

Introduction

1.1. DEFINITION

Theory of Machine is that branch of science which deals with the study of relative motion between the various parts of a machine, and forces which act on them. Theory of machine may be divided into *kinematics* and *dynamics*.

Kinematics is that branch of theory of machine which deals with the study of relative motion between the various parts of the machines. Here the various forces involved in the motion, are not considered. Thus kinematics is the study to know the displacement, velocity and acceleration of a part of the machine.

Dynamics is that branch of theory of machine which deals with the study of various forces involved in the various parts of the machine. The forces may be either static or dynamic.

Dynamics is further divided into *kinetics* and *statics*. Kinetics is that branch of theory of machine which deals with various forces when the body is moving whereas statics is that branch of theory of machine which deals with various forces when the body is stationary as shown in Fig. 1.1.

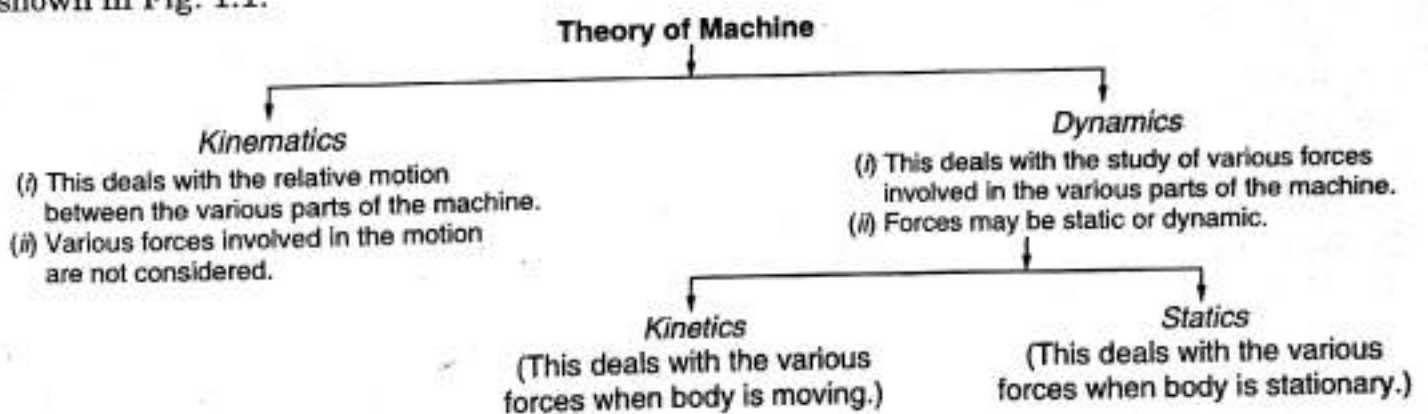


Fig. 1.1

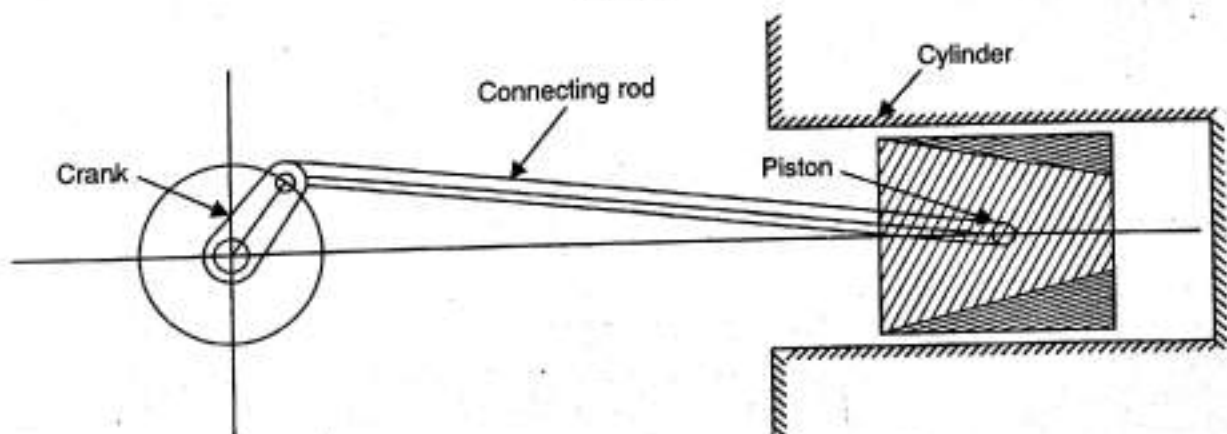


Fig. 1.1 (a)

MECHANISM :- Combination of a number of bodies assembled in such a way that the motion of one causes constrained and predictable motion to the others.

* Mechanism transmit and modify motion.

* eg - Slider Crank Mechanism, Whitworth Quick return mechanism etc.

Machine :- is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into desired work.

* It is neither a source of energy nor a producer of work but helps in proper utilization.

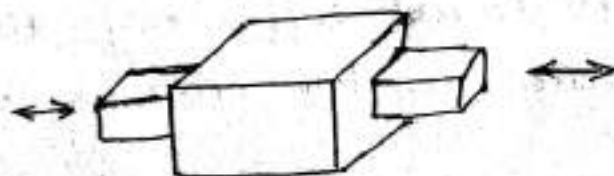
eg → Lathe, Drill Machines etc, Internal combustion engine.

* Types of Constrained Motion

↳ There are three types of constrained Motion:-

1) Completely Constrained Motion:-

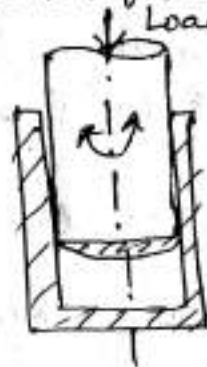
Motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied. ex - Sliding pair & turning pair etc.



2) Successfully Constrained Motion :-

When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using external force in fixed direction.

eg → Foot step bearing



3) Incompletely Constrained Motion :-

When the motion between two elements of a pair is possible in more than one direction and depends upon the direction of the force applied, it is known as incomplete constrained motion.

* Each Motion is independent of the other.

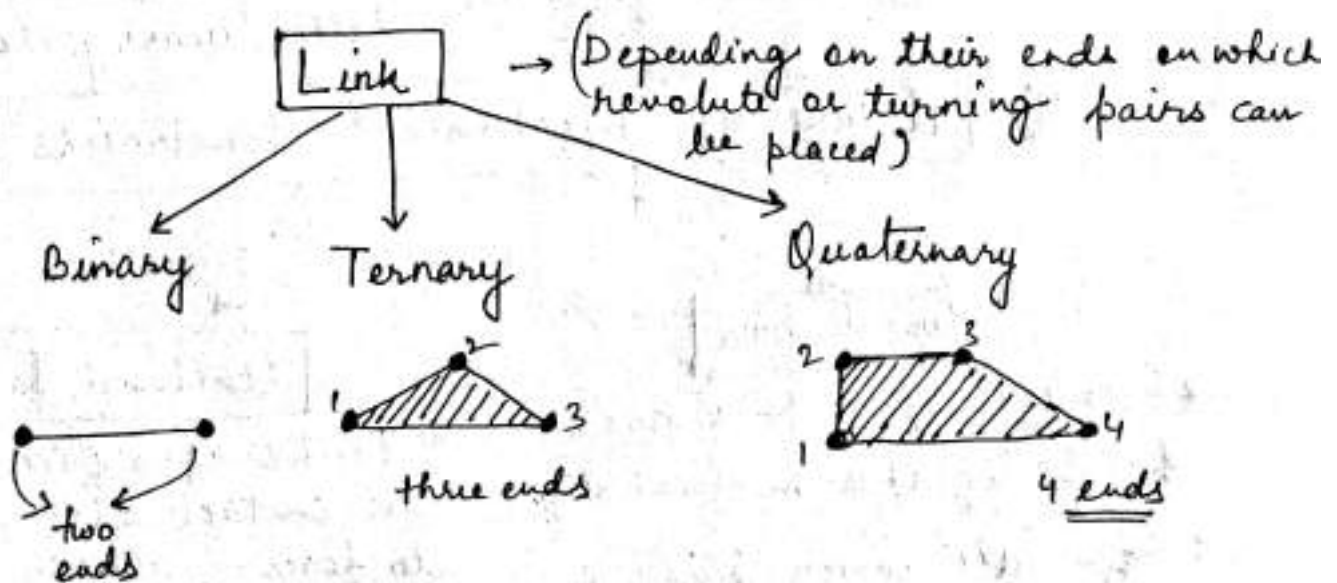
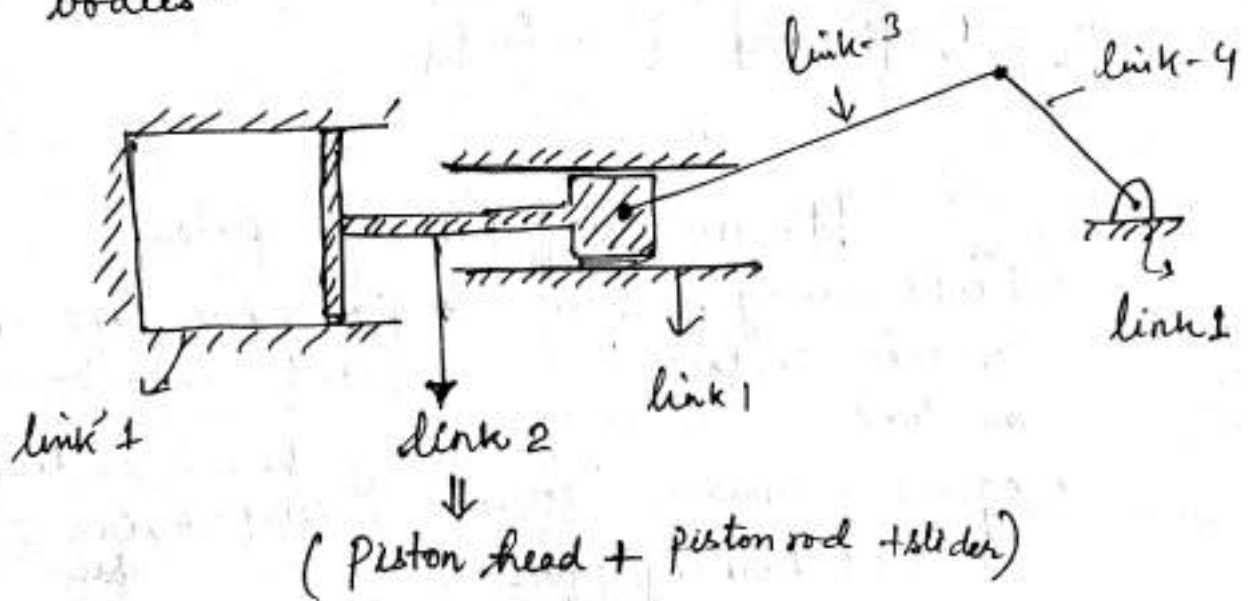


Rigid Bodies & Resistant Bodies :-

A body is said to be rigid if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.

LINK:- A resistant body or a group of resistant bodies with rigid connections preventing their relative movements is known as a link.

* Link may consist of one or more resistant bodies.

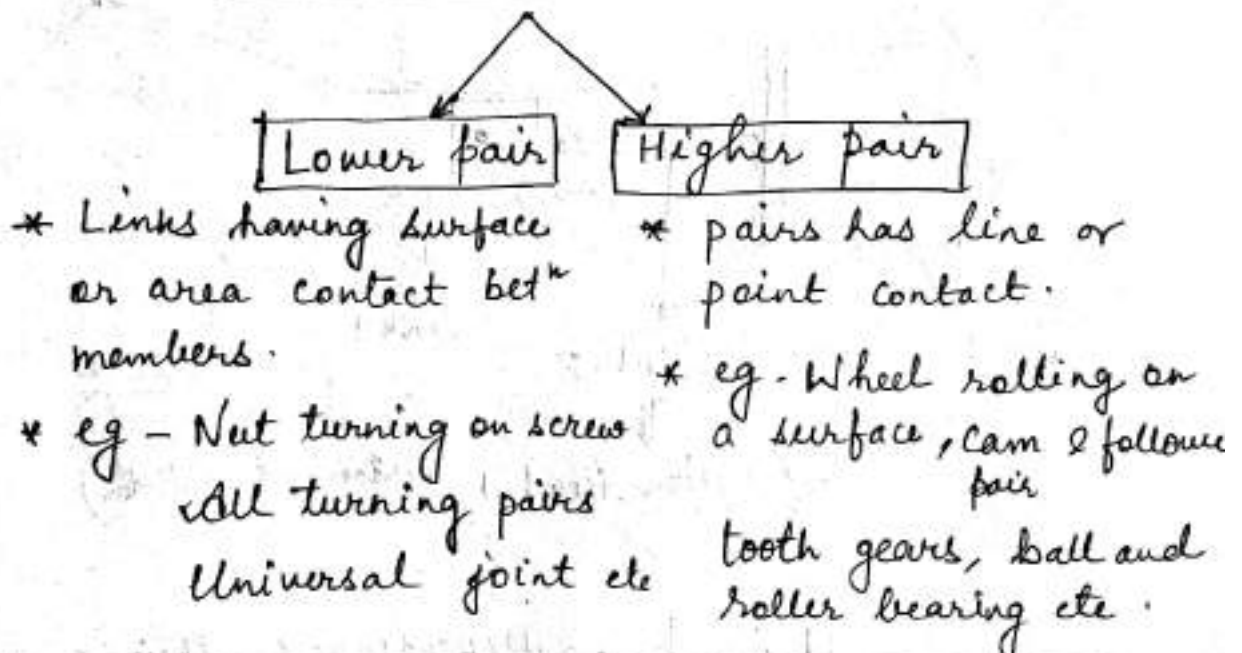


* There is no relative motion between the points within the link.

KINEMATIC PAIR:- A joint of two links having relative motion between them is a kinetic pair.

Types of ^{kinematic} kinetic pair :-

Kinematic pair can be classified according to 1) Nature of Contact



2) Nature of Mechanical Constraints

Closed pair

- * Elements of a pair are held together mechanically
- * Eg - All lower pairs & Some higher pair

Unclosed pair

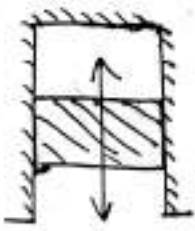
- * Links of a pair are in contact either due to force of gravity or some spring action.
- * eg - Cam and follower, footstep bearing.

Q)

Q3) Nature of Relative Motion

Sliding pair

- * Sliding motion relative to each other
- * eg → piston & cylinder



* Lower pair

Turning pair

- * Revolute pair
- * Turning motion relative to the other.
- * eg → Circular shaft rotating in a bearing

* Lower pair

Rolling pair

- * Rolling motion relative to each other
- * eg - Ball & rollers

* Higher pair

Screw pair

- * Turning & sliding motion
- * eg - lead screw & the nut of a lathe.

Spherical pair

- * One link in the form of a sphere turns inside a fixed link
- eg Ball & Socket-joint.



Types of Joints :-

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

<1> Binary joint :- two links are joined at the same connections.



<2> Ternary Joint :- Three links are joined at a single connection.

* It is equivalent to two Binary joint

$$1 \text{ TJ} = 2 \text{ BJ}$$



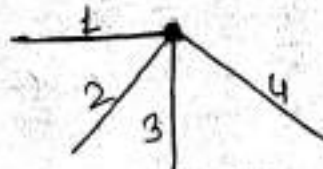
1-2 $\rightarrow$ 1 binary joint

2-3 $\rightarrow$ 1 binary joint

1-2-3 $\rightarrow$ 1 ternary joint

<3> Quaternary Joint :- 4-links are joined at a connection.

* It is equivalent to 3 - Binary joint.



$$1 \text{ QJ} = 3 \text{ B.J}$$

(4)

Kinematic Chain :- is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite. eg -

- Four bar chain
- Single slider crank chain
- Double slider crank chain.

* In case the motion of a link results in indefinite motions of other links, it is a Non-kinematic chain.

* A redundant chain doesnot allow any motion of a link relative to the other.

Linkage, Mechanism and Structure :-

* A Linkage is obtained if one of the links of a kinematic chain is fixed to the ground.

* If the motion (Input) of any of the moveable links results in definite motion (constrained or success-fully constrained motion) of the others, the linkage is known as a mechanism.

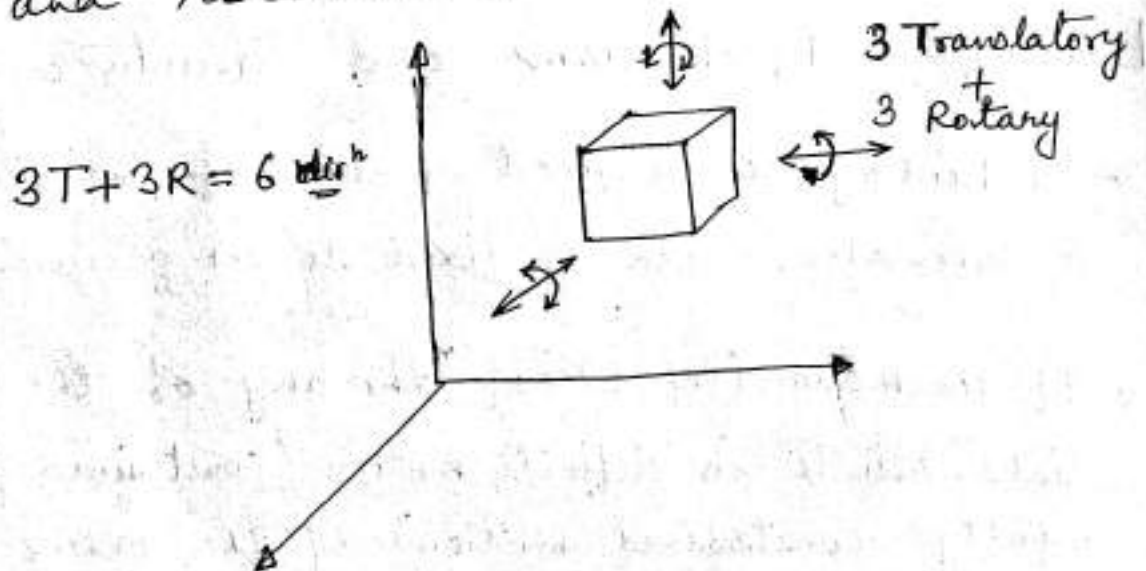
* If ^{of the} one link of redundant chain is fixed, it is known as a structure or frame or locked system.

- * Degree of freedom for structures is zero.
- * Degree of freedom for Mechanism is 1 or in some cases more than 1
- * A structure with negative degree of freedom is known as a superstructure.

Mobility of Mechanisms :-

Degree of Freedom :-

- * For pairs it is defined as the number of independent relative motions, both translational and rotational, a pair can have.



$\therefore$ Degree of freedom = 6 - Number of restraints.

Degree of freedom for plane Mechanisms.

means the number of inputs (i.e. number of independent co-ordinates) needed to ^{obtain} ~~determine~~ the definite motions of some of the links of a linkage (or mechanism).

* If only one input is necessary ^{to} ~~that~~ determines the motions of other links, then the mechanism have ^{single} ~~one~~ degree of freedom.

* It is also defined as minimum number of independent Co-ordinates needed to determine the configuration or position of the all the links of the mechanism with respect to fixed link.

* It is also known as movability of mechanisms.

* Plane Mechanism → All links are in same plane (2-Dimensional)

* Space Mechanism → Links of Mechanism are in different planes.
(3-Dimensional)

Degree of freedom of a mechanism in space:-

Let $N \rightarrow$ total Number of links in a mechanism.

$F \rightarrow$ Degree of freedom.

$P_1 \rightarrow$ Number of pairs having 1 D.O.F.

$P_2 \rightarrow$ " " " " 2 " "

" " " " " " "

" " " " " " "

$P_5 \rightarrow$ " " " " " " "

* In a Mechanism, One link is fixed

$\therefore$ Number of movable links = $(N-1)$

$\therefore$ Number of D.O.F with of $(N-1)$ links will be given by = $6(N-1) - \text{①}$

But each pair having 1 D.O.F will impose 5 restr and so reducing its D.O.F of mechanism by 5 P_1 .

Similarly, other pairs having 2, 3, 4 and 5 D.O.F reduces the degree of freedom of mechanism by $4P_2, 3P_3, 2P_4, 1P_5$ respectively.

$\therefore$ D.O.F of a mechanism in space is given by

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

* According to the number of general restraints a mechanism is classified into different orders.

Zero-Order Mechanism $\rightarrow$ No restraints ($D.O.F = 6$)

First-order Mechanism $\rightarrow$ 1 restraints ($D.O.F = 5$)

$\vdots$

Fifth-order Mechanism $\rightarrow$ 5 restraints ($D.O.F = 1$)

Sixth order " $\rightarrow$ 6 " ($D.O.F = 0$)
(Structure)

Degree of freedom for planar Mechanism:-

In planar mechanisms 3-restraints are present $\therefore D.O.F = 3$

$\therefore D.O.F (Max) = 3(N-1)$

But Degree of freedom in a plane with restraints is given by

$$F = 3(N-1) - 2P_1 - 1P_2$$

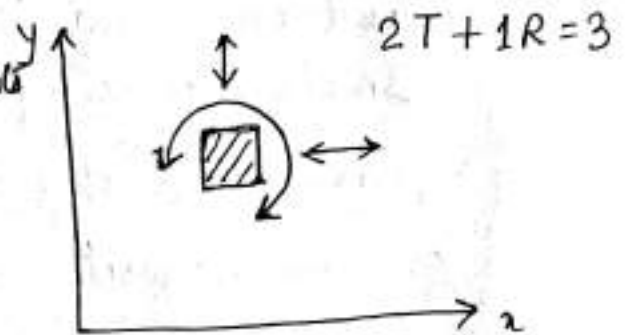
$\downarrow$ no. of pairs having single D.O.F
 $\downarrow$ No. of pairs having 2 D.O.F

known as Kutzbach's Criterion.

* If a mechanism having all pairs with a 1 D.O.F, then, above equⁿ reduces to-

$$F = 3(N-1) - 2P_1$$

known as Gruebler's Criterion.



Most mechanism are expected to have D.O.F=1
then,

$$F = 3(N-1) - 2P_1$$

$$\Rightarrow 1 = 3(N-1) - 2P_1$$

$$\Rightarrow 2P_1 = 3N - 4$$

$$\Rightarrow \boxed{P_1 = \frac{3}{2}N - 2} \quad \text{or} \quad \boxed{j = \frac{3}{2}L - 2}$$

Again; ~~Gruebler's~~ ^{Kutzbach's} Criterion is given by.

$$\boxed{F = 3(N-1) - 2P_1 - 1P_2}$$

and we know that All lower pairs have single degree of freedom & higher pairs have Two degree of freedom. So above equⁿ can be rewrite as

$$F = 3(N-1) - 2(L.P) - 1(H.P)$$

↓
number of
Lower pairs
present in the mechanism

↘ number of higher
pairs present in
the mechanism

$$= \boxed{F = 3(N-1) - 2j - 1h}$$

Note:- If $\text{DOF} = 1 \Rightarrow$ it is a mechanism

b) $\text{DOF} < 1 \Rightarrow$ it is a ~~super~~structure

c) $\text{DOF} > 1$ it is a unconstrained chain

d) DOF is negative $\Rightarrow$ it is Superstructure

Q) For the kinematic linkage shown in fig 1.18, calculate

- a) Number of Binary links (N_B)
- b) " Ternary links (N_T)
- c) " Quaternary links (N_Q)
- d) " Total links N
- e) " Loops (L)
- f) " joints or pairs ($L.P$) (P_i or j)
- g) " Degree of freedom (F)

a) $N_B = 4$ b) $N_T = 4 \neq N_Q = 2$

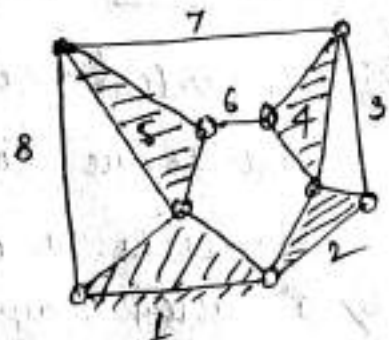
c) $N_Q = 0$ d) $N = 8$

e) 4

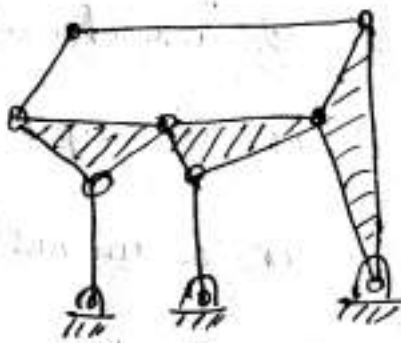
f) $P_i = j = 8 \parallel \left[\frac{4 \times 2}{2} + 4 \times 2 \right]$

g) $\text{D.O.F} = 3(N-1) - 2P_i$
 $= 3(8-1) - 2 \times 8 \parallel$
 $= 21 - 22$

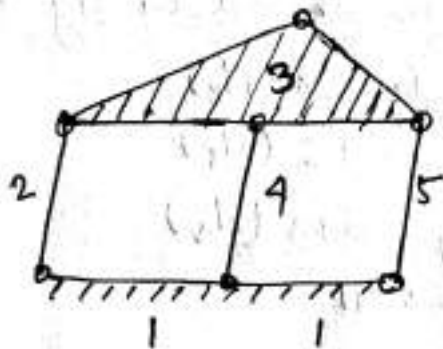
DOF = -1 $\rightarrow$ negative dof means this structure is super structure.



b)



Note
c)



$$\begin{aligned} D.O.F &= 3(N-1) - 2P_1 \\ &= 3 \times (5-1) - 2 \times 6 \\ &= 12 - 2 \times 6 \end{aligned}$$

$\boxed{D.O.F = 0}$ means structure.

Note: a) Empirical relations can give incorrect results. This is due to the reason that the lengths of the links or other dimensions.

b) a system may have one or more links which don't introduce any extra constraints, such links are known as redundant link and should not be counted to find D.O.F.

$$\text{So } \boxed{D.O.F = 3(N-1) - 2P_1 - 1P_2 - F_r}$$

where F_r is number of redundant degree of freedom.

$$\text{So } D.O.F = 3(5-1) - 2 \times 6 - 0 - 1 = 1 \rightarrow \text{mechanism}$$

Kinematic Chain:-

* is the combination of kinetic ^{links} parts joined in such a way that the relative motion between them is completely constrained.

eg → four bar chain, single slider crank chain etc

* Two eqⁿ for lower pairs are available

$$L = 2p - 4 \quad \text{--- (I)}$$

no. of links number of pairs

$$2 \quad f = \frac{L}{2} - 2 \quad \text{--- (II)}$$

for kinematic chain $L.H.S = R.H.S$

Mechanism → When one link of kinematic chain is fixed, it is known as mechanism.

* Mechanisms are of two types a) Simple - 4 links
b) Compound - more than 4 links

Inversion of mechanism → is the process of obtaining different mechanisms by fixing the different links of the same kinematic chain.

- * Thus we ^{can} get as many mechanism as the number of links in a kinematic chain.
- * Relative motion between various links is not changed.
- * Absolute motion of links can change drastically.

Simple kinematic chain ...

① Four Bar Chain

* Also known as Quadric cycle Chain.

* It has 4 links ^{different length} and 4 turning pairs.

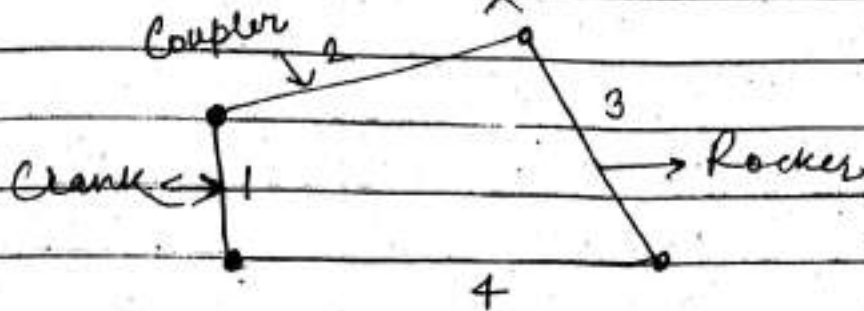


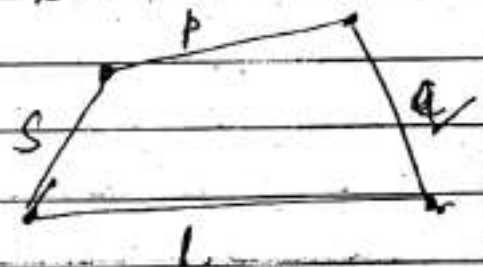
Fig: Four bar Chain.

Grashof's Law :-

* According to this law in the 4-bar mechanism the summation of shortest and longest lengths should not be greater than the summation of other two links for continuous relative motion between the links in the mechanisms.

$$S + L \leq P + Q$$

↓ ↓
shortest link longest link



* If Grashof's Law is not satisfied

i.e. $S + L > P + Q$

the four bar chain will be locked (Double rocker)

$$S + L < P + Q$$

$$S + L = P + Q$$

$$S + L = P + Q$$

$$S + L > P + Q$$

i) S-fixed.

⇒ double crank.

ii) S → adjacent to fixed link.

⇒ Crank-Rocker

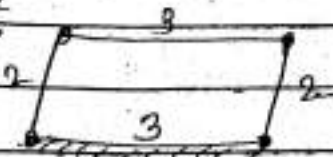
iii) S is coupler.

⇒ Double rocker

Same

i) S → fixed.

Case a)



* Parallelogram linkage

* Double Crank

Case b)



* Deltoid linkage

⇒ Crank-Rocker

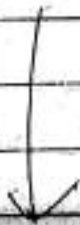
ii) S-fixed.

Double Crank



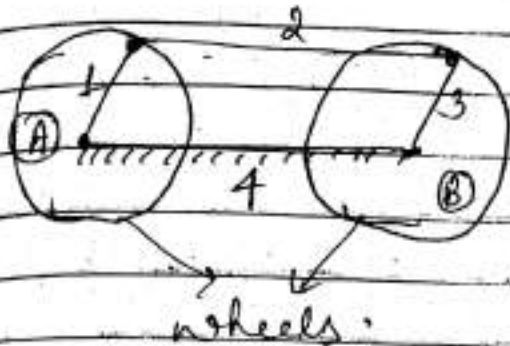
Double

Rocker



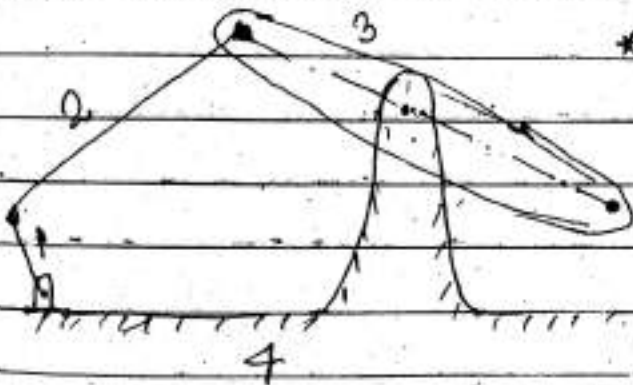
Inversions of four bar chain.

First Inversion $\rightarrow$ Coupled wheels of locomotive.



- * Double crank.
- * Opposite links are equal in length.
- * $\text{link } 1 = \text{link } 3$ & $\text{link } 2 = \text{link } 4$.
- * Link 1 & link 3 works as crank.
- * Rotary motion of wheel A is transmitted to wheel B.
- * Link 4 is fixed (frame) & link 2 is coupler or coupling rod.

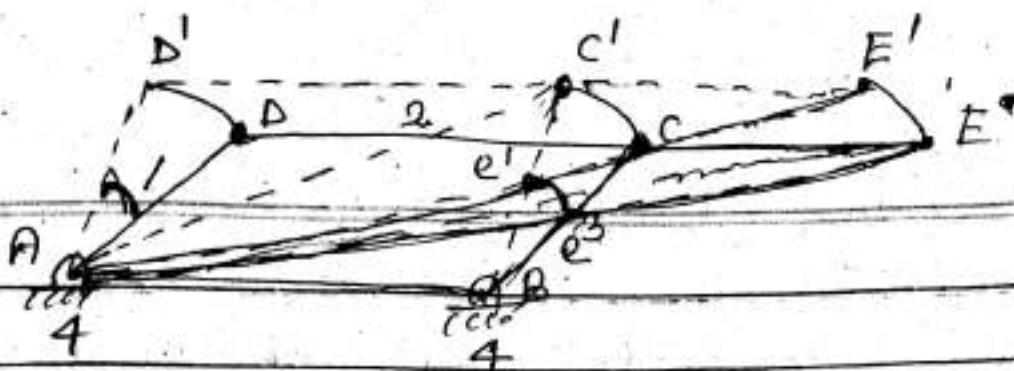
2nd Inversion $\rightarrow$ Beam engine



- * Also known as crank and lever mechanism.
- * Crank-rocker.
- * Link 1 $\rightarrow$ shortest (Crank).
- * Link 2 $\rightarrow$ rigid coupler.
- * Link 3 $\rightarrow$ oscillates (Rocker).
- * Used to convert rotary motion into reciprocating motion.

3rd Inversion $\rightarrow$ Pantograph $\rightarrow$ is a device used to duplicate some motions exactly but to a reduced or enlarged scale.

- * It is used for duplicating drawings, maps etc.
- * It is basically a quadrilateral of links in the form of parallelogram.
- * Used in pantograph milling machines.



* $\triangle Aeb \cong \triangle CEe$

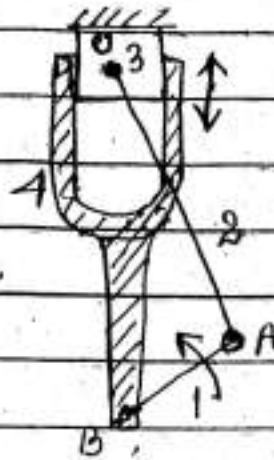
* Therefore, the ratio of motion of E' to the motion of F will be $\frac{Ae}{AE}$.

* Thus motion of E will be duplicated by e' .

2nd Inversion - Hand Pump

* Link '3' of slider crank chain is fixed.
(Slider)

* The shapes of piston & cylinder are exchanged.



* Link 2 oscillates about O.

* Link 4 → reciprocates

* Link 1 → Crank

* Used in hand pump.

3rd Inversion →

* Crank is fixed.

* It has two applications

• Oscillating Cylinder Engine & Rotary engine
Withworth's Whitefly Crank and slotted lever Quick return Motion.

a) Oscillating Cylinder Engine
Rotary Engine.

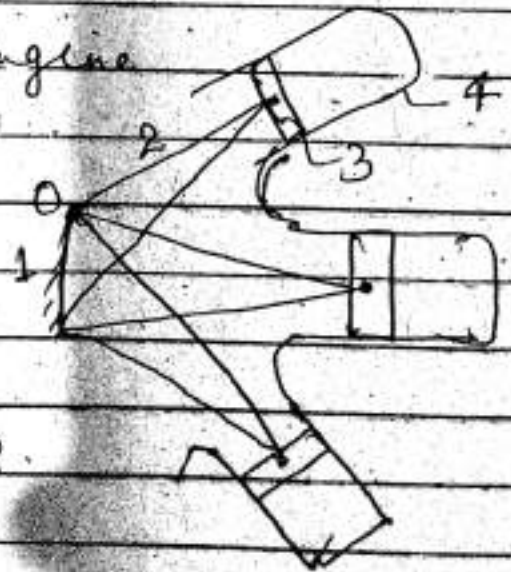
* Link 1 - crank - fixed

* A slider '3' mounted in the cylinder.

* Link - 2 Connecting rod connects link 1 & link 3.

* The complete assembly of cylinders & crank case rotate about the centre O.

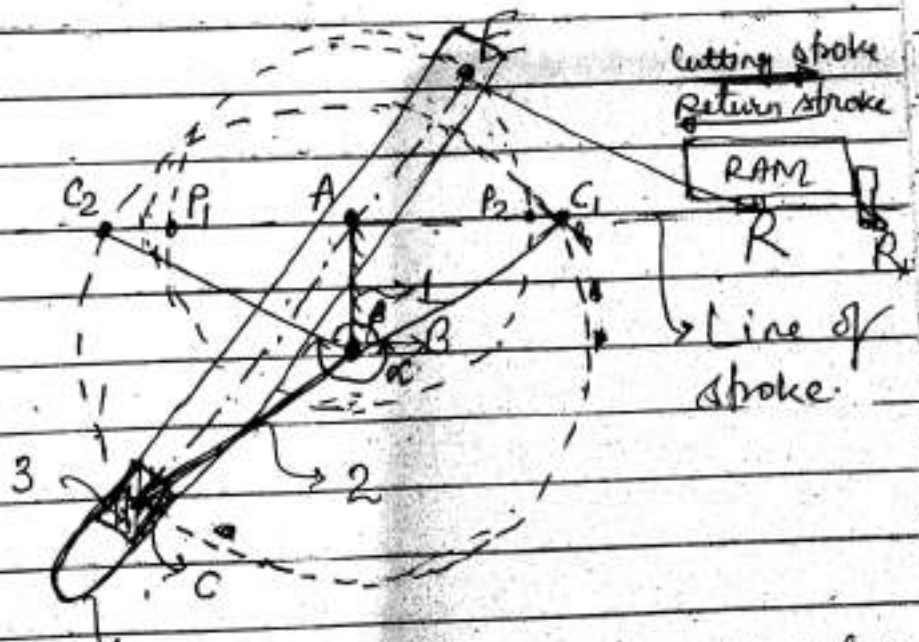
* Its construction is compact and used for supplying steam to the cylinder.



- * Instead one, seven or nine cylinders are placed symmetrically at regular intervals in the same plane.
- * At present these engines have become obsolete since they have been replaced by gas-turbines.
- * Earlier this engine was used in an engine.

b) Whitworth Quick Return Motion Mechanism

- * It is a mechanism used in workshop to cut metals.
- * The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.



- * Driving link 2 rotates at uniform angular speed.
- * Link 3 attached to ^{link 2} ~~crank~~ pin at 'C' slides along slotted bar CE (link 4) which oscillates at pivoted point A.

* Slider 3 rotates in a circle ^{with uniform velocity} about point B and slides on link 4.

* Link 4 has point E where a bar ER is pivoted which carries the ram at R, to which a cutting tool is fixed.

* Slider subtends angle α during cutting stroke & angle β during returning stroke at B.

* Time taken during cutting stroke is more than the time taken during the return stroke

* Cutting velocity $V_c = \frac{\text{distance travelled}}{\text{time}} = \frac{2AE}{(\frac{\alpha}{\omega})}$

$$V_c = \frac{2AE\omega}{\alpha}$$

$$V_R = \frac{2AE\omega}{\beta}$$

$$\text{so } V_c < V_R \quad (\because \alpha > \beta)$$

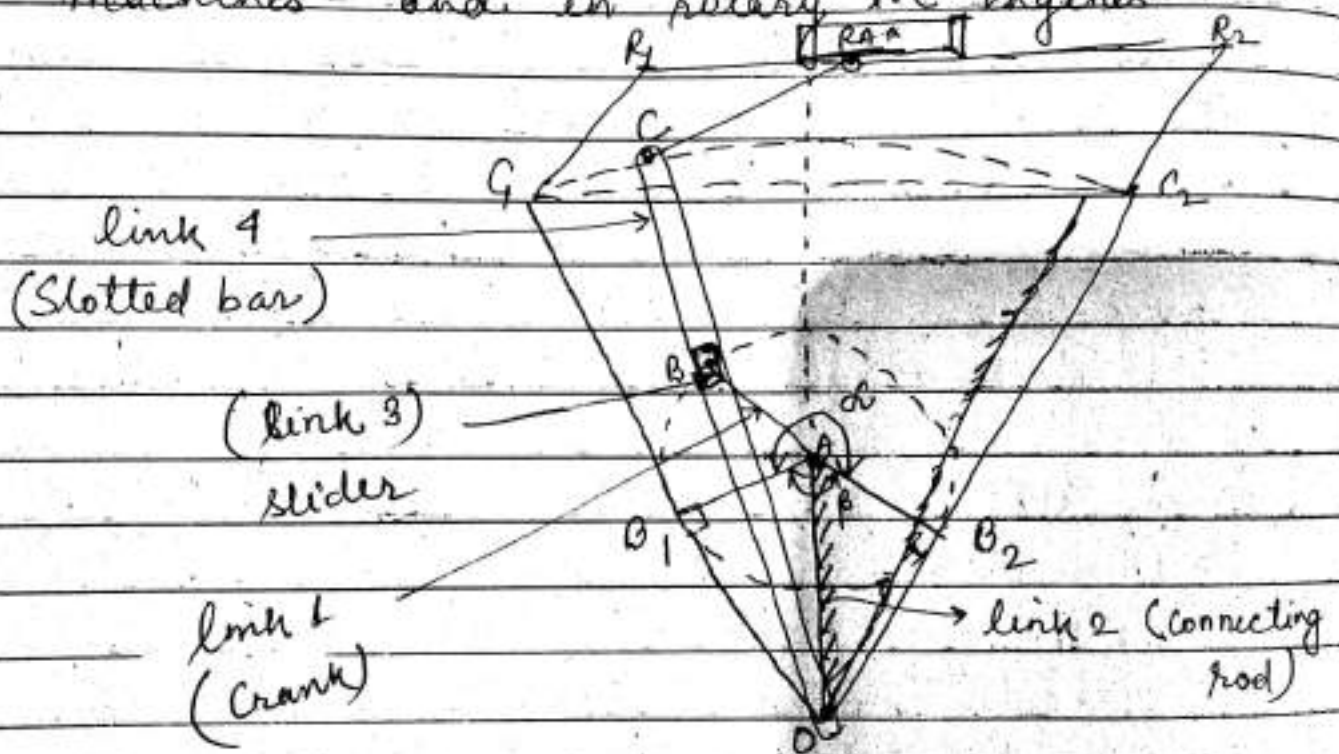
$$\text{also } \frac{\text{Time of cutting stroke}}{\text{Time of returning stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

4th Inversion of Single slider crank chain

- * Connecting Rod is fixed
- * It has two applications
 - a) Crank and slotted lever Q.R.M.
 - b) Oscillating Cylinder Engine

a) Crank and slotted lever Q.R.M.

- * Mostly used in shaping Machines, slotting machines and in rotary I.C. engines



- * link 2 (connecting rod) is fixed
- * Slider (link 3) reciprocates in oscillating slotted lever (link 4).
- * link 1 (crank) rotates about A.
- * When Crank moves angle α i.e. from B₁ to B₂, cutting tool cuts the metal, it is known as forward stroke
- * During Crank rotation from B₂ to B₁ i.e. (angle β)

tool comes back and does not cut the metal, it is called idle stroke.

$$* \frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{(360 - \alpha)}$$

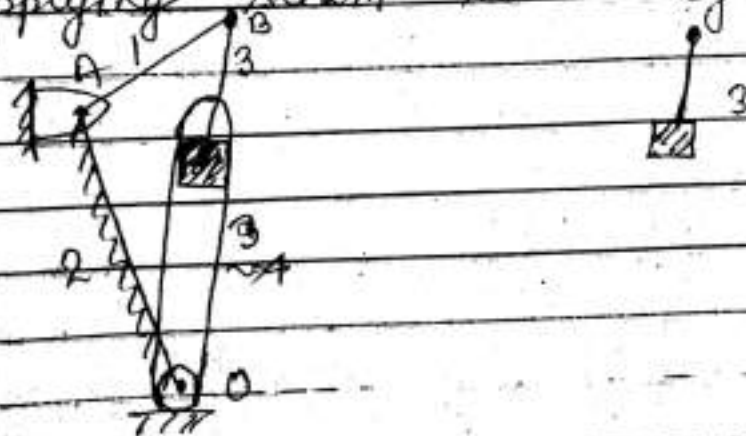
$$\alpha > \beta$$

Time of cutting > Time of return

→ Quick return motion.

b) Oscillating Cylinder engine

* Its construction is compact and used for supplying steam to the cylinder.



* Link 2 is fixed (connecting rod)

* Link 4 (Oscillating cylinder) oscillates about O as the crank (link 1) rotates about A.

* Link 3 (slider) reciprocates inside oscillating cylinder.

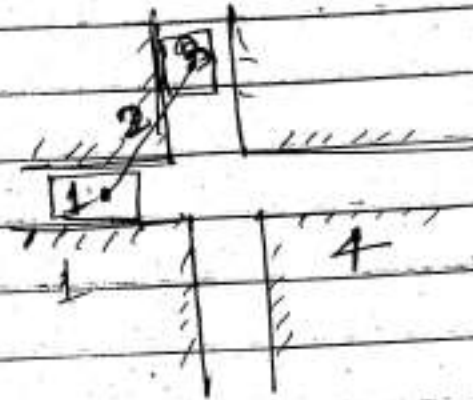
Double - Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as double slider crank chain.

link 4 & link 3 $\rightarrow$ sliders

link 2 $\rightarrow$ Connecting rod
or Crank

link 1 $\rightarrow$ Slotted bar



* There are three conversions available of this chain.

(1) Elliptical Trammel :-

* Link 4 (slotted bar or frame) is fixed.

* Two sliding blocks 1 and 3 are connected with a link 2, and the blocks are free to move inside the slotted frame (4).

* This mechanism is used for drawing the ellipse.

* Any point 'C' on the extension of line AB produces ellipse.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

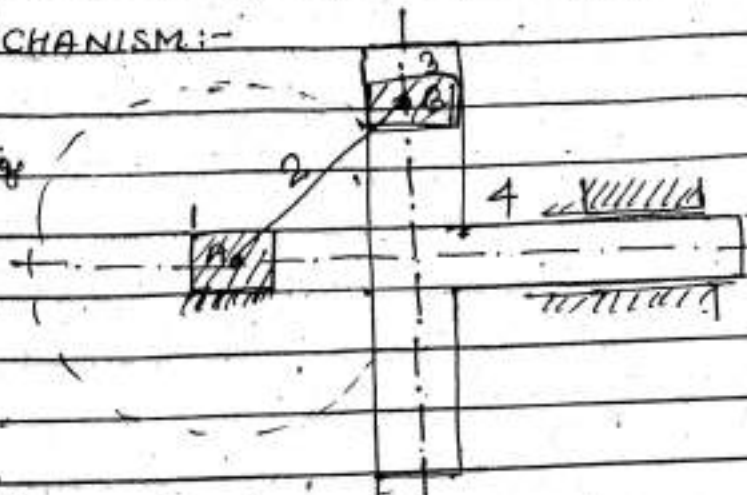
a $\rightarrow$ semi minor axis

b $\rightarrow$ semi major axis

② Second Inversion (

SCOTCH YOKE MECHANISM:-

- * One of the two sliding blocks A and B is fixed.



- * link 2 behaves as crank rotates about A.

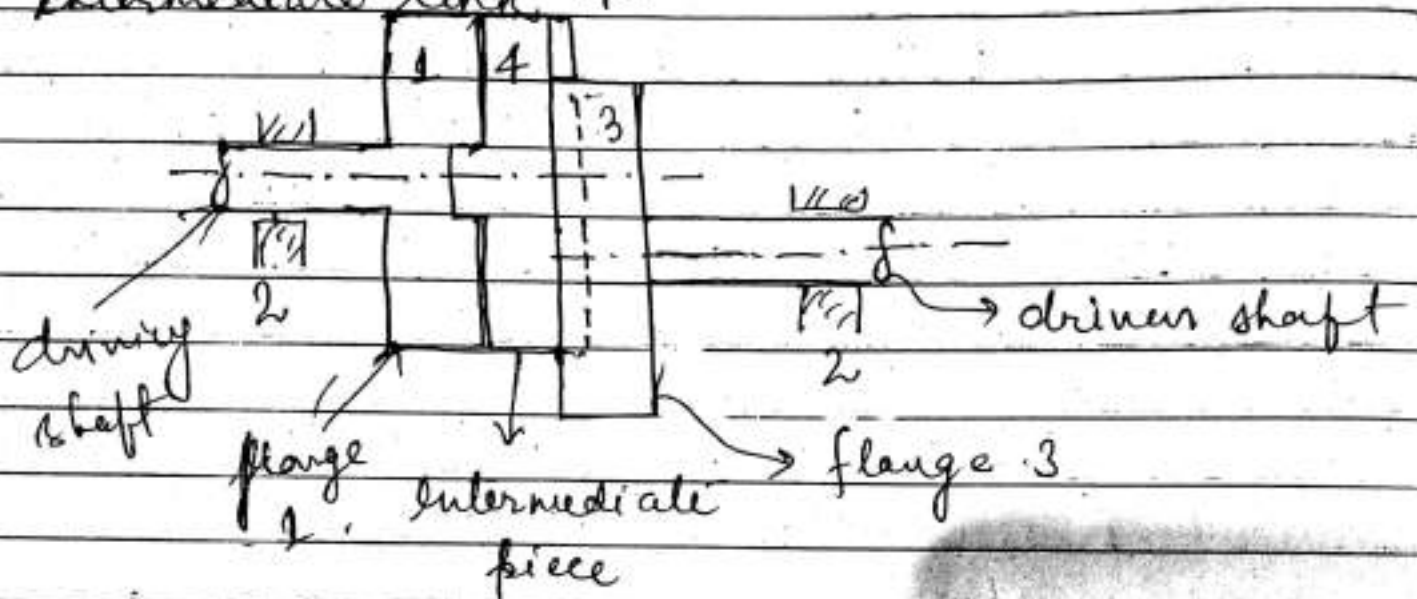
- * link 4 (slotted frame) reciprocates horizontally.
- * link 3 reciprocates in the slotted frame.

- * It converts rotary motion of link 2 into reciprocating motion of link 4.

③ Oldham's Coupling

- * 3rd inversion of double slider crank chain.
- * Used to connect two parallel shafts whose axes are at a small distance apart.
- * link 2 (crank is fixed)
- * The shafts to be connected have two flanges (link 1 and link 3).
- * link 1 & link 3 form turning pairs with link 2.
- * Link 4 (intermediate piece) is a circular disc have two tongues (diametrical projection) on each face at right angles to each other.
- * link 4 can slide in the slots in the flanges.

* When driving shaft rotates through certain angle. the driven shaft also rotates through same angle. Motion is transmitted through intermediate link 4.

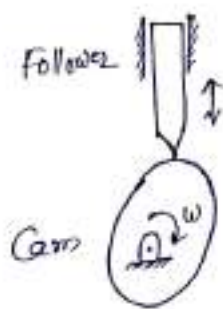


Maximum sliding speed of each tongue

$$v = \omega r$$

CAM and Follower

①



Cam is always
drives.
Follower is
driven

Both parts
are continuously
touching to each
other.

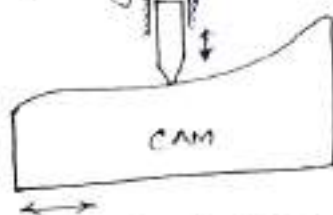
The rotation motion of Cam is going converted into reciprocating motion of follower.

Rotation or reciprocation motion of CAM is going converted into Reciprocation or oscillation motion of Follower.

- CAM and follower is also called as Exact function generator.
- To create the Profile of CAM is very difficult so it is very costly. (So CAM and follower are used where exactly time is required.)

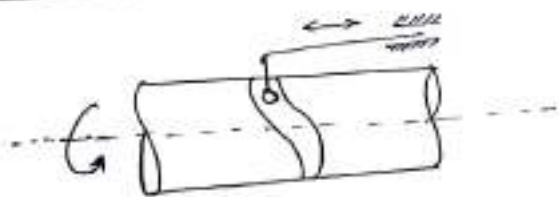
* Classification of CAM :-

(1) Translating / wedge cam.



Axis of motion of both Cam and follower are $\perp$ to each other.

(2) Cylindrical CAM / Barrel cam :-



This is used in automobile where gear is shifting.

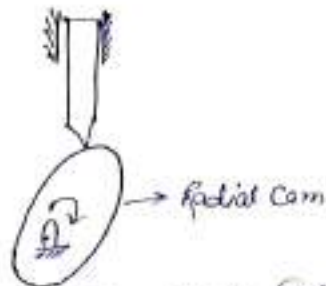
(3) Globoidal Cam:-

Concave
shape Convex
shape.

→ This is used in bottling plant
where bottle is filled.



(4) Radial Cam / Disc / plate cam.



Axis of motion of Cam and follower are $\perp$ to each other.

(2)

* Follower:-

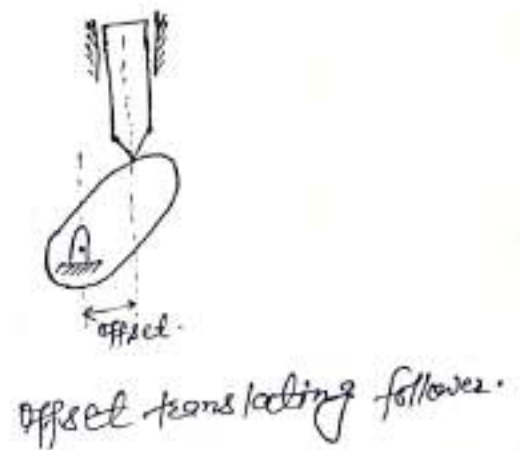
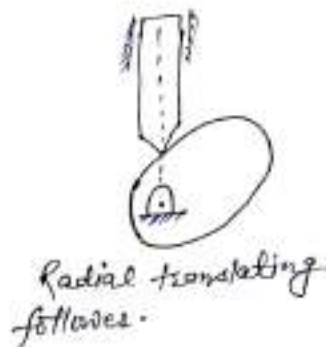
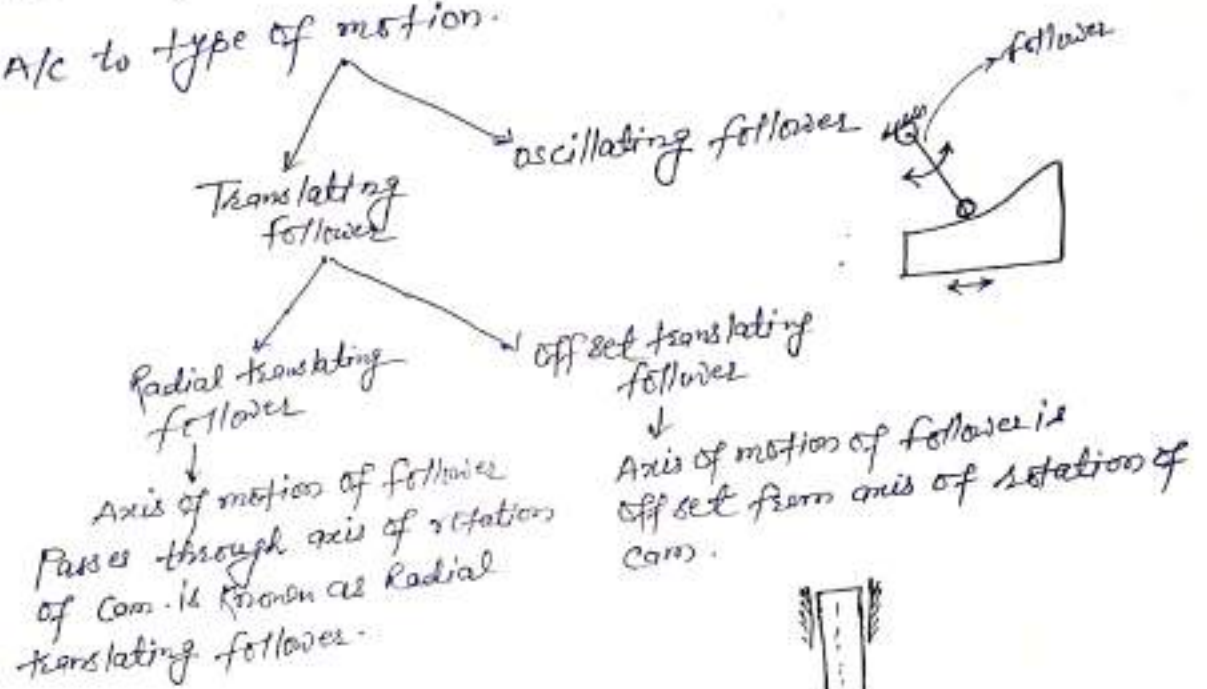
Follower is that device which follow the exactly motion of the Cam via direct contact.

* Classification of Follower:-

(A) A/c to type of motion.

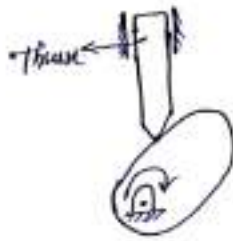
(B) A/c to type of Contact.

(A) A/c to type of motion.



(B) A/c to type of ~~Follower~~ Contact

(1)

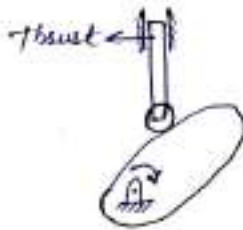


Knife edge follower

- (1) Due to sharp shape, wear rate is very high.
- (2) Side thrust is very high.
- (3) very high friction.

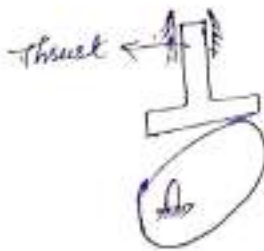
(2) Roller follower.

used in
gas turbine,
steam engine.



- Rolling friction $\ll$ Kinetic friction.
- 1) wear rate is very less.
 - 2) rolling friction is very less.
 - 3) very less thrust

(3) Flat face follower.



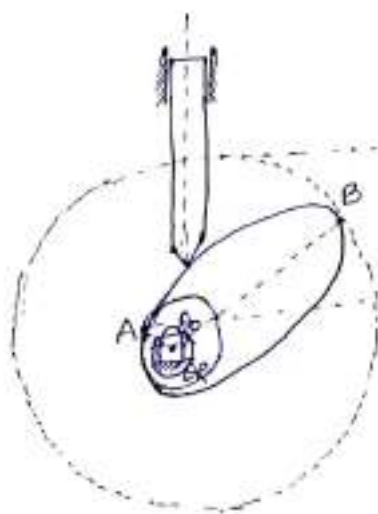
- (1) Rate of wear are less as compared to knife edge follower.
- (2) Kinetic friction involve.
- (3) Surface distortion.

(4) spherical / mushroom follower.



Contact area,

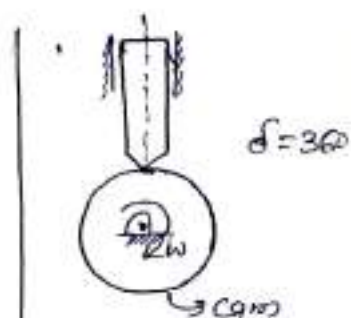
Flat face $>$ Spherical $>$ Knife edge.



θ_o = out stroke angle / Ascent angle.

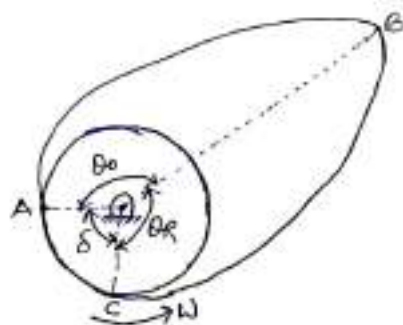
θ_r = Return stroke angle / Descent angle.

δ = Dwell angle. (Angle of rotation of Cam for which follower remains stationary.)



∴ Follower is at rest motion at this position because shape of cam is circle.

Cam is for certain rotation and follower motion at other times that particular stationary angle is called dwell angle.



δ → Dwell angle (∴ Follower stationary position).

θ_o → out stroke angle.

(∴ Follower lowest position to highest position or vice versa).

θ_r → Return stroke angle.

(∴ Follower highest position to lowest position or vice versa).

ω → Angular speed of Cam.

→ CAM and Follower are used only where Exact timing (crucial timing is required)

CAM

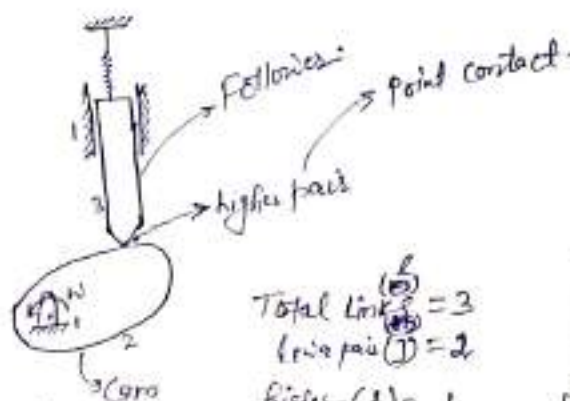
* Cam is a mechanical device which drives another mechanical element (Follower) by direct contact.

→ Contact may be point/line contact

→ Cam and follower are very costly device.

→ use only where timing is crucial.

→ $DOF = 1$



$$\therefore DOF = 3(l-1) - 2j - h$$

$$= 3(3-1) - 2 \times 2 - 1$$

$$= 3 \times 2 - 5$$

$$DOF = 1$$

hence DOF of Cam and follower is always 1.

CAM → Drive

Follower
↳ Driven

Cam and Follower 2 links
point contact
off to cam & follower
Line contact off
to cam & follower

• एडजस्टेबल
Accuracy knife
edge & follower
के बिना है बल्कि
स्ट्रॉक वॉश कर
होने के बाद ही
इसका फॉलोवर यूज
करेंगे

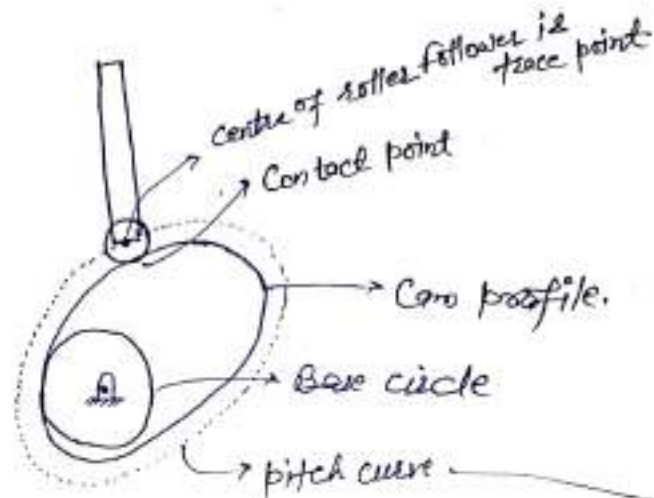
→ line contact

1 → Lower pair
(1) और 2 के बीच
Lower pair है
और 3 और 1 के बीच
कोई Lower pair है

• Follower on Tip
Contact higher pair
कहेगा है

Cam and follower
का DOF, हमेशा एक होता
है। यानी 1-7 के बिना
हमेशा Cam & follower
का DOF ही 1 होगा।
Cam की Rotation है तो
Follower को फिर दिखाना
more करेगा

* Terminology used in CAM/Follower.

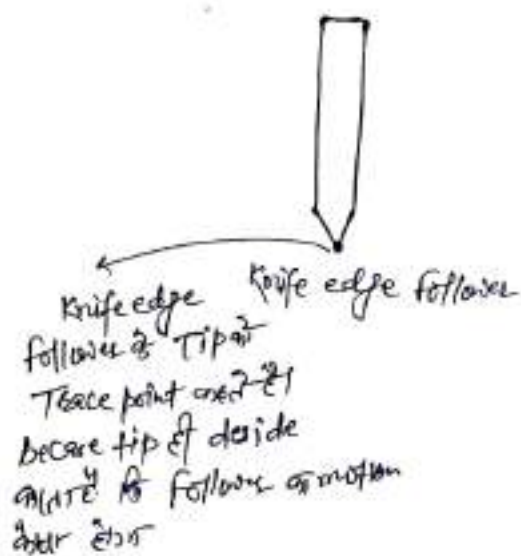


Axis of Reflection
 If min drawn
 circle of base
 circle of $\frac{1}{2}$
 smallest circle

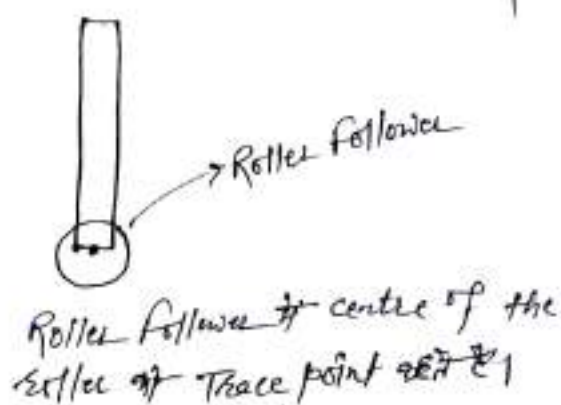
→ Knife edge
follows ϕ can
be contact point
at trace point
same as ϕ is
→ ~~knife~~ trace
point off of
locus follow ϕ
yet that is
pitch curve.

→ smallest circle which can be drawn inside the cam profile about axis of rotation is known as base circle.

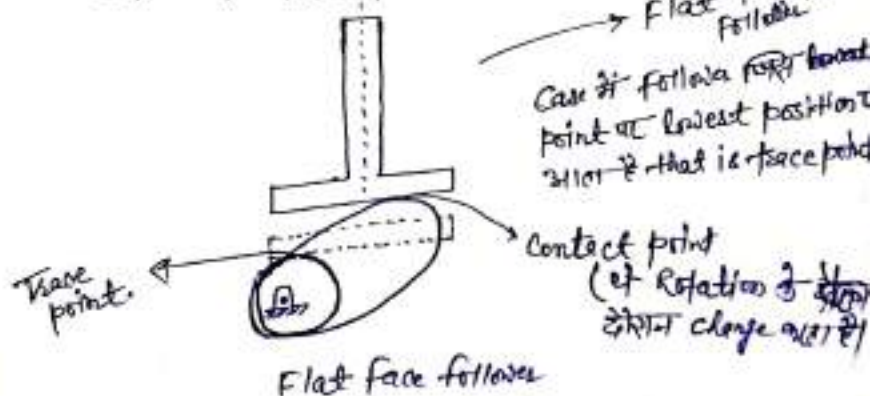
→ Trace point: → It is the theoretical point on follower which describe motion of follower.



Knife edge
follower at Tip
Trace point on 2nd & 1st
because tip of disc
will follow motion
of 2nd & 1st



Roller follows $\frac{1}{2}$ centre of the
enter of Trace point $2E1$



point of contact
और trace point
दोनों अलग-अलग
थीन हैं दोनों
Concise कह सकती
हैं but ऐसा
Same नहीं हो सकती

Case 2: Follows **right hand** rule
point at lowest position
3101 is that is face point

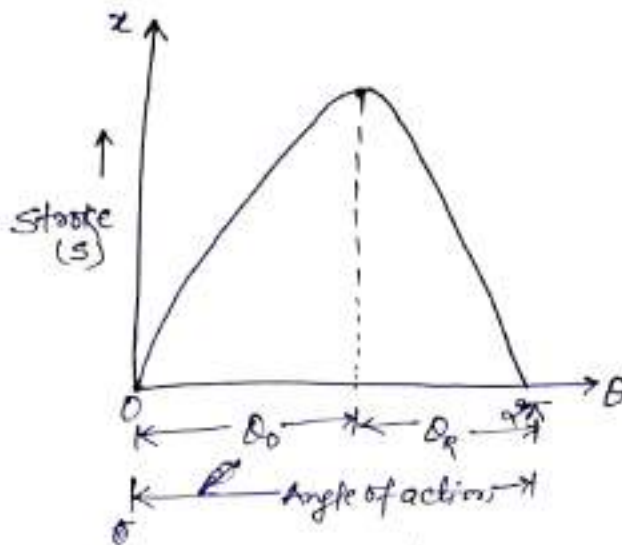
tack point
(ये Rotation के दौरान
कोसाज change करता है)

* Pitch curve :→ In case of ~~cam profile and~~
Knife edge follower, Cam profile & pitch curve are
Same. (Path traced by trace point).

• Prime circle :→ The smallest circle which can be
drawn from trace point.

Cam or
size, base
circle of
deside or

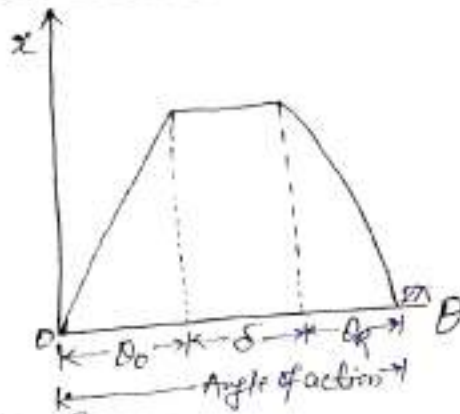
→ Rise-Return Cam.



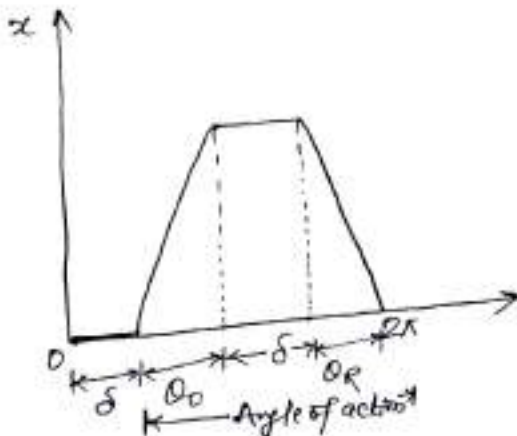
D_o = out stroke angle.

D_r = Return stroke angle.

→ Rise-Dwell-Return Cam.



→ D-R-D-R.

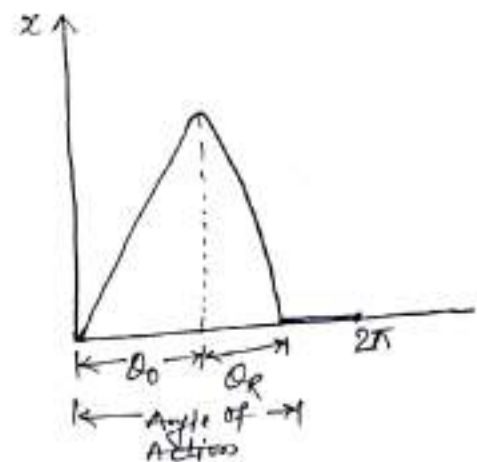


Follower initially zero height at start. When cam rotates, follower of height z is known as stroke. For certain angle of cam, follower of height z is known as stroke. This profile is known as Rise-Return Profile.

Cam at motor is connected to shaft, Cam rotates with constant angular speed. Cam for certain duration of 360° rotates. Follower angle is follower on cam motion. I am going to plot.

Start of rise at start end of return at angle of action end.

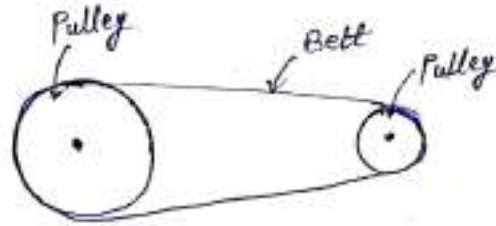
→ R-R-D-Cam



- Angle of action: - Angle turned by cam from start of rise to the end of return is known as Angle of action.

Belt drive.

Introduction:- The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotates at the same speed or at different speeds.



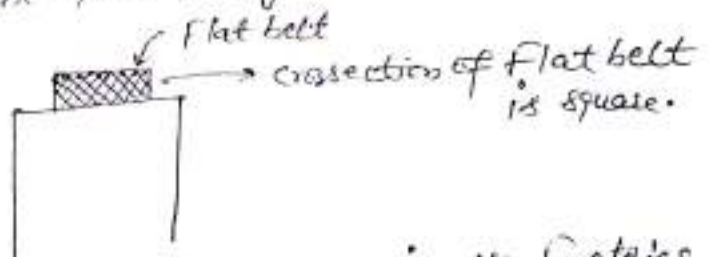
Draw back for belt drive is it slip occurs if both pulley rotate same speed then slip occurs less and if both pulley speeds are different then slip will be high.

- Generally belt drive used for long distance power transmission.
- Due to slip in belt, it is called -ve drive.

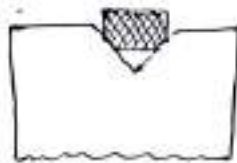
* Types of belts:-

- (1) Flat belt: - The flat belt is mostly used in the factories and workshop where a moderate amount of power is to be transmitted. The two pulleys are not more than 8 meters apart.

conveyor belt

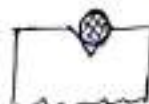


- (2) V-belt: → The V-belt is mostly used in the factories and workshop where a greater amount of power is to be transmitted from one pulley to another pulley when the two pulleys are very near to each other. The included angle for the V-belt is $30 - 40^\circ$.



Standard V-belt.

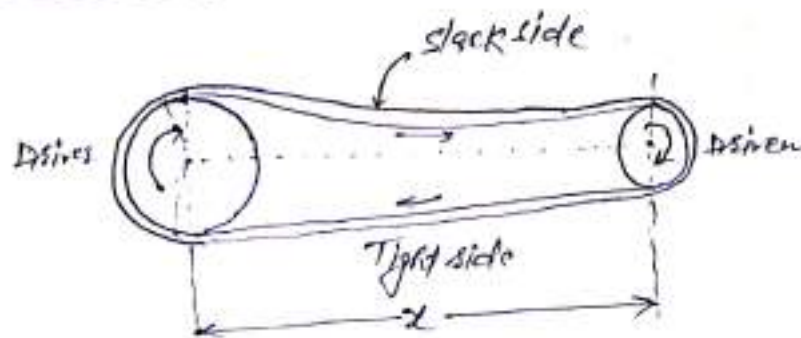
- (3) Circular belt or ropes: - The circular belt or ropes are mostly used in factories and workshop where a greater amount of power is to be transmitted. This is used where both pulleys are more than 8 meters apart.



Circular belt.

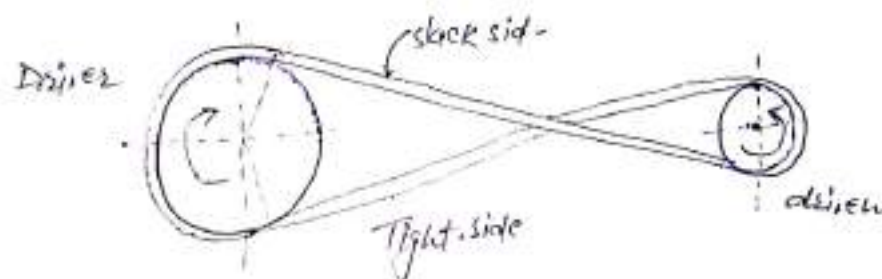
* Types of belt drives.

① Open belt drive



(2) Crossed or twist belt drive.

The crossed or twist belt drive is used with shaft arranged parallel and rotating in the opposite direction.



* Length of open belt drive.

$$L_{open} = (\pi r_1 + \pi r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

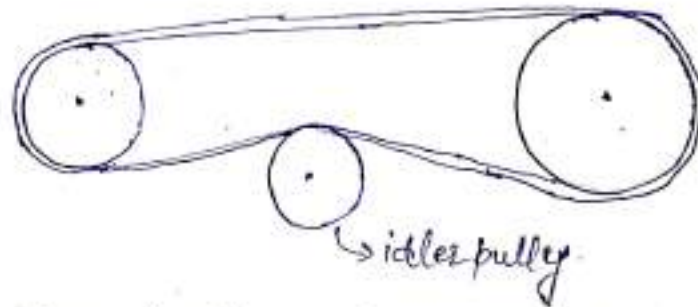
$$L_{open} = \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

* Length of cross-belt drive:-

$$L_{cross} = \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

* Slip of the belt :-

The motion of belts and pulleys assuming a firm frictional grip between the belts and the pulleys. But some times the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt.



→ By the using of idler pulley the angle of lap in the pulley increases and slipping of belt decreases. idler pulley increases the tension in the belt.

* Creep of belt :-

When the belt passes through from the slack side to the tight side, a certain portion of the belt extended and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surface. This relative motion is termed as creep.

* Ratio of Driving Tensions for flat belt drive.

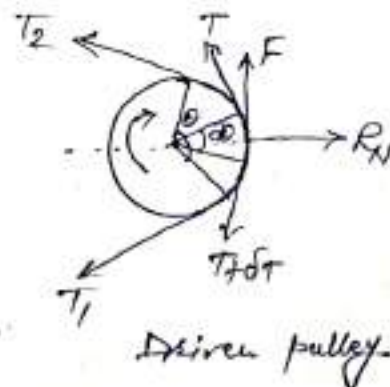
$$\frac{T_1}{T_2} = e^{\mu \theta}$$

where T_1 = Tension in the belt in the tight side.

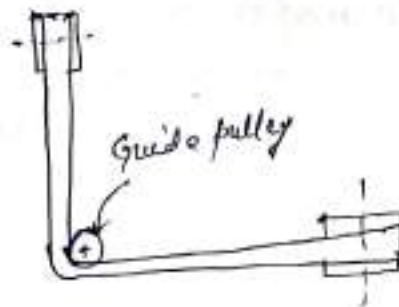
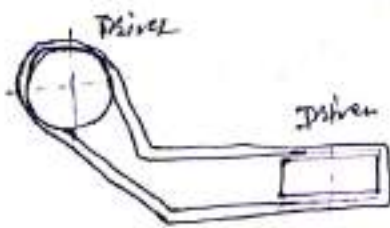
T_2 = Tension in the belt in slack side

μ = coeff of friction b/w belt and pulley

θ = Angle of lap.

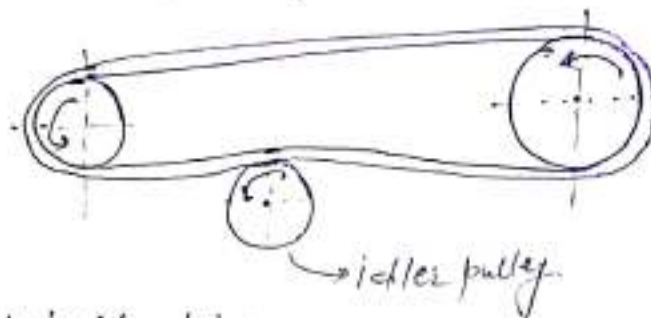


Quarter turn belt drive :-

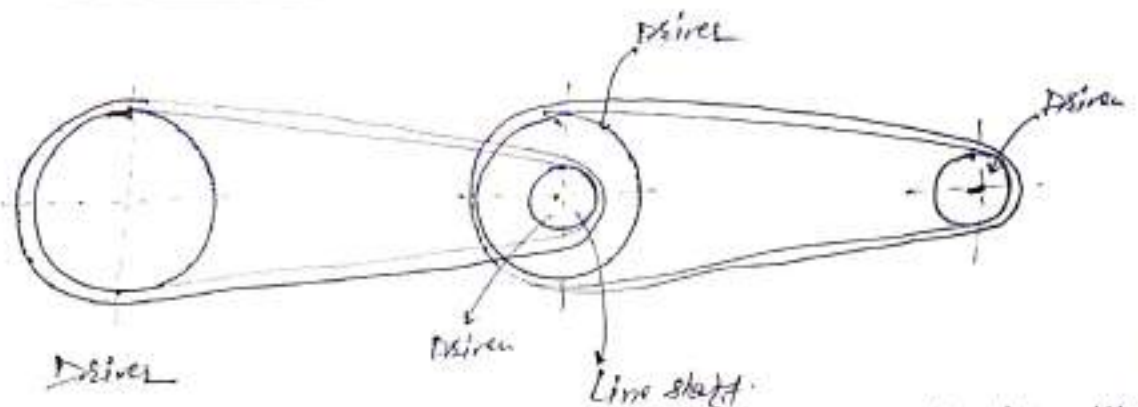


Belt drive with idler pulley.

It is also known as Jockey pulley drive. This type of drive is provided to obtain required belt tension.



Compound belt drive.



A compound belt drive is used when power is transmitted from one shaft to another through a number of pulleys.

* velocity ratio of a belt drive :-

It is the ratio of between the velocities of the driver and the follower or driven. It may be expressed -

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

where d_1 = dia. of driver
 d_2 = " " follower
 N_1 = speed of driver in rpm
 N_2 = " " follower " "

$$\left| \begin{aligned} V &= \frac{\pi D N}{60} \\ \frac{V}{N} &= \frac{\pi D}{60} \end{aligned} \right|$$

* Advantages of belt drive:-

1. Belts absorb shock and vibration in the driving machine due to sudden over load.
2. No casing is required as in gear drive.
3. Belt drives can be used for transmission of power between shafts at short and long distances.
4. The flexibility of belts permits transmission of power between non-parallel shaft also.
5. Flat belt efficiency, nearly 98%.
6. Comparatively cheaper than chain or gear drive.

* Disadvantages of belt drive:-

- (a) Slip and creep in the belt make the belt drive less efficiency than the chain or gear drive.
- (b) Belt drive requires more space comparing with chain or gear drive.

Note:- Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in minute, therefore,

$$\pi d n = \pi D N$$

$$d n = D N$$

$$\boxed{\frac{d}{D} = \frac{N}{n}}$$

where d = Dia. of the driver

D = " " " follower

n = speed of the driver

N = " " " follower

Velocity Ratio (i)

$$\boxed{i = \frac{D}{d} = \frac{n}{N}}$$

When the thickness of belt is considered.

$$\boxed{i = \frac{n}{N} = \frac{n+t}{N+t}}$$

where t = thickness of belt.

* Power transmitted by a belt:-

$$P = (T_1 - T_2) V$$

where V = velocity of the belt.

Q: For a flat open belt drive, the belt speed is 880 m/min and power transmitted is 22.5 kW. What is the difference b/w the tight side and slack side of the belt drive.

Sol.

$$\therefore P = (T_1 - T_2) V$$

$$22.5 \times 10^3 = (T_1 - T_2) \frac{880}{60}$$

$$T_1 - T_2 = \frac{22.5 \times 10^3 \times 3}{22}$$

$$T_1 - T_2 = 1540 \text{ N. Ans}$$

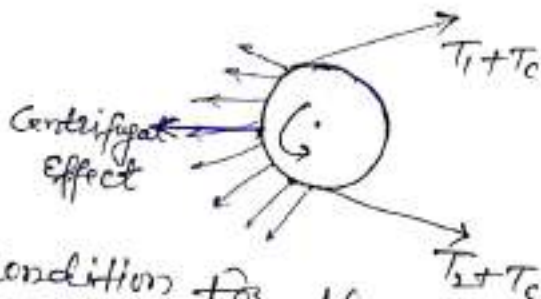
* Effect of Centrifugal force on Tension:-

Since the belt continuously runs over the pulleys, therefore some centrifugal force is caused, whose effect is to increase the tension on both tight as well as slack sides. The tension caused by centrifugal force is called centrifugal tension.

$$T_c = mv^2$$

where m = Mass of belt per unit length.

V = Linear velocity of belt.



* Condition for the Transmission of Maximum Power:-

Let T = Max^m tension in the belt = $(T_1 + T_c)$

We can show that when the power transmitted is maximum $\frac{1}{3}$ rd of the maximum tension is absorbed as centrifugal tension,

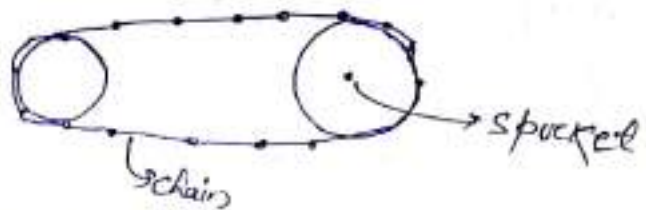
$$T_c = \frac{T}{3}$$

$$\text{So, } T_1 = T - \frac{T}{3} = \frac{2T}{3} \Rightarrow \boxed{T_1 = \frac{2T}{3}}$$

Chain drive.

Chain drives are used to transmit power between two parallel shafts comparatively at a longer distance.

The chain drive is in between the belt drives and gear drives.



Chain drive consist of two sprockets and an endless chain.

smaller sprocket is called pinion and bigger one is called wheel.

The sprockets are toothed wheels of special profile for teeth.

* Types of chain.

- (i) Bushed Roller
- (ii) Bush
- (iii) Bush Roller with belt plate
- (iv) Hooked link.
- (v) Stud link.

* Characteristics of different types of chains:-

<u>Types of chain.</u>	<u>Applications.</u>
(i) Silent	→ Main drive in various machine.
(ii) Bush Roller	→ Motor cycle.
(iii) Bush	→ General motor cycle.
(iv) Bush Roller with belt plates	→ Heavy duty.
(v) Hooked link belt	→ Agriculture machinery
(vi) Stud link belt Chain	→ Drives under impact load.

* Polygonal effect or chordal action of chain:-

- When the chain passes over the sprocket, it moves as a series of chords instead of a continuous arc as in the case of belt drive results in varying speed of chain drive. This phenomenon is called as polygon effect or chordal action.
- To reduce the chordal effect the number of teeth on the sprocket should be increased. The number of teeth on the sprocket affects the velocity ratio during its rotation through the pitch angle.
- The recommended minimum number of teeth for smooth operation is 17.
- Higher number of teeth 19 to 21 will give better life to the chain drive with lesser noise during operation.

* Back sliding of chain:-

The wear of chain results in the elongation of the chain and increase the pitch length. This makes the chain to ride out of the sprocket teeth resulting in faulty chain sprocket engagement. This is known as back sliding of chains.

* Failure of chain

- (1) chain elongation.
- (2) failure of joints and plates due to dynamic load.
- (3) wear of sprocket teeth.

* Design procedure of chain drive.

1. Determine the design power
$$= \text{Rated Power} \times \text{service factor } (K_s)$$

$$\text{Service factor } K_s = K_1 K_2 K_3 K_4 K_5 K_6$$

where K_1 = Load factor

K_2 = Factor for distance regulation.

K_3 = factor for centre distance.

K_4 = factor for position of sprocket

K_5 = Lubrication factor

K_6 = Rating factor.

2. Determine the transmission ratio.

$$i = \frac{Z_2}{Z_1} = \frac{\text{No's of teeth on sprocket wheel.}}{\text{No's of teeth on sprocket pinion.}}$$

$$= \frac{n_1}{n_2} = \frac{\text{speed of pinion.}}{\text{speed of wheel.}}$$

3. Based on the power, ~~transmitted~~ transmission ratio and use of select the type of chain (Roller chain or bush chain) and number of strands and note the specification of chain (from data book).

4. Specification of the selected chain

P = pitch (mm)

D_g = Roller diameter (mm)

W = width between roller inner plates (mm)

Q = Breaking load (kgf)

w = weight per unit length (kgf/m)

Gear drive.

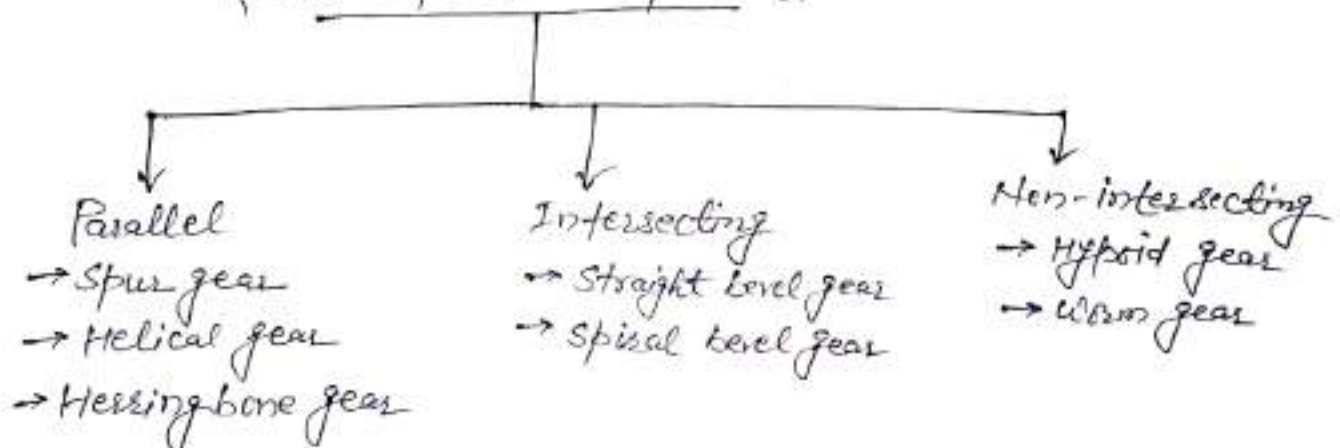
Gear: - Gear is defined as a machine element used to transmit power between rotating shafts by means of progressive engagement of projection called teeth.

Gears are toothed members which transmit power/motion between two shafts by meshing without any slip. Hence, gear drives are also called positive drives.

→ Gears operate in pairs, the smaller of the pair called the pinion and the larger is gear.

Types of gear drives.

Relative position of shafts.



* Spur gear :-

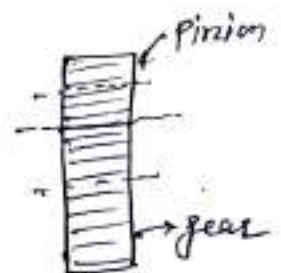
- spur gears have their teeth parallel to the axis and are used for transmitting power between two parallel shafts.
- They are simple in construction, easy to manufacture and cost is less.
- They have highest efficiency and excellent precision rating.

Depending of size and type.

Maximum power = 18 kW, speed = 10,000 rpm

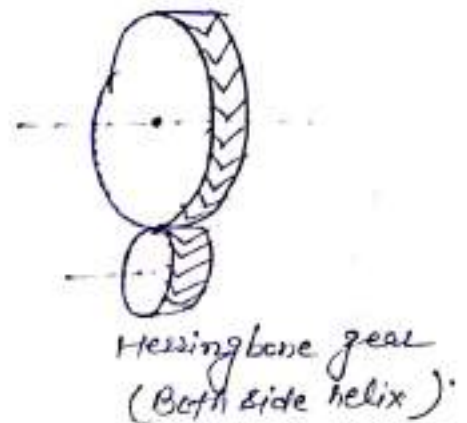
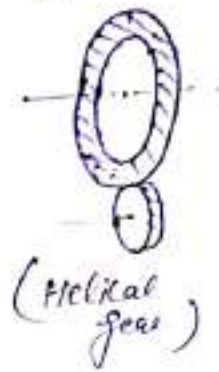
Circumferential velocity = 200 m/sec.

Overall efficiency = 96 - 99%



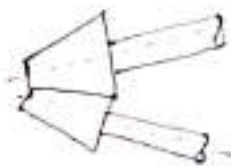
* Helical gear :-

Have teeth inclined to the axis of rotation. Helical gears are not so noisy because more gradual engagement during meshing.



* Bevel gear :-

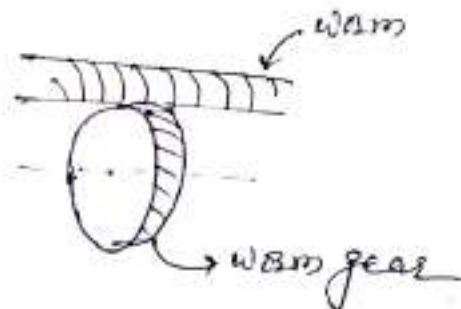
Have teeth formed on conical surfaces and are mainly used for transmitting power between intersecting shafts.



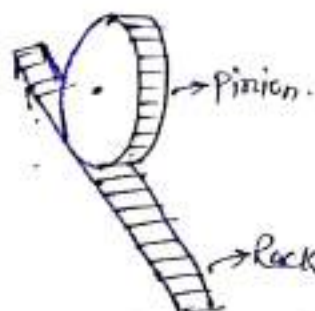
(used in Automobile).

* Worm and worm gear :-

worm gear resembles a screw.



* Rack and pinion



used in drilling machine.

International organization for standardization (ISO) recommendations.

ISO No. 888

international vocabulary of gears.

ISO No. R701

International Gear Notations

Symbols for Geometrical data.

* Law of gearing:-

The fundamental law of gearing states that the angular velocity ratio between the gears of a gear set must remain constant throughout the mesh.

$$\frac{n_1}{n_2} = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

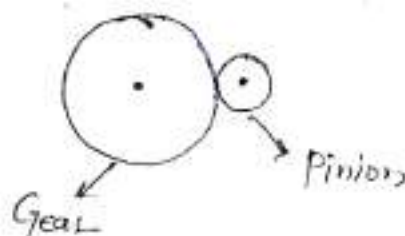
where

n = speed

d = diameter

z = number of teeth

ω = angular speed



* Gear Profile.

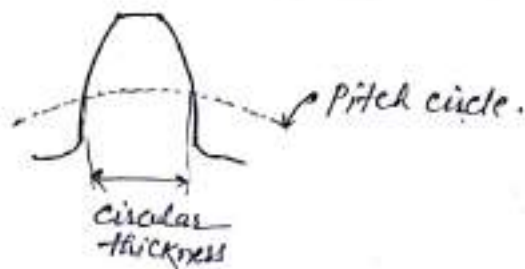
Profiles which can satisfy the law of gearing are-

1. Involute
2. Cycloidal
3. Circular arc or Novikov.

For producing constant velocity ratio, the curved profiles of the mating teeth must be such that the law of gearing is satisfied.

Involute Profile.

* Advantages of involute profile



- (i) Variation in centre distance does not affect the velocity ratio.
- (ii) Pressure angle remains constant throughout the engagement which results in smooth running.
- (iii) Straight teeth of basic rack for involute admit simple tools. Hence manufacturing becomes simple and cheap.

* Advantages of cycloidal gears:-

- (i) cycloidal gears do not have interference.
- (ii) cycloidal tooth is generally stronger than an involute tooth owing to spreading flanks in contrast to the radial flanks of an involute tooth.
- (iii) Because of the spreading flanks, they have high strength and compact drives are achievable.
- (iv) cycloidal teeth have longer life since the contact is mostly rolling which results in less wear.

* Flywheel.

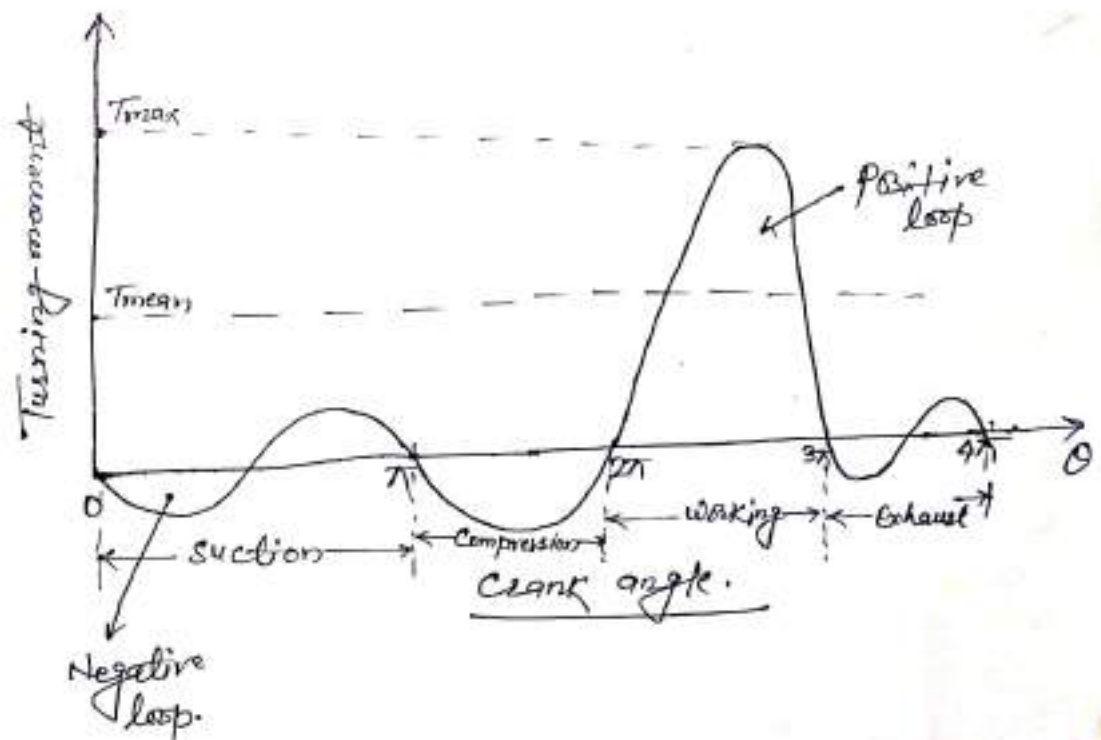
A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the ~~requirement~~ supply. Ex - Steam engine, I.C engine, reciprocating compressor etc.

A flywheel controls the speed variations caused by the fluctuation of the ~~energy~~ ^{engine} turning moment during each cycle of operation.

* Turning moment diagram:-

The turning moment diagram (also known as crank effort diagram) is the graphical representation of the turning moment or crank-effort for various position of the crank.

* Turning moment diagram for a Four stroke cycle internal combustion engine.



A turning moment diagram for a four stroke cycle internal combustion engine as shown in fig. We know that in a four stroke cycle internal combustion engine, there are one working stroke after the crank has turned through the two revolutions. i.e 720° or 4π . Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in fig.

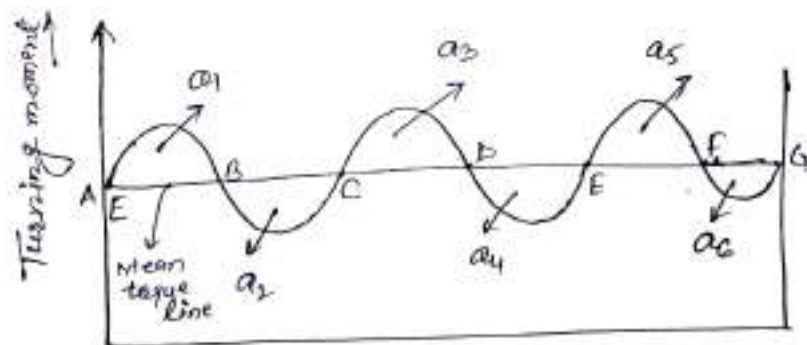
* Coefficient of Fluctuation of Energy :-

It may be defined as the ratio of the maximum Fluctuation of energy to the work done per cycle. Mathematically, Coefficient of Fluctuation of energy (C_E).

$$C_E = \frac{\text{Maximum Fluctuation of Energy}}{\text{Work done per cycle}}$$

$$\text{Work done} = T_{\text{mean}} \times \theta$$

* Determination of Maximum Fluctuation of Energy :-



Let Energy at point A = E

" " " B = E + a₁

" " " C = E + a₁ - a₂

∴ Maximum Fluctuation of Energy = Maximum Energy - Minimum Energy

* Coefficient of Fluctuation of speed:-

The difference between the maximum speed during and minimum speed during a cycle is called the maximum fluctuation of speed. The ratio of maximum fluctuation of speed to the mean speed is called Coefficient of fluctuation of speed.

Let N_1 = Maximum speed in r.p.m

N_2 = Minimum speed in r.p.m

$$N = \text{Mean speed in rpm} = \frac{N_1 + N_2}{2}$$

Coefficient of fluctuation of speed (C_s)

$$C_s = \frac{N_1 - N_2}{N}$$

$$C_s = \frac{\omega_1 - \omega_2}{\omega}$$

Note:- The reciprocal of the Coefficient of Fluctuation of speed is known as Coefficient of steadiness and is denoted by m (m).

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}$$

* Governors:-

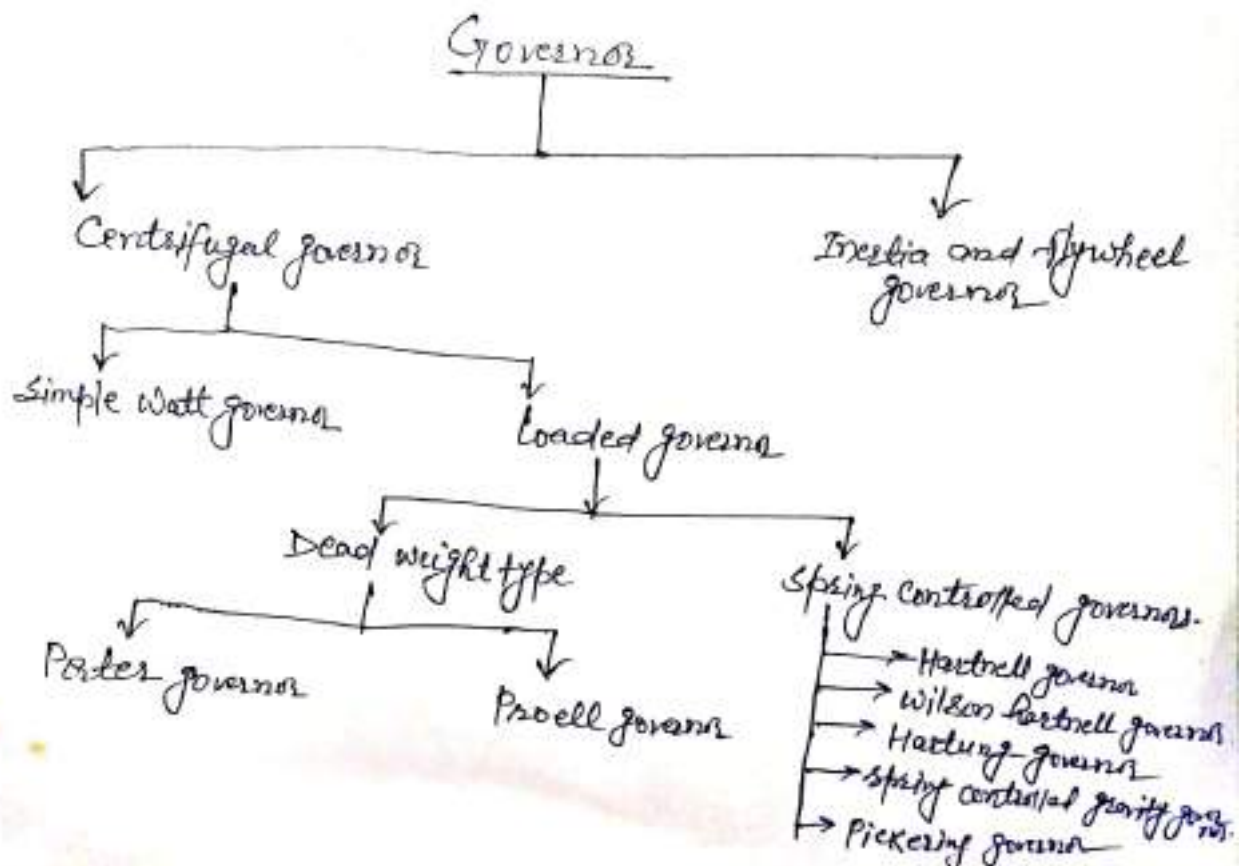
The flywheel controls the cyclic fluctuations in speed due to fluctuation of energy. The internal combustion engines come across the change in speed due to change in load which cannot be controlled by the flywheel. When the load on the engine decreases the speed of the engine increases. Similarly, when the load on the engine increases, the speed of the engine decreases.

The speed variation in speed occurring due to variation in load is controlled by making variation in fuel supply. This function is achieved by a mechanical device called as governor.

→ Thus the function of governor is to automatically maintain the speed of an engine within the prescribed limits for varying load conditions.

*

Types of Governors.



* Comparison between flywheel and governor.

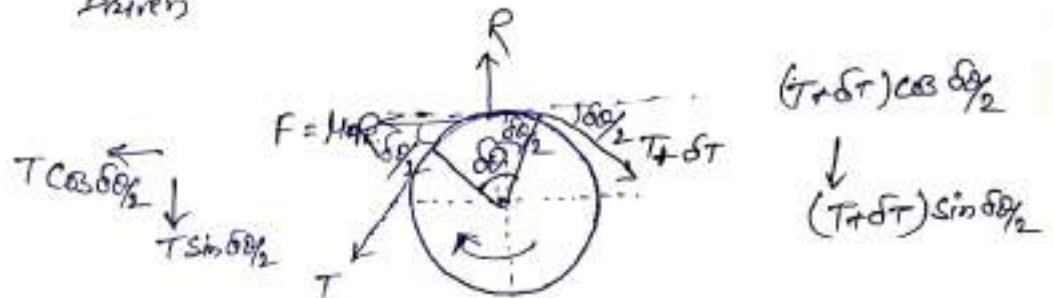
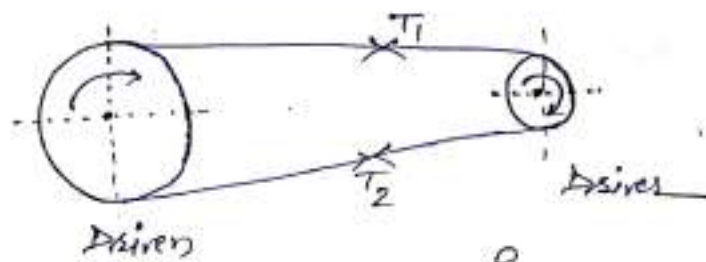
Flywheel.

- (1) The function of flywheel is to control the speed variations caused by the fluctuations of turning moment during a cycle.
- (2) Mathematically, a flywheel controls $\frac{\Delta N}{\Delta t}$.
- (3) A flywheel stores the energy and gives up the energy when required during a cycle.
- (4) It regulates the speed during one cycle only.
- (5) A flywheel has no control over quantity of charge.

Governor

- (1) The function of a governor is to keep the variation in mean speed of the engine within prescribed limits due to the fluctuations in load over a period of time.
- (2) Mathematically, a governor controls ΔN .
- (3) A governor regulates the speed by regulating the quantity of charge of prime mover.
- (4) It regulates the speed over a period of time.
- (5) A governor takes care of quantity of working fluid.

* Ratio of tension for Flat belt:-



Let T_1 = Tension in tight side

T_2 = Tension in slack side

μ = Coefficient of friction b/w belt and pulley

θ = angle of lap

R = Normal reaction

Resolving the forces horizontally and vertically.
in vertically

$$T \sin \frac{\theta}{2} + (T + \delta T) \sin \frac{\theta}{2} = R$$

$$T \frac{\theta}{2} + \theta (T + \delta T) \frac{\theta}{2} = R$$

$$T \frac{\theta}{2} + T \frac{\theta}{2} + \delta T \cdot \frac{\theta}{2} = R$$

$$2T \frac{\theta}{2} = R$$

$$T \theta = R \quad \text{--- (1)}$$

$$\left| \begin{array}{l} \sin \frac{\theta}{2} = \frac{\theta}{2} \\ \frac{\theta}{2} \cdot \delta T \approx \text{neglect} \end{array} \right.$$

In horizontally:-

$$(T + \delta T) \cos \frac{\theta}{2} - T \cos \frac{\theta}{2} = \mu R$$

$$T \cancel{\cos \frac{\theta}{2}} + \delta T \cos \frac{\theta}{2} - T \cancel{\cos \frac{\theta}{2}} = \mu R$$

$$\delta T \cos \frac{\theta}{2} = \mu R$$

$\cos \delta \theta_2$ is very small so $\cos \delta \theta_2 = 1$

$$\therefore \delta T = \mu R \quad \text{--- (2)}$$

From eqn (1) & (2)

$$\delta T = \mu T \delta \theta$$

integrating

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^\theta \delta \theta$$

~~ln T~~

$$\left[\ln \right]_{T_2}^{T_1} = \mu \theta$$

$$\ln T_1 - \ln T_2 = \mu \theta$$

$$\ln \left(\frac{T_1}{T_2} \right) = \mu \theta$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu \theta}}$$

* Condition for maximum power transmission by a belt:-

We know that $P = \frac{(T_1 - T_2)V}{1000}$

and $\frac{T_1}{T_2} = e^{\mu \theta} \Rightarrow T_2 = \frac{T_1}{e^{\mu \theta}}$

$$\therefore P = \frac{\left(T_1 - \frac{T_1}{e^{\mu \theta}} \right)}{1000} \times V = \frac{T_1 \left(1 - \frac{1}{e^{\mu \theta}} \right)}{1000} \times V$$

For a belt drive, $\left(\frac{1 - \frac{1}{e^{\mu \theta}}}{1000} \right) = \text{Constant}$

$$\therefore P = K T_1 V$$

$$T_{\max} = T_1 + T_c$$

$$T_1 = T_{\max} - T_c$$

$$\therefore P = K (T_{\max} - T_c) V \quad \text{a}$$

$$\text{Also } T_{\max} = mv^2$$

$$\therefore P = K(T_{\max} - mv^2)v$$

$$P = K(T_{\max}v - mv^3)$$

For maximum power, $\frac{dP}{dv} = 0$

$$K(T_{\max} - 3mv^2) = 0$$

$$\boxed{T_{\max} = 3mv^2}$$

$$T_{\max} = 3T_c$$

$$\therefore \boxed{T_c = \frac{1}{3} T_{\max}}$$

This is the condition for maximum power transmission.

Simple Watt Governor :

(S-02, S-03, S-05)

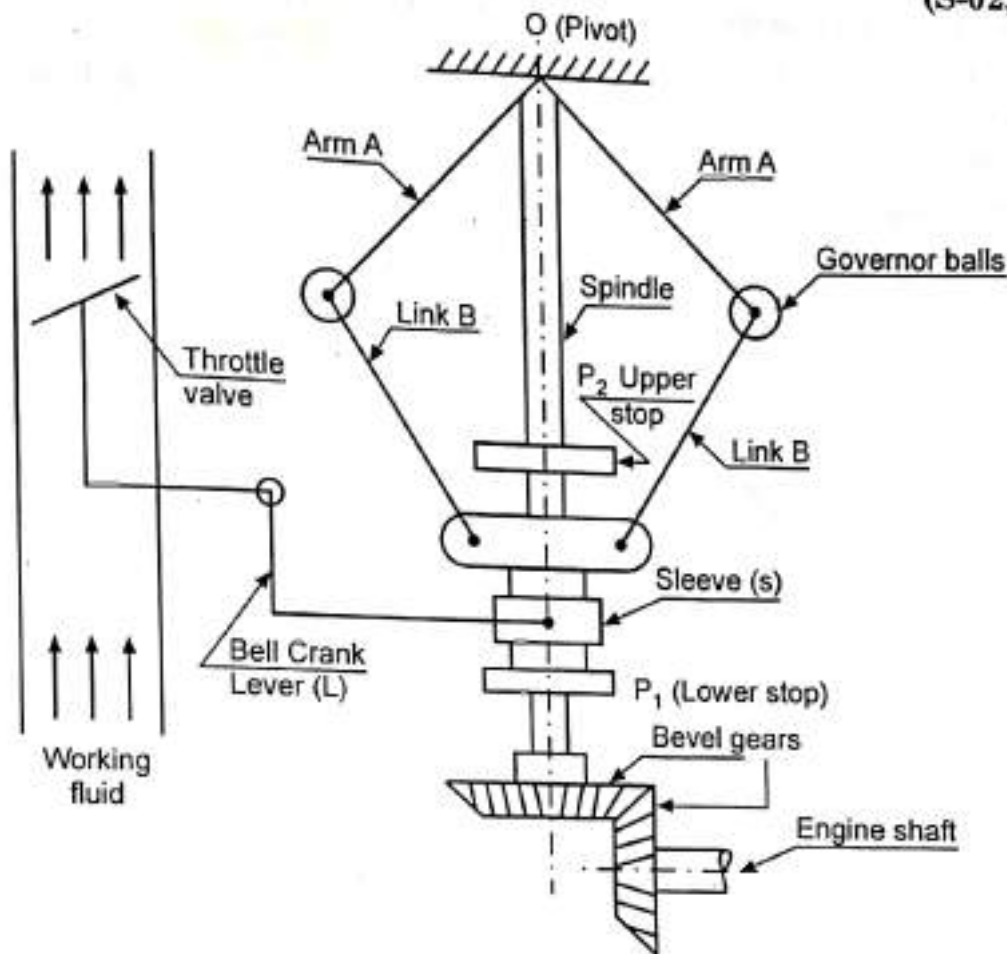


Fig. 5.4

Fig. 5.4 shows simple watt governor. It has a governor spindle which is connected to the engine shaft through bevel gears.

- A sleeve is mounted on the spindle which is free to slide on the spindle and rotates with the spindle. The sleeve can slide between the two stoppers P_1 and P_2 .
- The two ball masses are connected to the sleeve by lower arms. They are connected to the governor head by upper arms.
- The throttle valve is connected to the sleeve with the help of mechanical linkages.

Working :

- As engine shaft rotates with governor spindle whole assembly will rotate. When the output of the engine is equal to the load the governor assembly will rotate at mean speed, therefore ball masses will achieve a certain radius and keep on rotating on same radius.
- As the load on the engine decreases, the speed of the engine and governor will increase. Therefore, the centrifugal force acting on ball masses will increase, which results in movement of ball masses away from the axis. Therefore, the sleeve will move upwards. This movement will be transmitted to the throttle valve through the mechanical linkage such that the throttle valve will be partially closed. Therefore, the fuel supply will be reduced and the speed is brought back to mean speed level.
- As the load on the engine increases the governor speed will decrease, decreasing the centrifugal force acting on the ball masses. Therefore, the ball masses will come closer

pushing the sleeve downwards. This downward movement of the sleeve will operate the mechanical linkage to open the throttle valve partially. The fuel supply will be increased and the engine speed will brought back to mean speed. Thus, this governor will work on principle of centrifugal force.

2. Porter Governor :

(S-03, S-04, S-09)

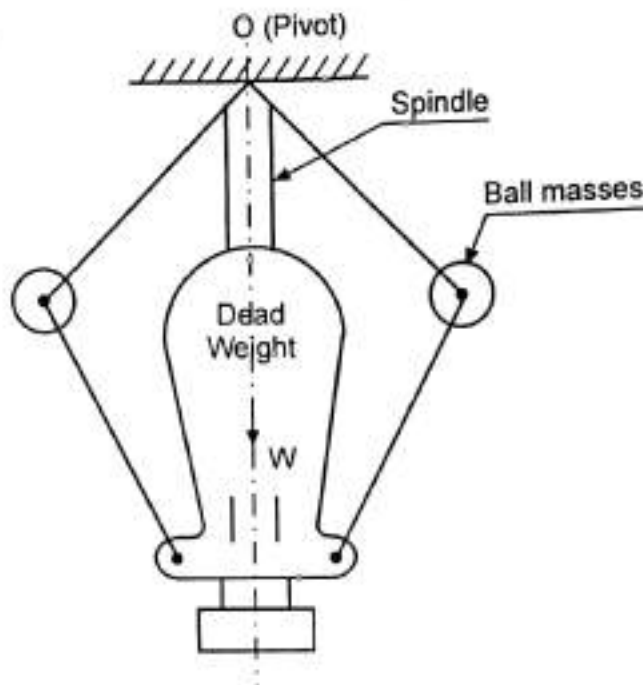


Fig. 5.5 : Porter Governor

Fig. 5.5 shows the Porter Governor which is the modification of simple governor.

- Its construction and working is similar to the simple governor. The only change is, the heavy weight is attached to the sleeve.
- This will help the governor to become a stable governor.
- The Porter Governor can be used for high speed applications.

3. Proell Governor :

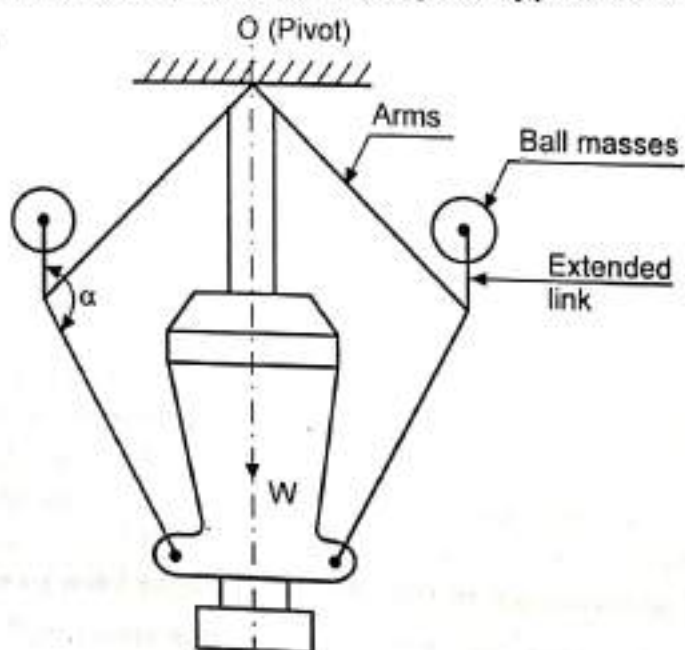


Fig. 5.6 : Proell Governor

- Fig. 5.6 shows the Proell Governor which is the modification of simple governor.
- Its construction and working is similar to simple governor except two changes :
 - (i) A heavy weight is attached to the sleeve.
 - (ii) The ball masses are mounted on the extension of the lower arms.
- This will help to balance the stability and sensitivity of the governor.

4. Hartnell Governor :

(W-04)

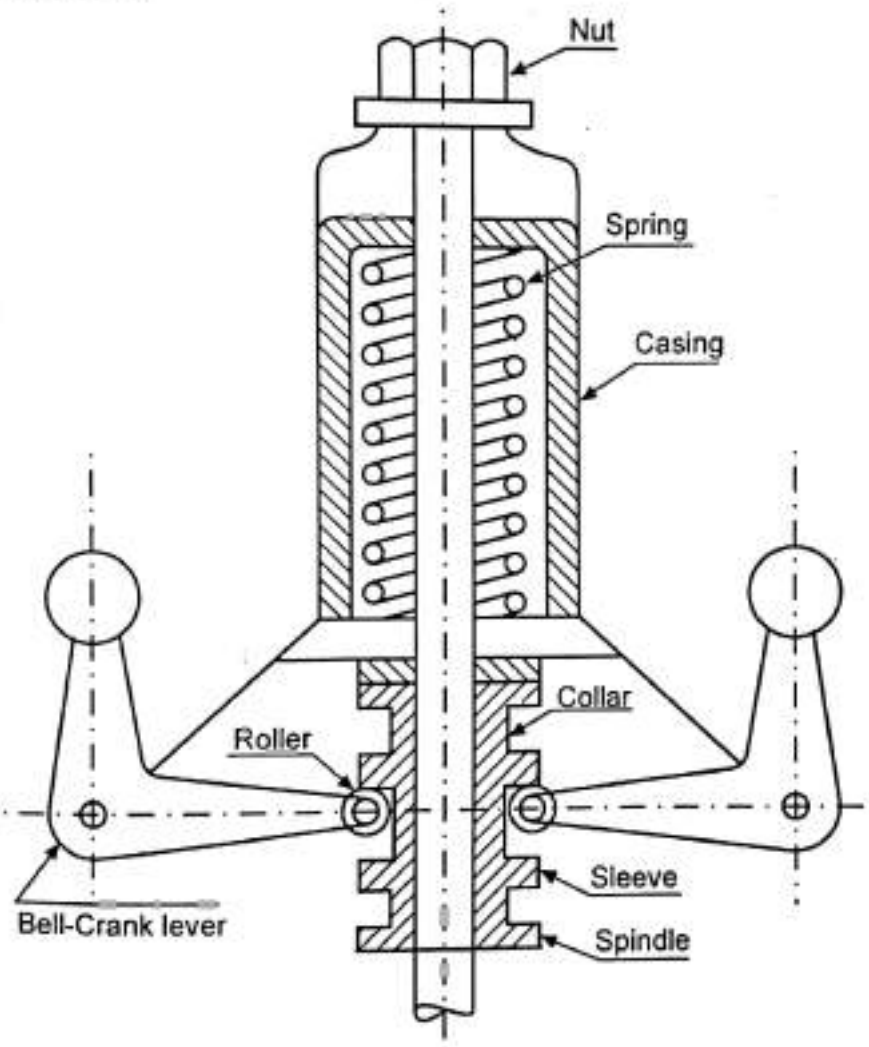


Fig. 5.7 : Hartnell Governor

- Fig. 5.7 shows a spring loaded Hartnell governor.
- It has a speed casing keyed to the spindle.
- The spindle is connected to the engine shaft through the bevel gear.
- A sleeve is mounted on the spindle which rotates alongwith the spindle as well as slides along the spindle.
- A compressed spring is mounted in the casing between the top of the casing and top of the sleeve.
- The ball masses are mounted on the bell crank lever.

note

Brakes & dynamometers

Power measuring device

PAGE NO.

DATE

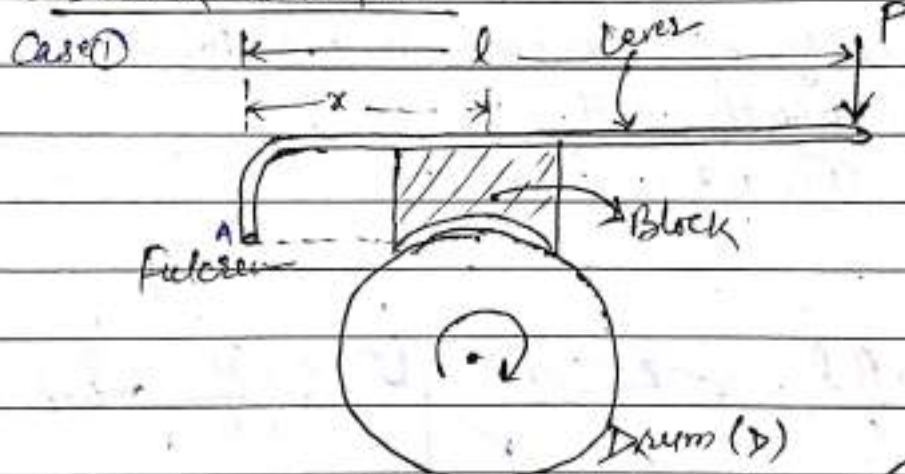
Brake is a device by which a braking torque can be applied to stop a rotating shaft or drum. If the same device is utilized to ~~measure~~ ^{measure} power also then it is known as dynamometer.

Types of brakes

- ① Block brakes
- ② Band brakes
- ③ Differential band brake
- ④ Band & Block brakes

- Types of brakes
- ① Hydraulic
 - ② Electric brake
 - ③ Mechanical brake

Block brakes.



$P \rightarrow$ Force applied at the end of the lever
 $R_N \rightarrow$ Normal force pressing the brake block on the wheel
 $r =$ Radius of the wheel
 $2\theta =$ Angle of contact surface of the block
 $\mu =$ Coeffⁿ of friction
 $F_t \rightarrow$ Tangential force or frictional force



$\therefore$ Drum or friction force
 rotation of opposite
 direction of block or
 direction of drum

Case ① when fulcrum is in line with point of contact or ~~to~~ line of action of frictional force.

Now Taking moment about A.

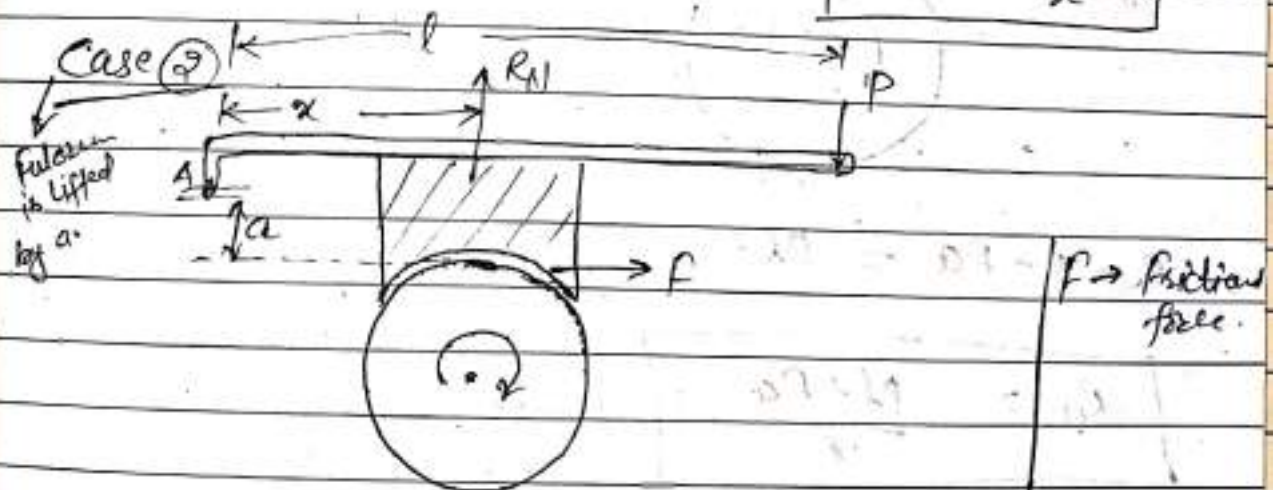
$$R_N \times x = P \times l$$

$$R_N = \frac{Pl}{x}$$

$$\text{Frictional force } F = \mu R_N = \frac{\mu Pl}{x}$$

$$\text{Frictional torque (braking torque)} = T_B = F \times x$$

$$T_B = \frac{FPl\mu}{x}$$



Taking moment about A.

$$R_N \cdot x + F \cdot a = Pl$$

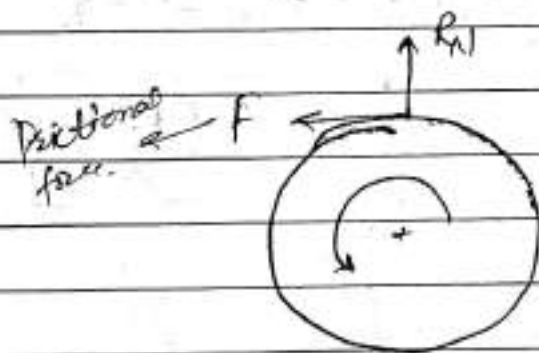
$$R_N = \frac{Pl - Fa}{x} \quad \text{--- (1)}$$

Frictional force $= F = \mu R_N$

Frictional torque $= T_B = F \times r$

$$\therefore T_B = \frac{\mu r (P_L - F a)}{x}$$

In this case the rotation of drum is going to effect the equation of R_N . if the drum is anticlockwise R_N is given by



with $R_N \cdot x - F a = P_L$

$$R_N = \frac{P_L + F a}{x} \quad \text{--- (2)}$$

If we examine these equations, then in case of clock wise rotation R_N equation (1) can be written as

$$R_N x = P_L - F a$$

$$R_N x = P_L - \mu R_N a$$

between the frictional force is ~~opposite~~ opposing

the applied force & hence if we want to make given brake

Self energizing the moment created by friction must support the moment due to applied load. & in this situation we can call then the brake is self energizing & this can happen when the dirⁿ of rotation is anticlockwise. which means eqn (2) is applicable

$$\text{i.e. } R_N \cdot x - f a = P l$$

$$\boxed{R_N = \frac{P l + f a}{x}}$$

Here the moment due to frictional force is supporting the applied load between less load is required to stop the drum and a situation may arise when the applied load becomes negligible.

$$\text{i.e. } P l \rightarrow 0$$

This can only happen if

self energizing $R_N \cdot x = P l + f a$

$$R_N \cdot x = P l + \mu R_N a$$

$$R_N (x - \mu a) = P l$$

which applied that $\boxed{x = \mu a}$ (3)

'Pl' becomes zero.

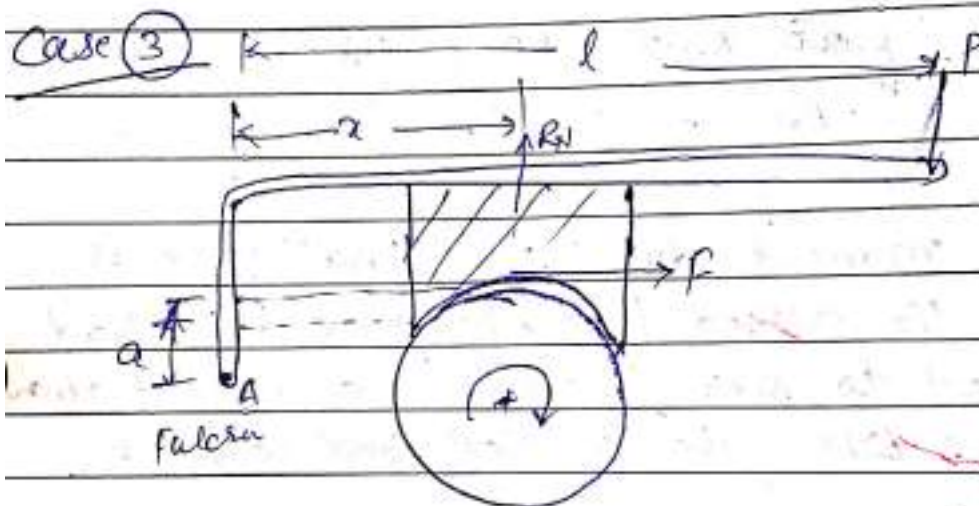
If $\frac{x}{a} = \mu$
 self locking
 if not self locking
 (moment)

$$\frac{x}{a} = \mu$$

self locking
 as self energizing brake
 self locking
 if not self locking
 (moment)

$\epsilon_{pu}^{(3)}$ represents extreme case of self energizing phenomenon which says that if ratio of 'x' and 'a' is equal to coeffⁿ of friction μ then the brake will become self locking which indicates that is anticlockwise dirⁿ with this $\epsilon_{pu}^{(3)}$

$x = \mu a$ not motion is possible.



$$R_H \cdot x - F a = P l$$

$$R_H = \frac{P l + F a}{x}$$

$$F = \mu R_H$$

Frictional force $= F = \mu R_H$

$\therefore$ Braking torque $= T_B = F \times r = \mu R_H \cdot r$

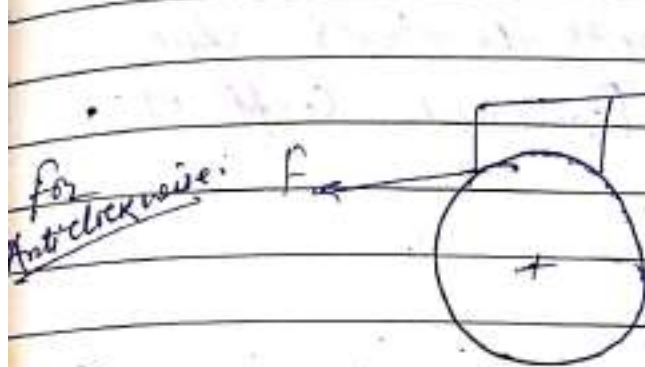
self locking $\epsilon_{pu}^{(3)}$

$$R_H \cdot x = P l + F a$$

$$R_H \cdot x - \mu R_H a = P l$$

$$R_H (x - \mu a) = PL$$

$$x = \mu a$$



$$R_H = \frac{PL}{x}$$

$$R_H = \frac{PL - Fa}{x}$$

$$R_H \cdot x + Fa = PL$$

$$R_H = \frac{PL + Fa}{x}$$

$$R_H = \frac{PL - Fa}{x}$$

$$T_B = \mu R_H$$

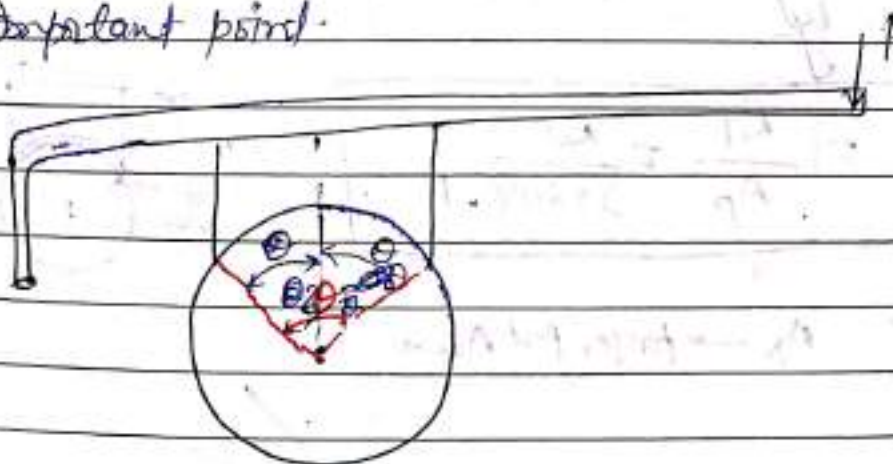
$$T_B = \mu R_H$$

$$x = \mu a$$

self locking

Important point:

*



In all above cases the angle of contact 2θ of block is taken less than 60° & due to this assumption the normal pressure b/w block & drum is

taken as uniform but if $2\theta \geq 60^\circ$ the normal pressure is less at the ends of the block & more at the centre, to make it uniform the block must be pivoted on the lever rather than ~~the~~ welding it with the lever due to this pivoted block Equivalent Coeffⁿ of friction becomes

$$\mu' = \frac{4H \sin \theta}{2\theta + \sin \theta}$$

where μ is actual Coeffⁿ coefficient of friction & the breaking torque (T_B) is given by

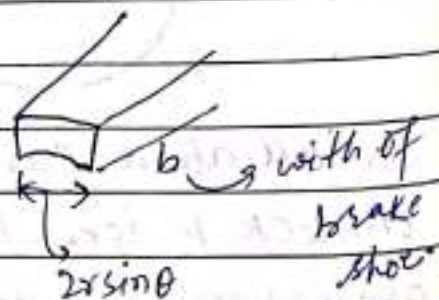
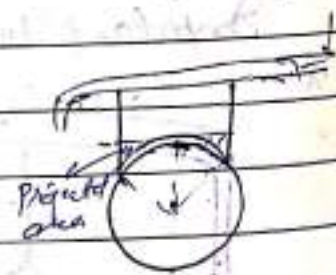
$$T_B = \mu' R_N r$$

* The bearing pressure on the block (brake shoe) is given by

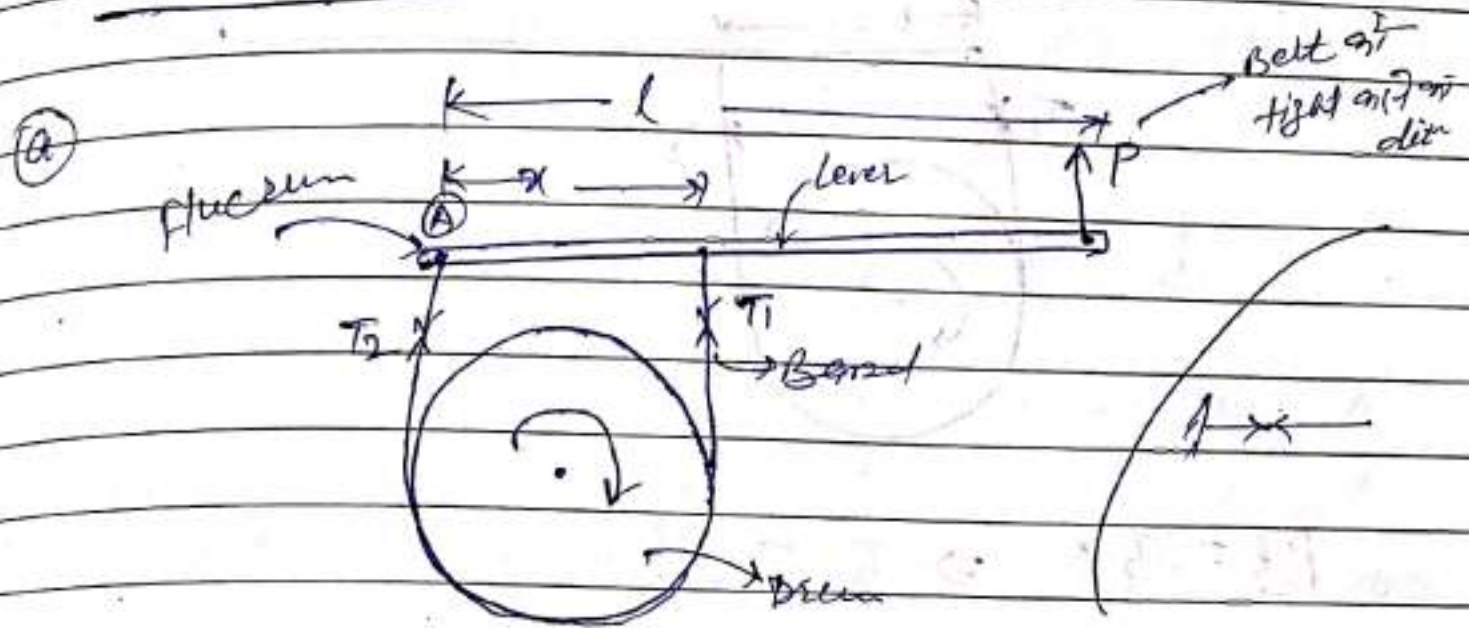
$$P_b = \frac{R_N}{A_p} = \frac{R_N}{2r \sin \theta \cdot b}$$

Bearing pressure

$A_p \rightarrow$ projected Area



Band brake



the disⁿ of P should always be such that it will try to make the belt tight

$T_1 \rightarrow$ Tight side tension

$T_2 \rightarrow$ Slack " " " " " "

Taking moment about fulcrum (A)

$$Pl = T_1 x$$

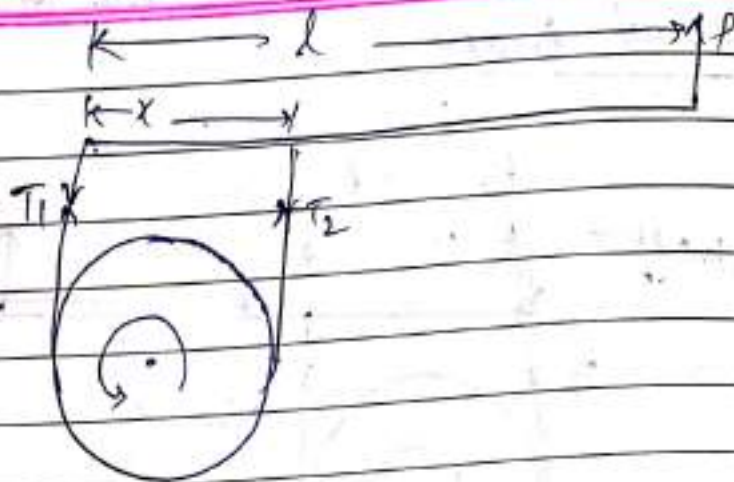
$$T_2 = (T_1 - T_2) r$$

Braking torque

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$T_1, T_2 \rightarrow$ forces

(b)



$$Pl = T_2 x \Rightarrow T_2 = \frac{Pl}{x}$$

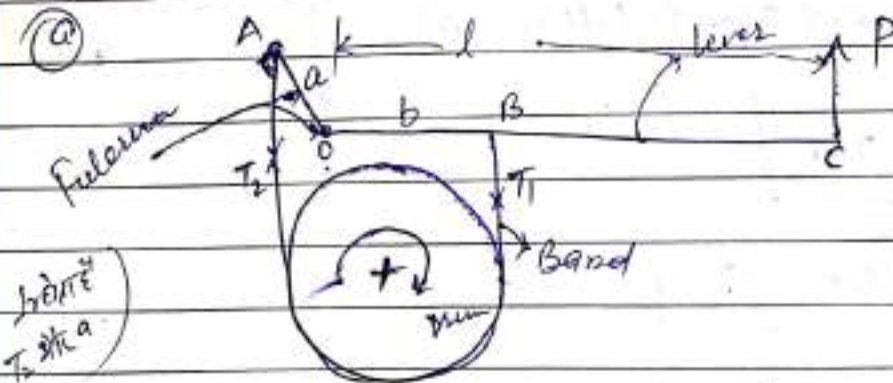
$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$T_3 = (T_1 - T_2)x$$

In a differential band brake, the ends of the two band are joined at A and B to a lever. AOC pivoted on a fixed pin or fulcrum O.

(3) Differential band brake

(a)



Point
T2 at a

* For breaking ① $OA > OB$ & 'p' should be $\downarrow$.

otherwise

② $OA < OB$ & p should be $\uparrow$.

Above relation is based on the effect that p should always be applied to move point A outward for situation ① & B outward for situation ②.

②

Taking moment about fulcrum.

$$T_2 a + p l = T_1 b$$

$$p l = T_1 b - T_2 a$$

p l moment
T₁ b and T₂ a
are 2 moments
∴ diffⁿ band
brake over it

② For anticlockwise dirⁿ of rotaⁿ of drum

$$T_1 a + p l = T_2 b$$

$$p l = T_2 b - T_1 a$$

$$\therefore \frac{T_1}{T_2} = e^{\mu \theta}$$

$$\therefore T_3 = (T_1 - T_2) \gamma$$

* For self locking in case (a)

i.e. Drum is rotating in clock wise dir

$$T_1 b \leq T_2 a$$

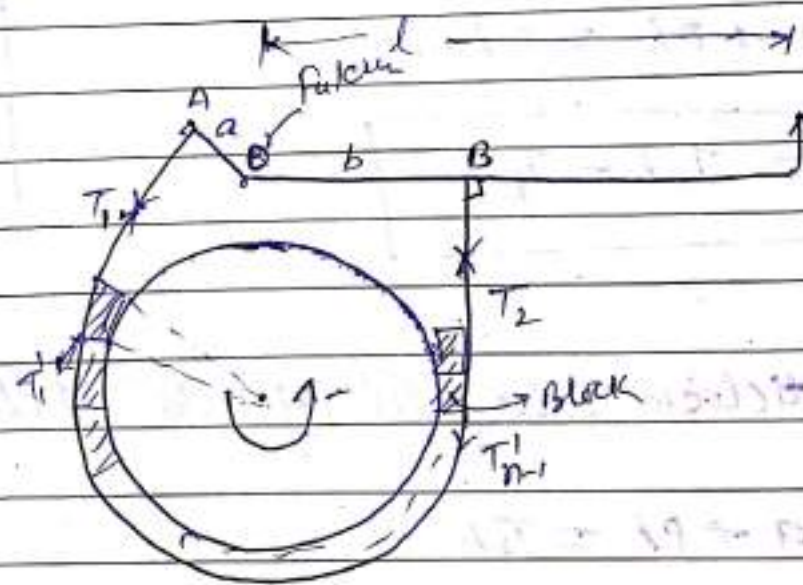
Zero or -ve
→ हो रहा है

For case (b)

$$T_2 b \leq T_1 a$$

(4) ~~Block & Band~~

Band & block break.



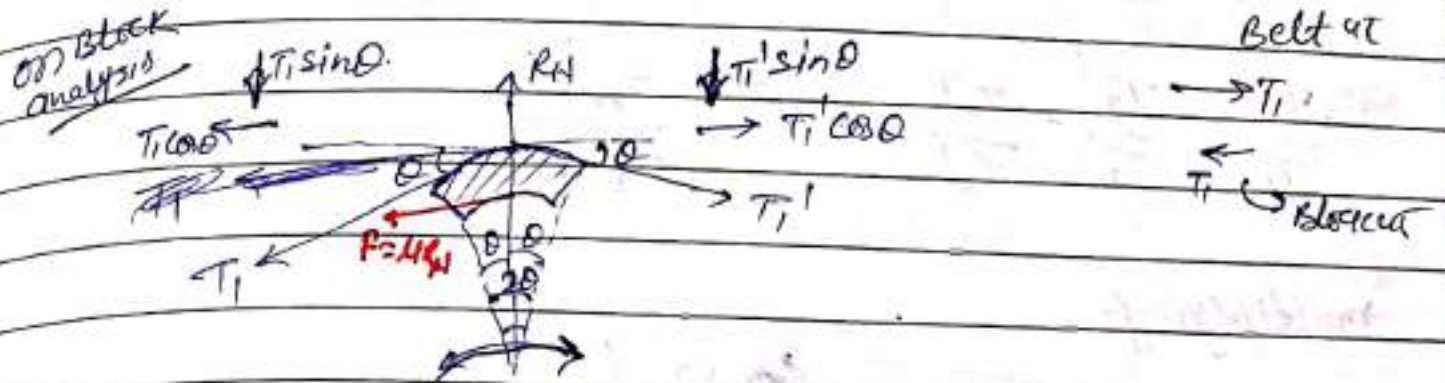
* If

$OA > OB$ then $P \downarrow$

$OA < OB$ " $P \uparrow$

Equation for breaking torque for a band and block brake.

Let $n = \text{no's of block's}$



Resolving the forces horizontally & vertically.

$$T_1 \cos \theta - T_1' \cos \theta + \mu R_N = 0$$

$$T_1' \cos \theta - T_1 \cos \theta = \mu R_N \quad \text{--- (1)}$$

$$T_1 \sin \theta + T_1' \sin \theta = R_N \quad \text{--- (2)}$$

$$(2) \div (1)$$

$$\frac{(T_1 + T_1') \sin \theta}{(T_1' - T_1) \cos \theta} = \frac{1}{\mu}$$

$$\frac{T_1 + T_1'}{T_1' - T_1} = \frac{1}{\mu \tan \theta} \Rightarrow \frac{T_1 + T_1' + T_1' - T_1}{T_1 + T_1' - T_1' + T_1} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

using compoundo & dividendo

Analysing every block

$$\frac{T_1}{T_1'} = \frac{T_2'}{T_2} = \frac{T_3'}{T_3} = \dots = \frac{T_{n-1}'}{T_n}$$

multiplying

$$\left[\frac{T_1}{T_2} = \left[\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right]^n \right] \quad \text{--- (I)}$$

The breaking torque is given

$$T_B = (T_1 - T_2) r$$

Where r is effective radius of system. To find out T_B , T_1 & T_2 both are required hence the second equation is

$$Pl = T_2 b - T_1 a \quad \text{--- (II)}$$

using (I) & (II), T_1 & T_2 can be calculated.

Clutch.

* Introduction:-

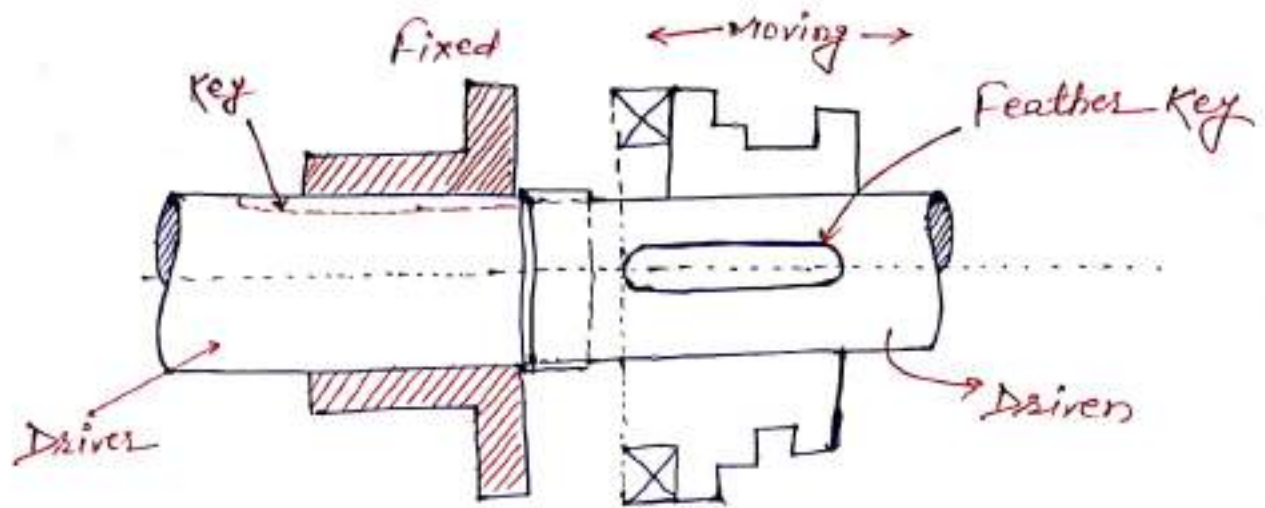
A clutch is a machine member used to connect a driving shaft to a driven shaft so, that the driven shaft may be started or stopped at will, without stopping the driving shaft. The uses of a clutch is mostly found in automobile. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. it is therefore necessary that the ~~the driven driving~~ driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

* Types of clutches.

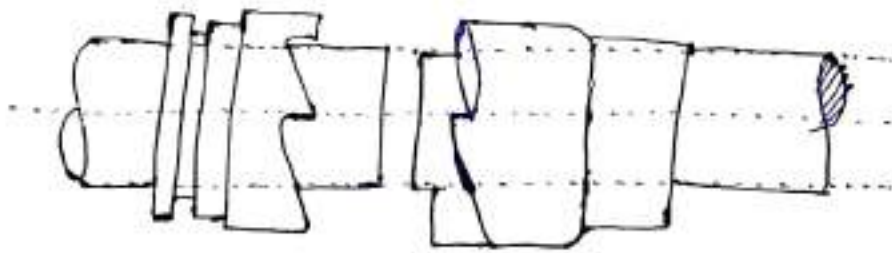
There are following two types of clutches are generally used. -

- (i) Positive clutch.
- (ii) Friction clutch.

Positive clutch :- the positive clutches are used when a positive drive is required. The simplest types of a positive clutch is a jaw or claw clutch. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws.



(a) Square Jaw clutch



(b) Spiral Jaw clutch.

A Square Jaw clutch is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The Spiral Jaw may be left hand or right hand, because power transmit by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of Jaw clutch is frequently applied to spool wheel, gears and pulley. In such a case the non-sliding part is made integral with the hub.

(ii) Friction clutches:-

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. its application is also found in case in which power is to be delivered to machines partially or fully loaded.

* Material for friction surfaces:-

The material used for lining of friction surface of a clutch should have the following characteristics -

- (i) It should have a high and uniform coefficient of friction.
- (ii) It should not be affected by moisture and oil.
- (iii) It should have the ability to withstand high temperature caused by slippage.
- (iv) It should have high heat conductivity.
- (v) It should have high resistance to wear.

The materials commonly used for lining of friction are as follows -

- Cast iron on cast iron or steel.
- ~~cast iron on cast iron or~~
- Hardened steel on hardened steel.
- Bronze on cast iron or steel.
- Pressed asbestos on cast iron or steel.

* Considerations in designing a friction clutch:-

The following consideration must be kept in mind while designing a friction clutch.

- (i) The suitable material forming the contact surface should be selected.
- (ii) The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed load.
- (iii) The clutch should not require any external force to maintain contact of the friction surfaces.
- (iv) The clutch should have provision for carrying away the heat generated at the contact surfaces.

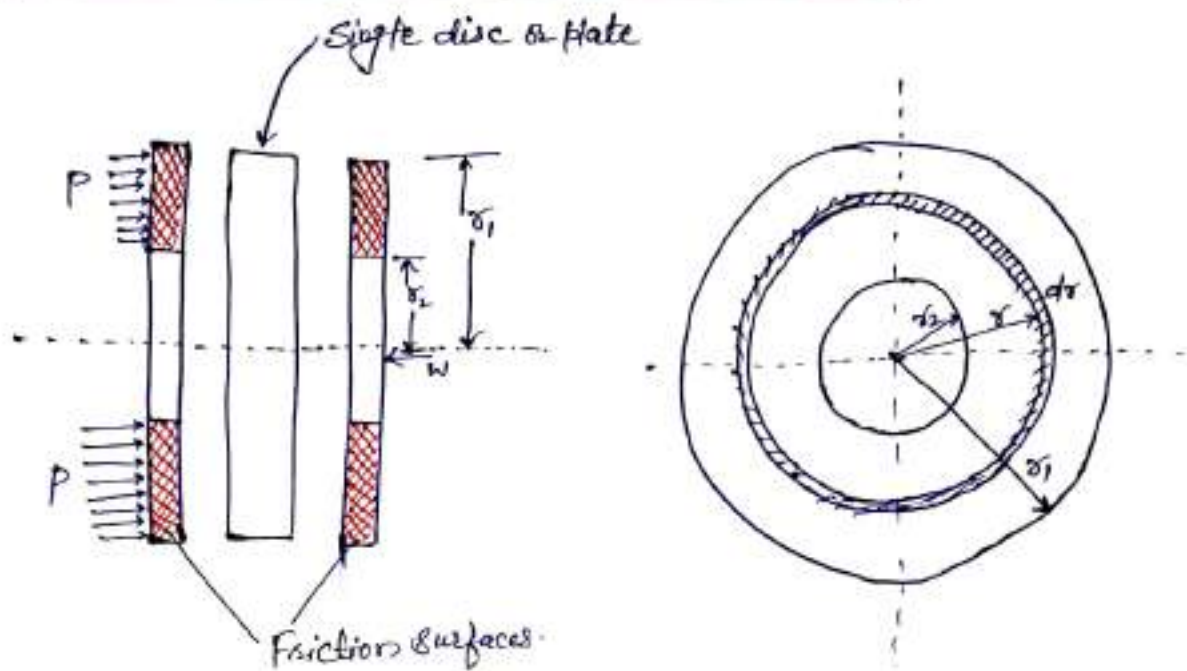
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* Types of friction clutches:-

- There are following types of friction clutches -
- (i) Disc or plate clutches (single disc or multiple disc clutches)
 - (ii) Cone clutches.
 - (iii) Centrifugal clutches.

The disc and cone clutches are known as axial friction clutches, while the centrifugal clutch is called radial friction clutch.

* Design of a Disc or plate clutch :-



Let T = Torque transmitted by the clutch
 P = Intensity of axial pressure with which the contact surfaces are held together.
 r_1 and r_2 = External and internal radii of friction faces.
 r = Mean radius of the friction face.
 μ = coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in fig.

We know that area of the contact surface or friction surface

$$= 2\pi r dr$$

$\therefore$ Normal or axial force on the ring

$$dW = \text{Pressure} \times \text{Area}$$

$= P \times 2\pi r dr = 2\pi P r dr$
 and the frictional force on the ring acting tangentially at radius r ,

$$F_r = \mu \times dW = \mu P \cdot 2\pi r dr$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu P \times 2\pi r dr \times r = 2\pi \mu P r^2 dr$$

We shall now consider the following two cases.

(i) when there is a uniform pressure

(ii) when there is a uniform wear

* Uniform pressure theory:-

When the pressure is uniformly distributed over the entire area of the friction face ~~as~~, then the intensity of pressure,

$$P = \frac{W}{\pi [r_1^2 - r_2^2]}$$

where W = Axial thrust with which the friction surfaces are held together.

We know that the frictional torque on the elementary ring of radius r and thickness dr is -

$$T_r = 2\pi H P r^2 dr$$

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi H P r^2 dr = 2\pi H P \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$T = 2\pi H P \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$T = 2\pi H \cdot \frac{W}{\pi [r_1^2 - r_2^2]} \cdot \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$T = \frac{2}{3} H W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$\boxed{T = H W R} \quad \text{where } R = \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

Considering uniform wearing:-

- The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction.
- The work of friction is proportional to the product of normal pressure (P) and the sliding velocity (V).
- Therefore,

Normal wear $\propto$ work of friction $\propto P \cdot V$

$$\therefore PV = K$$

$$\text{or, } P = \frac{K}{V}$$

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing process continues until the product PV is constant over the entire surface.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore,

$$Pr = C$$

$$\text{or } P = \frac{C}{r}$$

and the normal force on the ring,

$$\delta W = P \cdot 2\pi r dr = \frac{C}{r} \cdot 2\pi r dr = 2\pi C dr$$

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C dr = 2\pi C (r_1 - r_2)$$

$$C, C = \frac{W}{2\pi(r_1 r_2)}$$

Frictional torque acting on the ring

$$T_r = 2\pi \mu P r^2 dr = 2\pi \mu \frac{C}{r} r^2 dr$$

∴ Total torque acting on the friction surface

$$T = \int_{r_2}^{r_1} 2\pi \mu C r dr = 2\pi \mu C \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$T = \frac{\mu \cdot W}{2\pi(r_1 r_2)} (r_1 + r_2) (r_1 - r_2)$$

$$T = \mu W \frac{(r_1 + r_2)}{2} \Rightarrow \boxed{T = \mu W R} \text{ where } R = \frac{r_1 + r_2}{2}$$

Note: 1. In general, total frictional torque acting on the friction surfaces is given by:-

$$\boxed{T = n \mu W R}$$

where n = Numbers of pairs of friction surfaces

R = mean radius of friction surfaces

$$= \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] \rightarrow \text{For uniform pressure}$$

$$= \frac{r_1 + r_2}{2} \rightarrow \text{For uniform wear}$$



Note-2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (i.e. $n=2$)

Note-3. Since the intensity of pressure is maximum at the ~~outer~~ inner radius (r_2) of the friction or contact surface therefore,

$$P_{\max} \times r_2 = C \quad \text{or} \quad P_{\min} \times r_1 = C$$

$$\text{or} \quad P_{\max} = \frac{C}{r_2}$$

Note-4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore,

$$P_{\min} \times r_1 = C$$

Note-5 The average pressure (P_{av}) on the friction or contact surface is given by -

$$P_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}}$$

$$P_{av} = \frac{W}{\pi [r_1^2 - r_2^2]}$$

Note-6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch, the uniform wear theory is more approximate.

Note-7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

Q: Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

Solⁿ Given $W = 4 \text{ kN} = 4 \times 10^3 \text{ N}$, $r_1 = 100 \text{ mm}$, $r_2 = 50 \text{ mm}$

$$\therefore P_{\max} \cdot r_2 = C$$

$$\therefore C = 50 P_{\max}$$

We also know that total force on the contact surface (W).

$$W = 2\pi C (r_1 - r_2)$$

$$4000 = 2\pi \times 50 P_{\max} (100 - 50)$$

$$\therefore P_{\max} = 0.2546 \text{ N/mm}^2 \quad \underline{\text{Ans.}}$$

Also $P_{\min} \times r_1 = C$

$$\therefore C = 100 P_{\min}$$

We also know that,

$$W = 2\pi C (r_1 - r_2)$$

$$4000 = 2\pi \times 100 \cdot P_{\min} (100 - 50)$$

$$\therefore P_{\min} = 0.1273 \text{ N/mm}^2 \quad \underline{\text{Ans.}}$$

Average Pressure,

$$P_{\text{av}} = \frac{W}{\pi [r_1^2 - r_2^2]} = \frac{4000}{\pi [100^2 - 50^2]} = 0.17 \text{ N/mm}^2 \quad \underline{\text{Ans.}}$$

Q: A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 rpm. Determine the outer and inner diameters of frictional surfaces if the coefficient of friction is 0.255, ratio of dia. is 1.25 and the max^m pressure is not to exceed 0.1 N/mm². Also determine the axial thrust to be provided by spring. Assume the theory of uniform wear.

Solⁿ Given $n = 2$, $P = 25 \text{ kW} = 25 \times 10^3 \text{ W}$, $N = 3000 \text{ rpm}$
 $\mu = 0.255$, $\frac{r_1}{r_2} = 1.25$, $P_{\max} = 0.1 \text{ N/mm}^2$

We know that, the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^3 \times 60}{2\pi \times 3000} = 79600 \text{ N-mm.}$$

For uniform wear conditions,

$$Pr = C$$

$$P_{\max} \cdot r_2 = C \Rightarrow C = 0.1 r_2 \text{ N/mm}$$

and,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 0.1 r_2 (1.25 r_2 - r_2) = 0.157 r_2^2$$

Also for uniform wear theory,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2} = 1.125 r_2$$

Also Torque transmitted T

$$T = n H W R = 2 \times 0.255 \times 0.157 r_2^2 \times 1.125 r_2$$

$$r_2^3 = \frac{79.6 \times 10^3}{0.09} = 884 \times 10^3$$

$$\therefore r_2 = 96 \text{ mm. } \underline{\text{Ans}} \Rightarrow d_2 = 192 \text{ mm } \underline{\text{Ans}}$$

$$\therefore r_1 = 1.25 r_2 = 1.25 \times 96 = 120 \text{ mm } \underline{\text{Ans}}$$

$$\therefore d_1 = 240 \text{ mm } \underline{\text{Ans}}$$

Axial thrust,

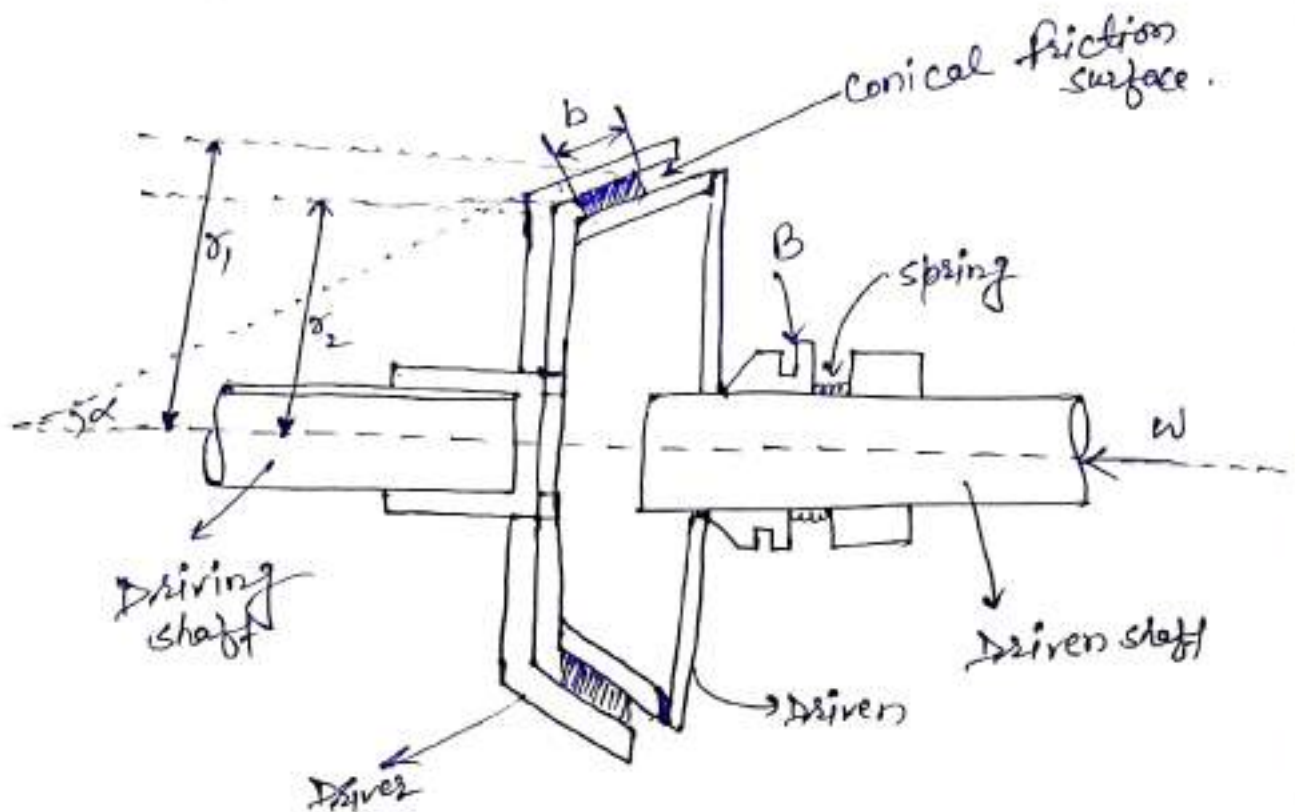
$$W = 2\pi C(r_1 - r_2) = 2\pi \times 0.1 \times 96(1.25 \times 96 - r_2)$$

$$W = 2\pi \times 0.1 \times 96 \times 0.25 \times 96$$

$$= 1447 \text{ N. } \underline{\text{Ans}}$$

Cone clutch :-

A Cone clutch was extensively used in automobile, but now a day it has been replaced completely by the disc clutch. it consist of one pair of friction surface only . . .



Design of a cone clutch :-

- * According to uniform pressure theory, Total torque transmitted by the clutch can be given as -

$$T = \frac{2}{3} \mu W \csc \alpha \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

- * According to uniform wear theory, Total torque transmitted can be given by -

$$T = \mu W \csc \alpha \left[\frac{r_1 + r_2}{2} \right]$$

$$T = \mu W R \csc \alpha \quad \text{where } R = \frac{r_1 + r_2}{2}$$

Note-1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

Note-2. If the clutch is engaged when one member is stationary and the other rotating (i.e. during engagement of the clutch) then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu W_n \csc \alpha$) act on the clutch which resist the engagement and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu W_n \csc \alpha = W_n \sin \alpha + \mu W_n \csc \alpha$$

$$W_e = W_n (\sin \alpha + \mu \csc \alpha)$$

Q: An engine developing 45 kW at 1000 rpm is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm^2 . Determine

- (i) The face width required
- (ii) The axial spring force necessary to engage the clutch.

Soln $P = 45 \text{ kW} = 45 \times 10^3 \text{ W}$, $N = 1000 \text{ rpm}$, $\alpha = 12.5^\circ$, $D = 500 \text{ mm}$
 $\mu = 0.2$, $P_n = 0.1 \text{ N/mm}^2$ $2R = 250 \text{ mm}$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{45 \times 10^3 \times 60}{2 \times \pi \times 1000} = 430 \text{ N-m}$$

We know that torque developed by the clutch (T),

$$T = 2\pi \mu P_n R^2 b$$

$$430 \times 10^3 = 2\pi \times 0.2 \times 0.1 \times 250^2 \times b$$

$$\therefore b = 54.7 \approx 55 \text{ mm} \quad \underline{\text{Ans}}$$

Normal force acting on the contact surface,

$$W_n = P_n \times 2\pi R \cdot b$$

$$= 0.1 \times 2\pi \times 250 \times 55 = 8640 \text{ N.}$$

$\therefore$ Axial spring force necessary to engage the clutch,

$$W_e = W_n (\sin \alpha + 0.25 \mu \cos \alpha)$$

$$= 8640 (\sin 12.5 + 0.25 \times 0.2 \times \cos 12.5)$$

$$= 2290 \text{ N.}$$

Ans.

Centrifugal clutch :-

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley as shown in fig. The outer surface of the shoes are covered by with a friction material. These shoes, which can move radially in guide are held against a boss on the driving shaft by means of springs.

