~40~~ ~~25~~ ~~5~~ 0	1	1	2 ←	2 ←	–	–
F_2	8	[30] 5	6	7	8	~~30~~ 0	1	2 ←		–	–	–
F_3	[15] 6	8	9	6	[5] 5	~~20~~ ~~5~~ 0	1	1	1	1	1	1
F_4	[10] 5	7	7	8	6	~~10~~ 0	1	1	1	1	1	–
Req:	~~30~~ ~~25~~ ~~15~~ 0	~~30~~ 0	~~15~~ 0	~~20~~ 0	~~5~~ 0							
P_1	1	1	2 ↑	1	1							
P_2	1	1	–	1	1							
P_3	1		–	1	1							
P_4	1	–	–	–	1							
P_5	1 ↑	–	–	–	1							
P_6	6 ↑	–	–	–	5							

∴ Minimum total transportation cost =

$7\times5 + 4\times15 + 5\times20 + 5\times30 + 6\times15 + 5\times5 + 5\times10$

$= 35 + 60 + 100 + 150 + 90 + 25 + 50$

(2) $= ₹\ 510$

Here the no. of allocated cells = 7 $\ne$ $m+n-1$ ($4+5-1=8$)

∴ This solⁿ is degenerate. [$7 < m+n-1$]

we add a ε in unoccupied cell, which is independent & cost matrix value is minimum.

where $\varepsilon > 0$ — Available

7 [5]	6	4 [15]	5 [20]	9	$u_1 = 7$
8	5 [30]	6 [ε]	7	8	$u_2 = 9$
6 [15]	8	9	6	5 [5]	$u_3 = 6$
5 [10]	7	7	8	6	$u_4 = 5$

Req. $v_1 = 0$ $v_2 = -4$ $v_3 = -3$ $v_4 = -2$ $v_5 = -1$

Iteration-1 of optimality test

Find u_i & v_j for all occupied cell

where $c_{ij} = u_i + v_j$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(3)

Substituting $v_1 = 0$ [Max^m allocation]

$c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 7 - 0 \Rightarrow u_1 = 7$

$c_{31} = u_3 + v_1 \Rightarrow u_3 = c_{31} - v_1 \Rightarrow u_3 = 6 - 0 \Rightarrow u_3 = 6$

$c_{41} = u_4 + v_1 \Rightarrow u_4 = c_{41} - v_1 \Rightarrow u_4 = 5 - 0 \Rightarrow u_4 = 5$

$c_{13} = u_1 + v_3 \Rightarrow v_3 = c_{13} - u_1 \Rightarrow v_3 = 4 - 7 \Rightarrow v_3 = -3$

$c_{14} = u_1 + v_4 \Rightarrow v_4 = c_{14} - u_1 \Rightarrow v_4 = 5 - 7 \Rightarrow v_4 = -2$

$c_{23} = u_2 + v_3 \Rightarrow u_2 = c_{23} - v_3 \Rightarrow u_2 = 6 - (-3) = 9$

$c_{22} = u_2 + v_2 \Rightarrow v_2 = c_{22} - u_2 \Rightarrow v_2 = 5 - 9 = -4$

$c_{35} = u_3 + v_5 \Rightarrow v_5 = c_{35} - u_3 \Rightarrow v_5 = 5 - 6 = -1$

	W_1	W_2	W_3	W_4	W_5	Available	u_i
F_1	7 [5]	6	4 [15]	5 [20]	9	40	7
F_2	8	5 [30]	6 [ε]	7	8	30	9
F_3	6 [15]	8	9	6	5 [5]	20	6
F_4	5 [10]	7	7	8	6	10	5
Req.	30	30	15	20	5		
v_j	$v_1=0$	$v_2=-4$	$v_3=-3$	$v_4=-2$	$v_5=-1$		

Find Δ_{ij} for all unoccupied cells, where

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$\Delta_{12} = c_{12} - (u_1 + v_2) = 6 - (7 - 4) = 3$

$\Delta_{15} = c_{15} - (u_1 + v_5) = 9 - (7 - 1) = 3$

$\Delta_{21} = c_{21} - (u_2 + v_1) = 8 - (9 - 0) = -1$

$\Delta_{24} = C_{24} - (u_2 + v_4) = 7 - (9 - 2) = 0$

$\Delta_{25} = C_{25} - (u_2 + v_5) = 8 - (9 - 1) = 0$

$\Delta_{32} = C_{32} - (u_3 + v_2) = 8 - (6 - 4) = 6$

(4) $\Delta_{33} = C_{33} - (u_3 + v_3) = 9 - (6 - 3) = 9 - 3 = 6$

$\Delta_{34} = C_{34} - (u_3 + v_4) = 6 - (6 - 2) = 6 - 4 = 2$

$\Delta_{42} = C_{42} - (u_4 + v_2) = 7 - (5 - 4) = 7 - 1 = 6$

$\Delta_{43} = C_{43} - (u_4 + v_3) = 7 - (5 - 3) = 7 - 2 = 5$

$\Delta_{44} = C_{44} - (u_4 + v_4) = 8 - (5 - 2) = 8 - 3 = 5$

$\Delta_{45} = C_{45} - (u_4 + v_5) = 6 - (5 - 1) = 6 - 4 = 2$

	W_1	W_2	W_3	W_4	W_5	Avai.	u_i
F_1	[5] 7(−)	(3) 6	[15] 4(+)	[20] 5	(3) 9	40	7
F_2	(−1) 8(+)	[30] 5	[ε] 6(−)	(0) 7	(0) 8	30	9
F_3	[15] 6	(6) 8	(6) 9	(2) 6	[5] 5	20	6
F_4	[10] 5	(6) 7	(5) 7	(5) 8	(2) 6	10	5
Req.	30	30	15	20	5		
V_j	$v_1 = 0$	$v_2 = -4$	$v_3 = -3$	$v_4 = -2$	$v_1 = -1$		

Now, choose the minimum negative value from all Δ_{ij} (opportunity cost) $= \Delta_{21} = -1$ & draw a closed path from S_2D_1

closed path is $S_2D_1 \rightarrow S_2D_3 \rightarrow S_1D_3 \rightarrow S_1D_1$

Now,

(5) Minimum allocated value among all negative position (-) on closed path = 2

Subtract 2 from all (-) & add it to all (+).

	W_1	W_2	W_3	W_4	W_5	Avainble
F_1	7 [3]	6	4 [17]	5 [20]	9	40
F_2	8 [2]	5 [30]	6 (-)	7	8	30
F_3	6 [15]	8	9	6	5 [5]	20
F_4	5 [10]	7	7	8	6	10
Req.	30	30	15	20	5	

Repeat the step 1 to 4 until an optimal Soln is obtained.

Iteration-2. of Optimality test

Find u_i & v_j for all occupied cells, where

$$c_{ij} = u_i + v_j$$

Substituting $v_1 = 0$

$c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 7 - 0 = 7$

$c_{21} = u_2 + v_1 \Rightarrow u_2 = c_{21} - v_1 \Rightarrow u_2 = 8 - 0 = 8$

$c_{31} = u_3 + v_1 \Rightarrow u_3 = c_{31} - v_1 \Rightarrow u_3 = 6 - 0 = 6$

$c_{41} = u_4 + v_1 \Rightarrow u_4 = c_{41} - v_1 \Rightarrow u_4 = 5 - 0 = 5$

$c_{13} = u_1 + v_3 \Rightarrow v_3 = c_{13} - u_1 \Rightarrow v_3 = 4 - 7 = -3$

$c_{14} = u_1 + v_4 \Rightarrow v_4 = c_{14} - u_1 \Rightarrow v_4 = 5 - 7 = -2$

$c_{22} = u_2 + v_2 \Rightarrow v_2 = c_{22} - u_2 \Rightarrow v_2 = 5 - 8 = -3$

$c_{35} = u_3 + v_5 \Rightarrow v_5 = c_{35} - u_3 \Rightarrow v_5 = 5 - 6 = -1$

(6)

	W_1	W_2	W_3	W_4	W_5	Avai	u_i
F_1	7 [3]	6	4 [17]	5 [20]	9	40	7
F_2	8 [2]	5 [30]	6 [illegible]	7	8	30	8
F_3	6 [15]	8	9	6	5 [5]	20	6
F_4	5 [10]	7	7	8	6	10	5
Req.	30	30	15	20	5		
v_j	$v_1=0$	$v_2=-3$	$v_3=-3$	$v_4=-2$	$v_5=-1$		

Find Δij for all unoccupied cells, where

$$\Delta ij = C_{ij} - (u_i + v_j)$$

$\Delta_{12} = 6-(7-3) = 6-4 = 2$

$\Delta_{15} = 9-(7-1) = 9-6 = 3$

$\Delta_{24} = 7-(8-2) = 7-6 = 1$

$\Delta_{25} = 8-(8-1) = 8-7 = 1$

$\Delta_{32} = 8-(6-3) = 8-3 = 5$

$\Delta_{33} = 9-(6-3) = 9-3 = 6$

$\Delta_{34} = 6-(6-2) = 6-4 = 2$

$\Delta_{42} = 7-(5-3) = 7-2 = 5$

$\Delta_{43} = 7-(5-3) = 7-2 = 5$

$\Delta_{44} = 8-(5-2) = 8-3 = 5$

$\Delta_{45} = 6-(5-1) = 6-4 = 2$

since all $\Delta ij \geqslant 0$

(7)

So, final optimal sol^n is

	W_1	W_2	W_3	W_4	W_5
F_1	7 [3]	6	4 [17]	5 [20]	9
F_2	8 [2]	5 [30]	6 [$\varepsilon-2$]	7	8
F_3	6 [15]	8	9	6	5 [5]
F_4	5 [10]	7	7	8	6

∴ Minimum transportation cost =

$$7\times3 + 4\times17 + 5\times20 + 8\times2 + 5\times30 + 6\times(\varepsilon-2) + 6\times15 + 5\times5 + 5\times10$$

$$= 21 + 68 + 100 + 16 + 150 - 12 + 6\varepsilon + 90 + 25 + 50$$

$$= 520 + 6\varepsilon - 12$$

$\Rightarrow$ ₹ 508 Ans [∵ ε is very small, so called zero]

Assignment Problem

Introduction
The objective of assignment problem is to assign a number of origins (jobs) to the equal number of destinations (persons) at a minimum time or minimum cost or maximize the profit.

Algorithm for Assignment problem
Step 1: Determine the cost table from given problem
I. If the number of source is equal to the number of destinations, go to step 3
II. If the number of sources is not equal to the number of destinations, go to step 2
Step 2: Add a dummy source or dummy destination, so that the cost table becomes a square matrix. The cost entries of dummy source/destinations are always zero.
Step 3: locate smallest element in each row of the given cost matrix and then subtract the same from each element of that row.
5
Step 4: in the reduced matrix obtained in step3, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have at least one zero.
Step 5: in the modified matrix obtained in step4, search for an optimal assignment as follows:
a. Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (x) all other zeros in its column. Continue in its manner until all the rows have been taken care of.
b. Repeat the procedure of each column of the reduced matrix.
c. If a row and/or column has two or more zeros and one cannot be chosen byt inspection, then assign arbitrary any one of these zeros and cross off all other zero of that row/column.
d. Repeat (I) through (III) above successively until the chain of assigning (□) or cross (x) ends. Solution is reached.
Step 6: if the number of assignments (□) is equal to n (the order of matrix) , an optimum solution is reached
If the number of assignment is less than n (the order of matrix), go to next step.
Step 7: Draw the minimum number of horizontal and/or vertical lines to cover all the zeros of the reduced matrix. This can be conveniently done be using a simple procedure:
a. Mark (√) rows that do not have any assigned zero.
b. Mark (√) columns that have zeros in the marked rows.
c. Mark (√) rows that have assigned zeros in the marked columns.
d. Repeat (b) and (c) above until the chain of making is completed.
e. Draw lines through all the unmarked rows and marked columns. This gives us the desired minimum number of lines.
Step 8: Develop the new revised cost matrix as follows:
a. Find the smallest element of the reduced matrix not covered by any of the lines.
b. Subtract this element form all the unmarked elements and add the same to all the Elements lying at the intersection of any two lines.
Step 9: Go to step 6 and repeat the procedure until an optimum solution is attained.

Hungarian Method :→

Step-① From a given problem, find out the cost table. Note that if the no. of origins is not equal to the no. of destinations then a dummy origin or destination must be added.

Step-② In each row of the table find out the smallest cost element, subtract this smallest cost element from each element in that row. So, that there will be at least one zero in each row of the new table.

→ This new table is known as First Reduced cost Table.

Step-③ In each column of the table find out the smallest cost element, subtract this smallest cost element from each elements in that column.

→ As a result of this, each row & column has at least one zero element.

→ This new table is known as Second Reduced cost Table.

Step-④ Now determine an assignment as follows

1. For each row or column with a single zero element cell that has not be assigned or eliminated, boxe that zero element as an assigned cell.

2. For every zero that becomes assigned, ... out all other zeros in the same row

and for column.

3. If for and for a column there are two or more zero & one can't be chosen by inspection, choose the assigned zero cell arbitrarily.

4. The above procedures may be repeated until every zero element cell is either assigned (boxed) or crossed out.

Step-⑤ An optimum assignment is found, if the no. of assigned cells is equal to the no. of rows (& column). In case we ~~add~~ had chosen a zero cell arbitrarily, there may be an alternate optimum. If no optimum solution is found i.e. some rows or columns without an assignment then go to step-6.

Step-⑥ Draw a set of lines equal to the no. of assignments which has been made in step 4, covering all the zeros in the following manner.

1. Mark check (✓) to those rows where no assignment has been made.

2. Examine the checked (✓) rows. If any zero element cell occurs in those rows, check (✓) the respective columns that contains those zeros.

3. Examine the checked column. If any assigned zero element occurs in those columns check (✓) the respective rows that contain those assigned zeros.

4. The process may be repeated until now more rows or column can be checked.
5. Draw lines through all unchecked rows & through all checked columns.

Step 7 → Examine those elements that are not cover by a line. choose the smallest of these elements & subtract this smallest from all the elements that do not have a line through them.

→ Add this smallest element to every element that lies at the intersection of two lines. Then the resulting matrix is a new revised cost table.

Q: → Assign the four tasks to four operators. The assigning costs are given in the table below.

Operators

Tasks	1	2	3	4
A	20	28	19	13
B	15	30	31	28
C	40	21	20	17
D	21	28	26	12

Solⁿ: → Step-1.
Given matrix is square matrix
Given matrix is square matrix row or column.
∴ No need to add dummy row or column.

Step-2
Selecting the smallest value in each row, & subtracting them to corresponding row

	1	2	3	4
A	7	15	6	0
B	0	15	16	13
C	23	4	3	0
D	9	16	14	0

Step-3

Reduce the new matrix given in the table by selecting the smallest value in each column & subtract them to the corresponding column.

Now,
column-wise reduction matrix

	1	2	3	4
A	7	11	3	0
B	0	11	13	13
C	23	0	0	0
D	9	12	11	0

Step-4 Draw a minimum no. of lines possible to cover all the zeros in the matrix.

Matrix with all zeros covered

	1	2	3	4
A	7	11	3	0
B	0	11	13	13
C	23	0	0	0
D	9	12	11	0

Step-8 Assign the tasks to the operator

Select a row that has a single zero &, assign by squaring it. Strike off remaining zeros if any in that row or column.

Repeat the assignment for other tasks.

Final assignment

Operators

Tasks	1	2	3	4
A	7	8	[0]	~~0~~
B	[0]	8	10	13
C	26	[0]	~~0~~	3
D	9	9	8	[0]

Therefore optimal assignment is

Task	Operator	Cost
A	3	19
B	1	15
C	2	21
D	4	12
Total cost = 67		

Ans.

Q → Solve the following assignment problem for minimum optimal cost

	1	2	3	4
1	0	30	80	50
2	40	0	140	30
3	40	50	0	20
4	70	80	130	0

Solⁿ: → This is a travelling salesman problem. In this problem, from city $1 \to 1, 2 \to 2$ etc. is not allowed.

Assign a large penalty $c_{ii} = \infty$ to all zero cost matrix.

	1	2	3	4
1	∞	30	80	50
2	40	∞	140	30
3	40	50	∞	20
4	70	80	130	∞

Now choose the minimum cost matrix & subtract them to the corresponding row.

Then.

	1	2	3	4
1	∞	0	50	20
2	10	∞	110	0
3	20	30	∞	0
4	0	10	60	∞

Now,

Reduce the new matrix given in the table by selecting the smallest value in each column & subtract them to the corresponding column.

	1	2	3	4
1	∞	0	0	20
2	10	∞	60	0
3	20	30	∞	0
4	0	10	10	∞

Now, Draw a minimum no. of lines to cover all the zeros.

	1	2	3	4
1	∞	0	0	20
2	10	∞	60	0
3	20	30	∞	0
4	0	10	10	∞

Here, the no. of lines drawn is not equal to the order of matrix

i.e. $3 \neq 4$

Now,

Taking smallest element of the matrix that is not cover by a single line which is 10. Subtract 10 from all not covered & add it to the intersection of line.

	1	2	3	4
1	∞	0	0	30
2	10	∞	50	0
3	20	20	∞	0
4	0	0	0	∞

Now, Draw minimum no. of lines to cover all zeros.

∞	0	0	30
10	∞	50	0
20	20	∞	0
0	0	0	∞

Here no. of lines draw = 3

i.e. $3 \neq 4$ [order of matrix]

Now,

Taking smallest element of the matrix that is 10. Subtract 10 from all not covered & add it to the intersection of lines.

	1	2	3	4
1	∞	0	0	40
2	0	∞	40	0
3	10	10	∞	0
4	0	0	0	∞

Here no. of lines = 4

& order of matrix = 4

... the optimal

Now,

	1	2	3	4
1	∞	~~0~~	[0]	40
2	[0]	∞	40	~~0~~
3	10	10	∞	[0]
4	~~0~~	[0]	~~0~~	∞

Note:- Escape max^{m} zero in row & select the minimum zero & cut the remaining zero row & column wise.

Now, A/c to the assignment, Salesman should visit city $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$

& this involves the cost of

$₹(80 + 90 + 80 + 40) = ₹\,220$ Ans

Sequencing problems

Introduction:

The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing.
A practical situation may corresponds to an industry producing a number of products, each of which is to be processed through different machines, of course, finite in number
The general sequencing problem may be defined as: Let there be n jobs to be performed one at a time on on each of m machines. The sequence (order) of the machines in which each job should be performed is given. The actual or expected time required by the jobs on each of the machines is also given. The general sequencing problem, therefore, is to find the sequence out of $(n!)^m$ possible sequences which minimize the total elapsed time between the start of the job in the first machine and the completion of the job on the last machine.

Assumptions:

• Each job once started on a machine, is to be performed up to completion on that machine.

• The processing time on each machine is known. Such a time is independent of the order of the jobs in which they are to be processed.

• The time taken by each job in changing over from one machine to another is negligible.

• A job starts on the machine as soon as the job and the machine both are idle and job is next to the machine and the machine is also next to the job.

• No machine may process more than one job simultaneously.

• The order of completion of job has no significance, i.e. no job is to be given priority. The order of completion of jobs is independent of sequence of jobs.

Basic Terms:

Number of Machines: It refers to the no. of service facilities through which a job must pass before it is assumed to be completed.
7
Processing Order: It refers to the order (sequence) in which given machines are required for completing the job.
Processing Time: It indicates the time required by a job on each machine.
Total Elapsed Time: It is the time interval between starting the 1st job and completing the last job including the idle time in a particular order by the given set of machines.

Idle time on a machine: It is the time for which a machine does not have a job to process, i.e. idle time from the end of job (i-1) to the start of jobs i.
Processing Time: It indicates the time required by a job on each machine.
No passing rule: It refers to the rule of maintaining the order in which jobs are to be processed on given machines.

Process 'n' jobs through two Machines:

Optimum Sequence of Algorithm:

1. List the jobs along with their processing times in a table.
2. Examine the rows for processing times on machines M_1 and M_2, and find the smallest processing time in each row, i.e. find out min.(t_{1j}, t_{2j}) for all j.
3. If the smallest processing time is for the first machine M_1, then place the corresponding job in the 1st available position in the sequence, otherwise place 2nd machine M_2.
4. If there is a tie in selecting the minimum of all the processing times, then there may be 3 situations

a) Minimum among all processing times is same for the machines, i.e., min.(t_{1j}, t_{2j})= t_{1k} = t_{2r}, then process the kth job first and the rth job last.
b) If the tie for minimum occurs among processing times t_{1j} on machine M_1 only, then select arbitrarily the job to process first.
c) If the tie for minimum occurs among processing times t_{2j} on machine M_2, then select arbitrarily the job to process last.

d) Cross off the jobs already assigned and repeat steps 1 through 4, placing the remaining jobs next to first or next to last, until all the jobs have been assigned

➢ Calculate idle time for machines M_1 and M_2:
Idle time for M_1 = Total elapsed time –(time when the last job in a sequence finishes on M_1.
Idle time for M_2= Time at which the 1st job in a sequence finishes on M_1 + (time when the jth job in a sequence starts on M_2) – {(time when the (j-1)th job in a sequences finishes on M_2)}
➢ The total elapsed time to process all jobs through 2 machines as under:

Process 'n' jobs through 'K' Machines:

Let there be 'n' jobs, each of with is to be processed through 'm' machines, says M_1, M_2, M_3,….. M_k, in order M_1, M_2, M_3,….. M_k,. the list of jobs with their processing times is:
job and before starting work on jth job.
I Time for which machine M remains idle after processing (j-1)th
Where, t Time required for processing jth job on machine M
t I
Total elapsed time Time when the nth job in a sequence finishes on machine M

Optimum Sequence of Algorithm:

The iterative procedure for determining the optimal sequence for 'n' jobs on 'k' machines can be summarized as follows:

Step 1: Find Min t_{1j}, Min t_{kj} and Maximum of each of $t_{2j}, t_{3j}, \ldots t_{k-1j}$ for all j = 1,2,…n
Step 2: Check the following
.

.

Step 3: If the inequalities of step 2 are not satisified, method fails. Otherwise go to next step.
Step 4: Convert the k machine problem into two-machine problem by introducing tow fictitious machine G and H, such that
1 2 3 1
2 3 4

….
Step 5: Determine the optimal sequence for the 'n' jobs and '2' machines equitant sequencing problems with the prescribed order GH in the same way as discussed earlier the resulting sequence shall be optimum for the given problem.

Processing Two Jobs through m machines

Example 1. Use graphical method to minimize the time needed to process the following jobs on the machines shown below also calculate the total time needed to complete both the jobs.

Job1 A B C D E
Job 2 C A D E B

Solution:

Step -1: First draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2.
Step 2: layout the machine time for two jobs on the corresponding axes in the technological order machine A takes 2 hours for job 1 and 5 hours for job 2.
10
Construct the rectangular PQRS for the machine A. Similarly, other rectangular for machine B,C,D and E are constructed as shown in fig.
Step 3: make a program by starting from the origin O and moving through various stages of Completion (points) until the point marked finish is obtained. Physical interpretation of the path thus chosen involves the series of segments which are horizontal or vertical or diagonal making an angle of 450 with the horizontal. Moving to the right means that job1 is proceeding while job 2 is idle and moving upward means that job 2 is proceeding while job 1 is idle and moving diagonally means that both the jobs are proceeding simultaneously. Further both the jobs cannot be processed simultaneously on the same machine. Graphically diagonal movement through the
blocked out (shaded) area is not allowed and similarly for other machines too.
Step 4: (To find an optimal path) An optimal path is one that minimizes idle for job1 (horizontal) and job 2 (vertical movement). Choose such a path on which diagonal moment is as much as possible.
Step 5: (to find the elapsed time). The elapsed time is obtained by adding the idle time either of the job to the processing time for that job. In this problem the

Sequencing Model

idle time for the chosen path is seen to be 3hrs for the 1 and 0 for the job 2. Thus the total elapsed time, 17 + 3 = 20hrs

Q→ Six jobs A, B, C, D, E & F have arrived at one time to be processed on a single machine. Assuming that no new jobs arrive thereafter, determine

Job :	A	B	C	D	E	F
Processing time (min) :	7	6	8	4	3	5

(i) optimal sequence as per S.PT rule.
(ii) completion times of the job.
(iii) Mean flow time.
(iv) Average in-process inventory.

Solⁿ:→ (i) A/c to S.P.T. rule

optimal sequence is

$E \rightarrow D \rightarrow F \rightarrow B \rightarrow A \rightarrow C$

(ii) completion of job times are [As per SPT]

3, 7, 12, 18, 25 & 33 mins resp.

(iii) Mean flow time $= \frac{3+7+12+18+25+33}{6}$

$= \frac{98}{6} = 16.33$ mins Ans.

(iv) No. of jobs waiting — Time duration

6 ——— (0 − 3) min

5 ——— (3 − 7) min

4 ——— (7 − 12) "

∴ Average in-process inventory =

$$\frac{6\times3+5\times4+4\times5+3\times6+2\times7+1\times8}{3+4+5+6+7+8}$$

$$=\frac{18+20+20+18+14+8}{33}=\frac{98}{33}=2.96$$

≈ 3 jobs.

* Processing 'n' jobs through 'two' machines →

Q→ A machine operator has to perform two operations, turning & threading on a no. of diff. jobs. The time required to perform these operations (in min) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

Job	Time for Turning (min)	Time for threading (min
1	3	8
2	12	10
3	5	9
4	2	6
5	9	3
6	11	1

Also find the total processing time & idle times for turning & threading operation.

∴ Total elapsed time = 43 mins.

Idle time for turning M/c = 1 min

& " " " threading M/c = 2+4 = 6 mins

Ans.

* Processing 'n' jobs through 'three' Machines →

Q → A machine operator has to perform three operations: turning, threading & knurling on a no. of diff. jobs. The time req. to perform these operations (in min) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time req. to turn out all the jobs. Also find the idle times for the three operations.

Job	Time for Turning (min)	Time for Threading (min)	Time for Knurling (min)
1	3	8	13
2	12	6	14
3	5	4	9
4	2	6	12
5	9	3	8
6	11	1	13

∴ Total elapsed time = 43 mins.

Idle time for turning M/c = 1 min

& " " " threading M/c = 2+4 = 6 mins

Ans.

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4	2	6	12
5	9	3	8
6	11	1	13

Sol^n:→ First of all convert three m/cs into two m/cs.

Job	G_i (Turning + Threading)	H_i [Thred. + Knurling]
1	11	21
2	18	20
3	9	13
4	8	18
5	12	11
6	12	14

[Note:- Adding processing time]

Now, By applying sequencing rule Sequence of Jobs are as follows.

M/c G_i → M/c-H_i

4	3	1	6	2	5

Now, elapsed time are as below.

Job	Turning op.		threading op.		Knurling op.	
	Time in	Time out	Time in	Time out	Time in	Time out
4	0	2	2	8	8	20
3	2	7	8	12	20	29
1	7	10	12	20	29	42
6	10	21	21	22	42	55
2	21	33	33	39	55	69
5	33	42	49	[illegible]	[illegible]	[illegible]

∴ Total elapsed time = 77 mins.

∴ Idle time for turning = 77 - 42 = 35 mins

∴ Threading = 2 + 1 + 11 + 3 + (77 - 45)

= 49 mins.

Knurling = 8 mins Ans

Replacement problems

Introduction: Replacement problems `are concerned with the situation that arise when some items such as men, machines, electric-light bulbs, etc. Need replacement due to their decreased efficiency, failure or break down. The deteriorating efficiency or complete breakdown may be either grzadual or sudden,
Broadly speaking, the requirement of a replacement may be in any of the following

Replacement of items that fail completely:

Some situations, failure of a certain item occurs all of a sudden, instead of gradual deterioration (e.g., failure of light bulbs, tubes, etc.).
The failure of the item may result in complete breakdown of the system. The breakdown implies loss of production, idle inventory, idle labour, etc. Therefore, an organization must prepare itself against these failures.
Thus, to avoid the possibility of a complete breakdown, it is desirable to formulate a suitable replacement policy. The following two courses can be followed in such situations.

• **Individual replacement policy**. Under this policy, an item may be replaced immediately after its failure.

• **Group replacement policy**. Under this policy, the items are replaced in group after a certain period, say t, irrespective of the fact that items have failed or not. If any item fails before its preventive replacement is due, then individual replacement policy is used.

1

GAME THEORY

Introduction:

A competitive situation will be called a "Game". Many practical problems require decision-making in a competitive situation where there are two or more opposing parties where the action of one depends up on the one taken by the opponent.
For example: candidates for an election advertising and marketing campaigns by competing business firms, countries involved in military bottles etc.
If Game has the following properties

➢ Each player has all relevant information available
➢ Each Game has finite number of players
➢ Each player can use finite number of strategies.

Some Basic Terms:

Player: the participants or competitors in the game are known as players. A player may be individual or group of individuals.

Strategy: A strategy for a player is defined as a set of rules of alternative courses of action available to him in advance, by which player decides the course of action that he should adopt strategy may be of two types.

➢ Pure strategy

➢ Mixed strategy

Pure strategy: If the player select the game strategy each time , then it is referred to as purestrategy.

The objective of the players is to maximize gains or to minimize losses.

Mixed strategy: When the player use a combination of strategies and each player always kept

guessing as to which course of action is to be selected by the other player at a particular occasion then this is known as mixed strategy. There is a probabilistic situation and objective of the player is to maximize expected gains or to minimize losses.

Optimum strategy: A course of action or play which puts the player in the most preferred position, irrespective of the strategy of his competitors is called an optimum strategy.

2

Pay-off matrix:

When the players select their particular strategies. The pay-off (gains of losses) can be represented in the form of a matrix called the payoff matrix.

11 12 13
21 22 23
31 32 33

Player B

a a a

Player A a a a

a a a

. .

. .

. .

. . . .

Let player 'A' have 'm' strategies A and player 'B' have 'n' strategies. It is assume that each player has his choices from amongst the pure strategies. Also it is assumed that player 'A' is always the gainer and player 'B' is always looser.

Value of the Game: It is the expected pay off of play when all the players of the game follow their optimum strategies. The game is called value of the game

Optimum strategy:

A course of action or play which puts the player in the most preferred position irrespective of the strategy of his competitors is called on optimum strategy.

Value of the game:

It is the expected pay off of play when all the players of the game follow their optimum strategies. The game is called fair is the value of the game is zero and unfair if it is non-zero.

Saddle Point:

The definition of pure strategy states that both the players use the same strategy every time and that is the best strategy from the points of both players. In such case, the strategy indicates the saddle point, which is the point maximum gain for A and minimum loss for B.

Rules for determine a saddle point:

We may now summarize the procedure of locating the saddle point of a pay off matrix as follows:

Step 1: select the minimum element of each row of the pay off matrix and mark them (*)

3

Step 2: select the maximum element of each column of the pay off matrix and mark them (+)

Step 3: if there appears an element in the pay off matrix in the pay off matrix marked (*) and (+) both, the position of that element is a saddle point of the pay off matrix

Methods form solving game without saddle points

When there are no saddle points in the game, pure strategies as optimal strategies are not available. Such games can be resolved by using mixed strategy and following methods can be adopted

1. Algebraic method.
2. Graphic method
3. Dominance method.
4. Linear programming method

Algebraic method:

Algebraic formula for matrix method:

The general formulae can be derived algebraically from the matrix in the following manner. The matrix so obtained from rule of dominance can be written as:

Value of the game

Where P = probability or chance of player A using strategy and q of player B using strategy.

Graphical Method:

Some games will be specialized nature like $2 \cdot n$ or $m \cdot 2$ m 2 games. The pay off matrix for the 2 n games will contain 2 rows and n columns, where as the pay off matrix of the m 2 game will contain 'm' rows and 2 columns. If there is no saddle point for these games, one can solve them using graphical method.

Algorithm for $2 \cdot n$ games:

If the given pay off matrix is $2 \cdot n$(no. of rows are 2 and no. of columns are more than 2). Then we follow the steps

Step 1: Plot the gain function on a graph assuming a suitable scale. Keep x on X-axis and gain on
y-axis.

Step 2: Since the player A is a maximum player, find the highest intersection point in lower boundary of the graph. Let it be maximum point

Step 3: if the number of lines passing through the maximum point is only two, from $2 \cdot 2$ pay off matrix form the original problem by retaining and go to step 8 otherwise go to step 7.

Step 4: identify any two lines with opposite slopes passing through that point. Then from $2 \cdot 2$ pay off matrix form the original problem by retaining only column corresponding to those two lines which is having opposite slopes.

5

Step 5: Solve $2 \cdot 2$ game using oddments and find strategies for player 'A' and 'B' and also value
of game.

Algorithm for $m \cdot 2$game:

If the given pay off matrix is $m \cdot 2$ (no. of rows are more than 2 and no. of columns are 2). Then we follow the steps

Step 1: plot the gain function on a graph function on a graph by assuming a suitable scale, keep
'y' on 'x-axis' and gain on 'y – axis'.

Step 2: since B is minimax player, find lowest intersection point in the upper boundary of the graph. Let it be the minimax point.

Step 3: if number of lines passing through minimax point is only two, form a $m \cdot 2$ pay off matrix form original problem by retaining only the row corresponding to those two lines and go to step 8, otherwise go to step 7.

Step 4: identify any two lines with opposite slopes passing through that point the form $2 \cdot 2$ pay off matrix form the original by retaining only the rows corresponding to two lines which are

Step 5: solve the $2 \cdot 2$ game using oddments and find strategies for player 'A' and player 'B' and also the value of the game.

Dominance Method:

Some times it is observed that one of pure strategies of either player is always inferior to at least one of remaining one. The superior strategies are said to dominate the inferior one. Clearly a player would have to incentive to use inferior strategies which are dominated by superior one. In such cases of dominance, we can reduce the size of pay off matrix by deleting those strategies which are dominated by others. Thus if each element in one row say k_{th} of pay off matrix (a_{ij}) is less than or equal to the corresponding elements in some other row say r_{th} then player A will never choose k th strategy in other words probability P_k =P (choosing the k_{th} strategy) is zero, if $a_{kj} < a_{rj}$ for all j =1,2,3,….,n

The value of the game and the non – zero choice of probabilities remain unchanged even after the deletion of k_{th} row from the pay off matrix. In such a case the k_{th} strategy is said to be dominated by r $_{th}$ one.

General rules of dominance are:

1) If all the elements of a row say r_{th} less than or equal to the corresponding elements of any other row say r_{th} then k_{th} is dominated by r the row.

6

2) If all elements of a column, say k_{th} are greater than or equal to the corresponding elements of any other column say r_{th} then k_{th} column is dominated by r_{th} column.

3) Dominated rows and columns can be deleted to reduce the size of pay off matrix is optimal strategies will remain unaffected.

The modified Dominance Property:

The dominance property is not always based on superiority of pure strategies only. A given strategy can also be said to be dominated if it is inferior to an average of two or more other pure strategies more generally. If some convex linear combination of some rows dominates the i_{th} row, then i_{th} row will be deleted similar arguments follow for columns.

Queuing Theory(Waiting Lines)

Queue called the waiting line in order to maintain a proper discipline. Here the arriving people are called customers and the persons issuing the ticket are called a Server.

Queuing System:

A Queuing system can be completely described by

I. The input (or arrival pattern)

II. The service mechanism (or service pattern)

III. The Queue discipline

IV. Customer’s behavior.

The input pattern (or arrival pattern)

The input describes the way in which the customers arrive and join the system.

Generally the customers arrive in more or less random fashion which is not worth making the prediction. Thus the arrival pattern can be described in terms of probability distribution for inter arrival times (the time between two successive arrivals) must be defined. We deal with those queuing system in which the customers arrive in ‘Poisson’ fashion. Mean arrival rate is denoted by λ.

The service pattern (Out pattern)

It described how many customer can be served at a time, what the statistical distribution of the service time is , and when the service is available. Service time may be a constant or a random variable. Distributions of service time which are important in practice are the exponential distribution. The mean service rate is denoted by μ.

The Queue Discipline:

The Queue discipline is the route determining the formation of the queue, the manner of the customer’s behaviors while waiting, and the manner in which they are chosen for service. The simplest discipline is “First come, first served”. Such type of Queue discipline is observed at a ration shop.

Some of the Queue service disciplines are

FIFO – “First in, First out”

LIFO – “Last in, First out”

SIRO – “Service in Random order”

Customer’s Behavior:

The customer generally behaves in 4 ways

1. Balking: A customer may leave the Queue, of there is no waiting space
2. Reneging: This occurs when the waiting customer leaves the Queue due to Impatience.
3. Priorities: In certain applications some customers are served before others regardless of their order of arrival.
4. Jockeying: Customers may jockey from one waiting line to another.

Queuing Models:

Generally Queuing Model may be completely specified in the following symbol form: (a/b/c: d/e)

Where ‘a’ = inter arrival time
‘b’ = inter service time
‘c’ = Number of service channels
‘d’ = system capacity
‘e’ = Queue discipline

Model I $\cdot\ M/M/1: \cdot / FIFO \cdot$

This model deals with a queuing system having single service channel. Poisson input, Exponential service and there is no limit on the system capacity while the customers are served on a “First in, First out” basis.

Characteristics of Model I

1. Average number of customers in system is given by

Model III $\cdot\ M/M/C: \cdot / FIFO \cdot$

This model deals with a queuing system having more than one service channels. Poisson input, Exponential service and there is no limit on the system capacity while the customers are served on a “First in, First out” basis.

Model IV $\cdot\ M/M/C: N / FIFO \cdot$

This model deals with a queuing system having more than one service channels. Poisson input, Exponential service and there is ‘N’ customers in the system capacity while the customers are served on a “First in, First out” basis.

Simulation

Simulation is a imitation of reality. It representation of reality through the use of a model or other device which will react in the same manner as reality under a set of condition.

Examples:

a. Testing of medicines on animals in laboratories. In this case, the animal responses simulate the human responses.

b. Driving automobiles on test tracks. The test track simulates the actual environment in which the automobiles are driven.

Monte-Carlo Technique or Monte – Carlo simulation

This technique involves the selection of random observations within the simulation model. It is constrained for application involving random numbers to solve deterministic and stochastic problems the underlying principle of this technique is

1) Replace the actual statistical universe by another universe described by some assumed probability distribution.

2) Sample from this theoretical population by mean of random numbers.

Steps in Simulation

Step 1: Clearly define the problem
a) Identify the objectives of the problem.
b) Identify the main factors which have the greatest effect on the objectives of the problem.
Step 2: Construct an appropriate model.
a) Specify the variable and parameters of the model
b) State the conditions under which the experiment is to be performed.
c) Define the relationship between the variables and parameters.
Step 3: Prepare the model for experimentation
a) Define the starting conditions for the simulation.
b) Specify the number of runs of simulation to be made.
Step 4: Using step 1 to 3, experiment with the model.
a) Define a coding system that will correlate the factors defined in step with the random numbers to be generated for the simulation
b) Select a random number generator and create the random numbers to used in the simulation.
c) Associate the generated random numbers with the factors identified in step 1 and coded in step 4 (a).
Step 5: Summarizes and examine the results obtained in step 4.
13
Step 6: Evaluate the results of the simulation

Advantages to Simulation:

• Can be used to study existing systems without disrupting the ongoing operations.

• Proposed systems can be "tested" before committing resources.

• Allows us to control time.

• Allows us to identify bottlenecks.

• Allows us to gain insight into which variables are most important to system performance.

• **Manufacturing:** Material handling systems, assembly lines, automated production facilities, inventory control systems, plant layout, etc..

• **Business:** Stock and commodity analysis, pricing policies, marketing strategies, cash flow analysis, forecasting, etc..

• **Government:** Military weapons and their use, military tactics, population forecasting, land use, health care delivery, fire protection, criminal justice, traffic control, etc.

Disadvantages to Simulation

• Model building is an art as well as a science. The quality of the analysis depends on the quality of the model and the skill of the modeler (Remember: *GIGO*)

• Simulation results are sometimes hard to interpret.

•

Simulation analysis can be time consuming and expensive should not be used when an analytical method would provide for quicker results.

Dynamic Programming

Introduction: Dynamic programming is to perform optimization (Minimization or Maximization). It is to divide the original problem into many sub problems and solve the sub problems individual obtain the optimum solution of the last problem by integrating the optimum solution of original problem.

Characteristic of DPP

The features which characterize the dynamic programming problem are as follows:

1. The problem can be divided into stages with a policy decision required at each stage.
2. Every stage consists of a number of states associated with it. The states are the different possible condition in which the system may find itself at that stage of the problem
3. Decision at each stage converts the current stage into state associated with the next stage.
4. The state of the system at a stage is described by a set of variables called state variables.
5. When the current state is known, an optimal policy for the remaining stages is independent of the policy of the previous ones.
6. The solution procedure begins by finding the optimal policy for each state to the last stage.
7. A recursive relationship which identifies the optimal policy for each state with n stages remaining, given the optimal policy for each state with (n-1) stages left
8. Using recursive equation approach, each time the solution procedure moves backward stage by stage by stage for obtaining the optimum policy of each state for the particular stage, till it attains the optimum policy beginning at the initial stage.

Dynamic Programming Algorithm

The computational procedure for solving a problem by dynamic programming approach can be summarized in the following steps:

Step 1: Identify the decision variables and specify objective function to be optimized under certain limitations, if any

Step 2: Decompose (or divide) the given problem into a number of smaller sub-problems (or stages). Identify the state variables at each stage and write down the transformation function as a function of the state variables and decision variables at the next stage.

Step 3 : write down a general recursive relationship for computing the optimal policy decide whether forward or Backward method is to follow to solve the problem

Step 4: Construct appropriate stages to show the required values of the return function.

Step 5: Determine the overall optimal policy or decisions and its value at each stage there may be more than one such optimal policy.

Applications of Dynamic Programming:

➢ In the production area, this technique has been used for production scheduling and employment smoothing problems

➢ It has been used to determine the inventory level and for formulating the inventory recording

➢ It can be applied for allocating the scarce resources to different alternative uses such as, allocating the salesmen to different sales districts etc.

➢ It is used to determine the optimal combination of advertising media(TV, Radio, News papers) and the frequency of advertising.

➢ It can apply in Replacement theory to determine at which age the equipment is to replace for optimal return from the facilities.

➢ Spare part level determination to guarantee high efficiency utilization of expensive equipment.

➢ Other areas: Scheduling methods, Monrovian decision models, infinite stage system, probabilistic decision problem etc.

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