

Module 1

● Introduction

- * plain cement concrete (pcc) - plain cement concrete is a hardened mass obtained from a mixture of cement, sand, gravel and water in a definite proportions. These ingredients are mixed together to form a plastic mass which is poured into desired shape moulds called as forms. This plastic mass hardens on setting and we get plain cement concrete. The hardening of this mixture is caused by a chemical reaction between cement and water.

plain cement concrete has good compressive strength but very little tensile strength, thus limiting its use in construction.

- * Reinforced cement concrete (RCC) :- plain cement concrete has very low tensile strength. To improve the tensile strength of concrete, some sort of reinforcement is needed which can take up the tensile stresses developed in the structure.

The reinforcing steel is placed in the forms and fresh concrete is poured around it. This solidified composite mass is called as Reinforced cement concrete and is abbreviated

as R.C.C. Thus, Reinforced Cement Concrete is a composite material which is made up of concrete and steel reinforcement.

The Composite action of steel and concrete in a reinforced concrete section is dependent on the following important factors.

- (i) The Bond between steel and concrete.
- (ii) prevention of corrosion of steel bars embedded in the concrete.
- (iii) practically equal thermal expansion of both concrete and steel.

Uses of R.C.C.

1. R.C.C. has innumerable use in construction. Some are -
- 1) Buildings
 - 2) Flyovers.
 - 3) Water tanks.
 - 4) Road and Rail Bridges.
 - 5) Chimneys and Towers.
 - 6) Retaining wall.
 - 7) Bunkers and Silos.

* Difference between working stress method (WSM) and Limit state method (LSM):-

WSM	LSM
1) Low Load is used.	1) Ultimate Load is used
2) No warning signs.	2) There is warning sign.
3) Sudden collapse without cracking warning.	3) It is shown the cracking sign before collapse.
4) It is used where the load does not exceed exceed. ex- water tank	4) Designed load fluctuating loads. ex- Building, Bridge etc.
5) Allows only on elastic limit to yield point	5) Allows to cross elastic limit & enter the plastic limit.
6) Low amount of material used.	6) High amount of material used.
7) Low risk and the member can't be fail in future having large life span.	7) The member can be fail in future having large life span.
8) It is uneconomical	8) It is economical

⊗ partial FOS (γ_f) for steel = 1.78

FOS of concrete = 3 [for WSM]

⊗ partial FOS (γ_f) for steel in LSM

FOS of steel = 1.15

FOS of concrete = 1.5

Q1. A RCC Beam (300mm x 700mm) is reinforced with 4 no. bar dia 20mm. The Centre of bars are 50mm from the under side of the beam. The maximum stress are not to exceed 7N/mm^2 for concrete and 140N/mm^2 for steel. Find the M.O.R of the section and satisfied the safety distributed load that can carry that can span is 6m.

$m = 13$

Solⁿ Given that

$b = 300\text{ mm}$

$D = 700\text{ mm}$

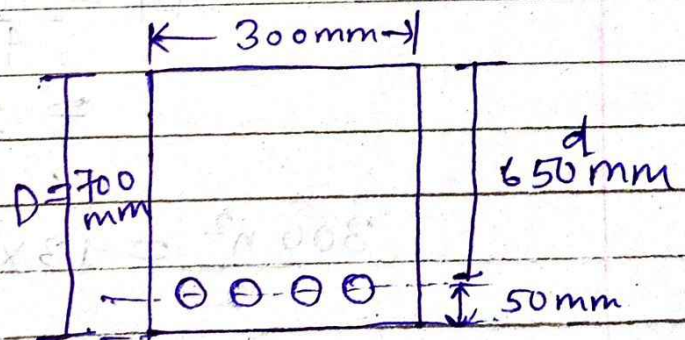
eff. Cover = 50mm

$d = 700 - 50 = 650\text{ mm}$

dia of bar = 20mm

No. of bar = 4

$\sigma_{st} = 140\text{ N/mm}^2$, $\sigma_{cbc} = 7\text{ N/mm}^2$



$$m = 13$$

$$L = 6m$$

⊛ Critical neutral axis (n_c)

$$\frac{m \cdot \sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d - n_c}$$

$$\frac{13 \times 7}{140} = \frac{n_c}{650 - n_c}$$

$$n_c = 256 \text{ mm}$$

⊛ Actual neutral axis (n)

$$\frac{b n^2}{2} = m \cdot A_{st} (d - n)$$

$$A_{st} = \frac{\pi}{4} \times 20^2 \times 4$$

$$= 1256.63 \text{ mm}^2$$

$$\frac{300 n^2}{2} = 13 \times 1256.63 (650 - n)$$

$$n = 217.12 \text{ mm}$$

Here,

$$n_c > n$$

So, under Reinforcement.

$$M_x = \sigma_{st} \cdot A_{st} \left(d - \frac{n}{3} \right)$$

$$M_x = 140 \times 1256.63 \left(650 - \frac{217}{3} \right)$$

$$M_x = 101627856.9 \text{ Nmm}$$

$$\boxed{M_x = 101.62 \text{ KNm}}$$

let $w \text{ KN/m}$ be the UDL

$$\text{maximum moment} = \frac{wl^2}{8} \quad (\text{UDL})$$

$$M_x = \frac{wl^2}{8}$$

$$101.62 = \frac{w \times 6^2}{8}$$

$$w = \frac{101.62 \times 8}{6^2}$$

$$w = 22.58 \text{ KN/m}$$

$$W = w_s + w_d$$

$$w_d = b \times D \times 1 \times 25$$

$$= \frac{300 \times 700 \times 1 \times 25}{1000 \times 1000}$$

$$w_d = 5.25 \text{ kN/m}$$

$$W = w_s + w_d$$

$$22.58 = w_s + 5.25$$

$$w_s = 22.58 - 5.25$$

$$w_s = 17.33 \text{ kN/m}$$

Q:2. A Rcc beam 250mm x 500mm is RIF with # 4-22mm ϕ bar with a clear span 25mm. The effective span is 5m. Find the concentrated load, the beam can support at the centre. Take M25 and Fe250 and $m=18$.

Sol/m:

Given Data,

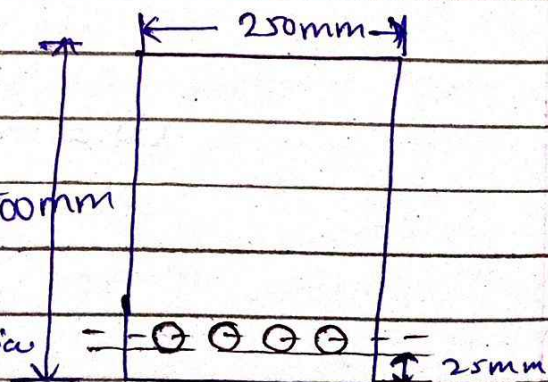
$$b = 250 \text{ mm}$$

$$D = 500 \text{ mm}$$

$$\text{clear cover} = 25 \text{ mm}$$

$$d = 500 - 25 - \frac{22}{2} \text{ bar dia} = 464 \text{ mm}$$

$$\boxed{d = 464 \text{ mm}}$$



$$m = 18$$

$$L = 5m$$

$$\sigma_{cbc} = 5 \text{ N/mm}^2$$

$$\sigma_{st} = 130 \text{ N/mm}^2$$

* Critical neutral axis (n_c)

$$\frac{m \cdot \sigma_{cbc}}{\sigma_{st}} = \frac{n_c}{d - n_c}$$

$$\frac{18 \times 5}{130} = \frac{n_c}{464 - n_c}$$

$$n_c = 189.8 \text{ mm}$$

* Actual neutral axis

$$\frac{bn^2}{2} = m \cdot A_{st} (d - n)$$

$$A_{st} = \frac{\pi \times 22^2 \times 4}{4}$$

$$\frac{250 n^2}{2} = 18 \times 1520.53 (464 - n)$$

$$= 1520.53 \text{ mm}^2$$

$$n = 227.53 \text{ mm}$$

Hence,

$$n > n_c$$

So, over R/F section.

$$M.O.R = \frac{1}{2} f_{cbc} \times b \times n \left(d - \frac{n}{3} \right)$$

$$= \frac{1}{2} \times 5 \times 250 \times 227.53 \left(464 - \frac{227.53}{3} \right)$$

$$M_x = 55198303.98 \text{ Nmm}$$

$$M_x = 55.2 \text{ KNm}$$

Let $w \text{ KN/m}$ be the UDL

$$\text{Maximum moment} = \frac{w l^2}{8} \quad (\text{UDL})$$

$$55.2 = \frac{w \times 5^2}{8}$$

$$w = \frac{55.2 \times 8}{5^2}$$

$$w = 17.664 \text{ KN/m}$$

$$w = w_s + w_d$$

$$w_d = b \times D \times 1 \times 25$$

$$= \frac{250}{1000} \times \frac{3000}{1000} \times 1 \times 25$$

$$w_d = 3.125 \text{ KN/m}$$

$$w = w_s + w_d$$

$$17.664 = w_s + 3.125$$

$$w_s = 14.539 \text{ kN/m}$$

Module 2: Limit State of collapse

* Assumptions in Limit State of collapse in flexure (Bending) (Refer C1.38, IS 456)

- ① plane sections normal to the axis remain plane after bending. It means that the strain at any point in the cross-section is proportional to the distance from the neutral axis.
- ② The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- ③ The relationship between the stress-strain distribution in concrete is assumed to be parabolic, for design purpose, the compressive strength f_{cd} of concrete is assumed to be 0.67 times the characteristic strength of concrete. The partial safety factor $\gamma_{mc} = 1.5$ shall be applied in addition to this

$$\text{Maximum compressive stress in concrete} = \frac{0.67 f_{ck}}{1.5}$$

4) Find ~~the~~ Moment of Resistance

(i) $\frac{x_u}{d} = \frac{x_{u\max}}{d}$ Balance section (Pg-96)

(ii) $\frac{x_u}{d} < \frac{x_{u\max}}{d}$ under-reinforced section.

(iii) $\frac{x_u}{d} > \frac{x_{u\max}}{d}$ over reinforced section

Q2. Determine the MOR of beam having 300mm x 550mm (effective). The beam is reinforced with 1963mm² of steel in tension Zone. Use M20 Concrete and Fe 415 and Fe 250 steel.

Solⁿ: Given data,

$$b = 300 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$A_{st} = 1963 \text{ mm}^2$$

$$f_y (\text{Fe}) = 415 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

for Fe 415 steel.

$$(i) \frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

$$x_u = \frac{0.87 \times 415 \times 1963}{0.36 \times 20 \times 300}$$

$$x_u = 328.12 \text{ mm}$$

(ii) Limiting Depth of neutral axis

$$\frac{x_{u\max}}{d} = 0.48$$

$$x_{u\max} = 0.48 \times 550$$

$$x_{u\max} = 264 \text{ mm}$$

Here

$$x_u > x_{u\max}$$

So,

Over Reinforced section

$$M_{ulim} = 0.36 \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d} \right) b d^2 f_{ck}$$

$$= 0.36 \times 0.48 \left(1 - 0.42 \times 0.48 \right) \times 300 \times 550^2 \times 20$$

$$M_{ulim} = 250403788.8 \text{ Nmm}$$

$$M_{ulim} = 250.40 \text{ kNm}$$

for fe 250

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

ii)
$$x_u = \frac{0.87 \times 250 \times 1963}{0.36 \times 20 \times 300}$$

$$x_u = 197.66 \text{ mm}$$

iii) Limiting Depth of neutral axis

$$\frac{x_{u\max}}{d} = 0.53$$

$$x_{u\max} = 0.53 \times 550$$

$$x_{u\max} = 291.5 \text{ mm}$$

$$x_{u\max} > x_u$$

Under R/F section

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$M_u = 0.87 \times 250 \times 1963 \times 350 \left(1 - \frac{1963 \times 250}{300 \times 550 \times 20} \right)$$

$$M_u = 199902718.4 \text{ Nmm}$$

$$M_u = 199.9 \text{ kNm}$$

Q2: A Rec beam (200 x 400) mm effective is reinforced with # 3 - 16 mm ϕ bars of Fe 415 steel. Find the ultimate uniformly distributed load which the beam can carry over a span of 5 m. Take M_{20} .

Soln: Given data,

$$b = 200 \text{ mm}$$

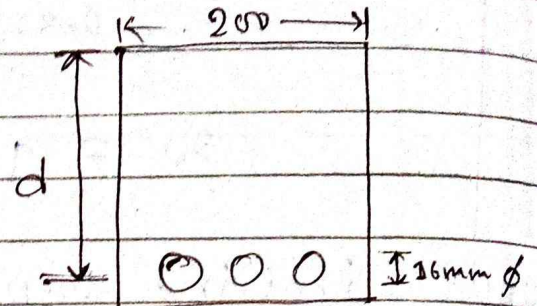
$$d = 400 \text{ mm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$L = 5 \text{ m}$$

$$\text{bar} = 16 \text{ mm } \phi (3)$$



$$A_{st} = \frac{\pi}{4} \times 16^2 \times 3 = 603.185 \text{ mm}^2$$

$$(i) \quad \frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b d}$$

$$x_u = \frac{0.87 \times 415 \times 603.185}{0.36 \times 20 \times 200}$$

$$x_u = 151.23 \text{ mm}$$

(ii) Limiting Depth of neutral axis ($x_{u\max}$)

$$\frac{x_{u\max}}{d} = 0.48$$

$$x_{u\max} = 0.48 \times 400$$

$$x_{u\max} = 192 \text{ mm}$$

$$x_{u\max} > x_u$$

So,

Under R/F section

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$= 0.87 \times 415 \times 603.185 \times 400 \left(1 - \frac{603.185 \times 415}{200 \times 400 \times 20} \right)$$

$$M_u = 73483212.15 \text{ N-mm}$$

$$M_u = 73.48 \text{ kNm}$$

Let $w \text{ kN/m}$ be the UDL on the beam

$$M = \frac{w l^2}{8}$$

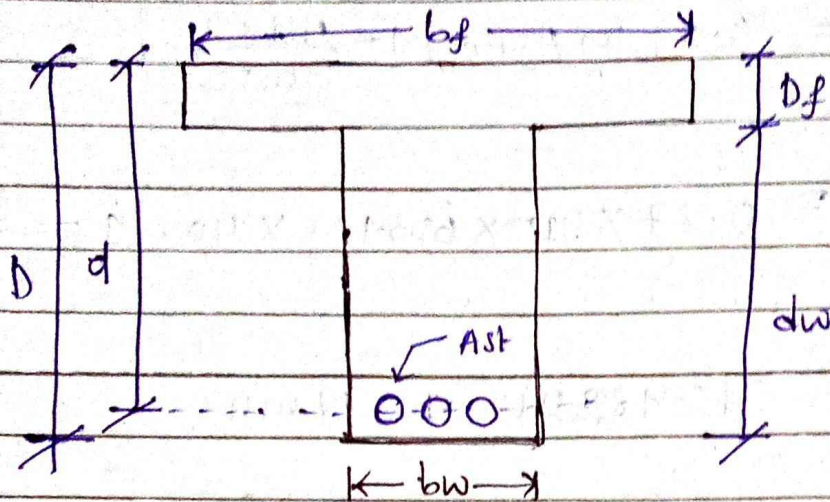
$$M = \frac{w \times 5^2}{8}$$

$$\frac{73.48 \times 8}{5^2} = w$$

$$w = 23.51 \text{ kN/m}$$

T-Beam

In RCC construction, slabs and beams are cast monolithically. In such construction, a portion of the slab acts integrally with the beam and bends along with the beam under the loads.



① Check the cross-sectional dimension →

Span Range

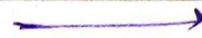
Span / Depth ratio (L/d)

3-4m



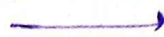
15-20

5-10m



12-15

>10m



12

* Adopt all extra dimensions.

② Effective span →

① centre to centre of bearing, (clear span + Bearing thickness)

$[D_f \geq (150 - 200) \text{ mm} \text{ lies always } \text{standard value}]$

(ii) Clear span + Effective depth = Effective span.

* Which one is less that taken for design.

(3) Loads $\rightarrow$ DL $\rightarrow$ (Self wt. of slab) $\times$ (Df $\times$ Spacing of 'T' beam $\times$ unit wt. of concrete).
IL $\rightarrow$ optional

$$DL = (b_w \times d_w \times \text{unit weight of concrete})$$

✓
(Self wt of rib)

* Design Ultimate Load $\rightarrow [1.5 \times (DL + IL)]$

(4) Ultimate moment $\rightarrow M_u = \frac{wl^2}{8}$

$$\text{Ultimate shear forces} = V_u = \frac{wl}{2}$$

(5) Effective width of flange (bf)

$$= \left[\frac{L_o}{6} + b_w + 6D_f \right] \quad [Pg - 37, IS 456]$$

* If $b_f <$ Spacing of T-beam (Hence ok)

* If $b_f >$ Spacing of T-beam (Re design).

~~* Balance shear (V_{us}) = [$V_u - (f_{cb} b_w d)$]~~

* Spacing of stirrups —

$$S_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}} \right]$$

6) Moment capacity of flange section →

$$M_{uf} = 0.36 f_{ck} b_f D_f (d - 0.42 D_f)$$

$$\text{If } M_{uf} < M_u \quad (\text{Redesigned})$$

$$\text{or } M_{uf} > M_u, \text{ then } b = b_f$$

For check the N.A

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

If $x_u < D_f$ then, max moment produced & the section treated as rectangular section for designing reinforcement.

or If $x_u > D_f$, then moment is less & section treated as square section.

7) Tension Reinforcement →

[From IS 456, Pg-96]

Determine the (A_{st}) (Total)

⑧ Shear reinforcement →

IS 456

- (i) Normal shear stress (τ_v) [Pg 72]
- (ii) Design shear strength of concrete (τ_c)
- (iii) percentage of steel (P_t) for web section.

* If $\tau_v > \tau_c$ [Shear reinforcement is required]
or If $\tau_v < \tau_c$ [Redesigned].

* Balance shear (V_{us}) = $[V_u - (\tau_c b w d)]$

* Spacing of stirrups —

$$s_v = \left[\frac{0.87 f_y A_{sv} d}{V_{us}} \right]$$

⑨ Check for deflection Control —

(i) percentage of steel (tension) $\left[P_t = \frac{100 A_{st}}{b f d} \right]$ for flange section.

(ii) Determine the K_f , K_e , K_f values from IS 456
pg - 38 & 39.

(iii) Determine $\left(\frac{L}{d}\right)_{max} = \left(\frac{L}{d}\right)_{basic} \times K_f \times K_e \times K_g$

or, $\left(\frac{L}{d}\right)_{provided}$

If $\left(\frac{L}{d}\right)_{max} > \left(\frac{L}{d}\right)_{provided}$, Hence OK

or, if $\left(\frac{L}{d}\right)_{max} < \left(\frac{L}{d}\right)_{provided}$, Hence redesigned.

Q1) A T-beam slab floor of reinforced concrete has a slab 150mm thick spanning b/w the T-beam which are spaced 3m apart. The beam have a clear span of 10m and the end bearing are 450mm thick walls.

$$L.L = 4 \text{ kN/m}^2$$

M20 ~~Concrete~~ Concrete

Fe 415

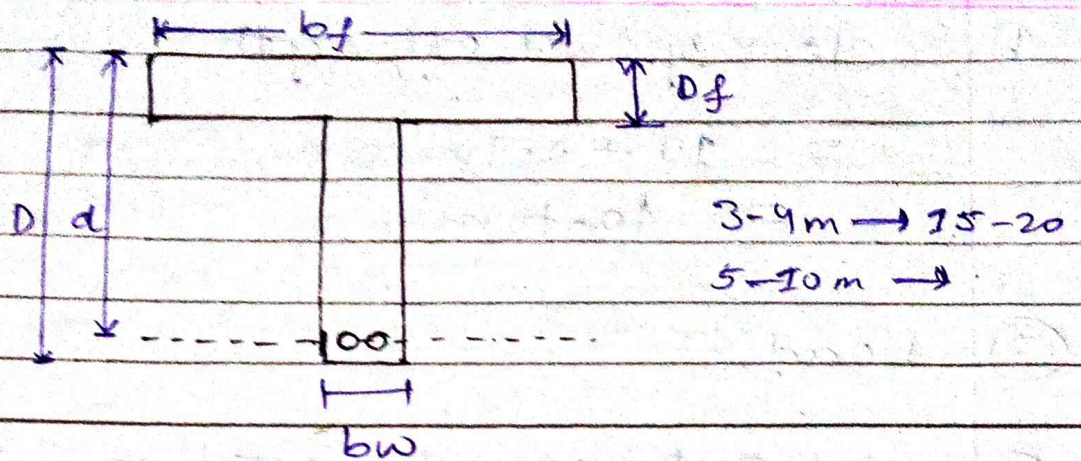
Soln:

$$D_f = 150 \text{ mm}$$

$$\text{Spaced} = 3 \text{ m}$$

$$\text{clear span} = 10 \text{ m}$$

$$\text{bearing} = 450 \text{ mm}$$



① Cross-sectional dimensional

Assuming effective depth (d)

$$= \left(\frac{\text{Span}}{15} \right) = \frac{10 \times 10^3}{15} = 666.66 \approx 700 \text{ mm}$$

Assuming effective cover = 50 mm

$$\text{breadth of the web} = \frac{700}{2} = 350 \approx 300 \text{ mm}$$

Take = 300 mm

$$\text{So, } bw = 300 \text{ mm}$$

$$d = 700 \text{ mm}$$

$$D = 700 + 50 = 750 \text{ mm}$$

② Effective span

① c/c of bearings

$$= (\text{clear span} + \text{bearing thickness})$$

$$= 10 \text{ m} + 0.45 \text{ m}$$

$$= \underline{\underline{10.45 \text{ m}}}$$

② Clear span + eff. depth

$$= 10 + 0.70$$

$$= 10.70 \text{ m}$$

③ Load ÷

① D.L = $D_f \times \text{spacing of T beam} \times \text{unit wt of concrete}$

$$= 0.15 \times 3 \times 25$$

$$D.L = 11.25 \text{ kN/m}$$

② Self wt of rib (web)

$$= (b_w \times d_w \times 25)$$

$$= 0.3 \times 0.6 \times 25$$

$$= 4.5 \text{ kN/m}$$

$$d_w = 750 - 150$$

$$= 600 \text{ mm}$$

$$= 0.6 \text{ m}$$

$$\text{Total Load} = 11.25 + 4.5$$

$$= 15.75 \text{ kN/m}$$

$$\text{Imposed load} = 4 \times \text{③} \text{ — spacing of T beam}$$

$$= 12 \text{ kN/m}$$

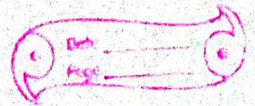
④ Design ultimate load (w_u)

$$= (I.L + D.L) \times 1.5$$

$$= (12 + 15.75) \times 1.5$$

$$= 41.625 \approx 45 \text{ kN/m}$$

$$w_u = 45 \text{ kN/m}$$



④ Ultimate moment & shear force :-

$$\begin{aligned}\text{a) Ultimate moment (Mu)} &= \frac{wl^2}{8} \\ &= \frac{45 \times (10.45)^2}{8} \\ &= 614.26 \text{ KN-m}\end{aligned}$$

$$\begin{aligned}\text{b) Shear force (Vu)} &= \frac{wl}{2} \\ &= \frac{45 \times 10.45}{2} \\ &= 2457.05 \text{ KN-m} \\ &= 235.125 \text{ KN}\end{aligned}$$

⑤ Effective width of flange (b_f)

$$\begin{aligned}&= \frac{L_0}{6} + b_w + 6d_f \quad \left[\begin{array}{l} \text{IS 456 23.1.2} \\ \text{Pg-37} \end{array} \right] \\ &= \frac{10.45}{6} + 0.300 + 6 \times 0.15 \\ &= 2.94 \text{ m}\end{aligned}$$

⑥ Ultimate moment capacity of flange section:

$$\begin{aligned}M_{uf} &= 0.36 f_{ck} b_f d_f (d - 0.42 d_f) \\ &= 0.36 \times 20 \times 2.94 \times 0.15 (0.700 - 0.42 \times 0.15) \\ &= 0.36 \times 20 \times 2940 \times 150 (700 - 0.42 \times 150) \\ &= 2022602400 \text{ Nmm} \\ &= 2022 \times 10^6 \text{ Nmm} \\ M_{uf} &= 2022 \text{ KNm}\end{aligned}$$

⑦ Tension Reinforcement

IS 456 Pg-36.
6-2.1, 6-1.1

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$614.26 \times 10^6 = 0.87 \times 415 \times A_{st} \times 700 \left(1 - \frac{A_{st} \times 415}{2940 \times 700 \times 20} \right)$$

$$A_{st} = 2493 \text{ mm}^2$$

Actual neutral axis

$$M_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b f}$$

$$M_u = \frac{0.87 \times 415 \times 2493}{0.36 \times 20 \times 2490}$$

$$x_u = 50.20 \text{ mm}$$

Hence,

$$D_f > x_u$$

$$A_{st} = 2493 \text{ mm}^2$$

Hence,

provide 2 bar of 32mm ϕ & 2 bar of 25mm ϕ .

$$a) \quad 2 \times \frac{\pi}{4} \times 32^2 = 1608.495 \text{ mm}^2$$

$$b) \quad 2 \times \frac{\pi}{4} \times 25^2 = 981.747 \text{ mm}^2$$

$$\therefore \text{Total } A_{st} = 1608.49 + 981.75 \\ = 2590 \text{ mm}^2$$

⑧ Shear Reinforcement

a) Nominal shear stress (τ_v) = $\frac{V_u}{bd}$ [IS 456 Pg-72: 40.1]

$$\tau_v = \frac{V_u}{bd}$$

$$\Rightarrow \frac{235.125 \times 10^3}{300 \times 700}$$

$$\Rightarrow 1.119 \text{ N/mm}^2$$

b) percentage of steel (ρ) [IS 456 Pg-73 Table 19]

$$\rho = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 2590}{300 \times 700}$$

$$= 1.23\%$$

Interpolating.

$$\tau_c = 0.62 + \left(\frac{0.67 - 0.62}{1.25 - 1.00} \right) \cdot (1.23 - 1)$$

$$\tau_c = 0.66 \text{ N/mm}^2$$

③ * $\tau_{c \max} = 2.8$ [IS 456 Table 20]
Pg-73
for M20 concrete.

$$\tau_{c \max} > \tau_c$$

$$\tau_v > \tau_c$$

Hence,

shear reinforcement is required

④ Spacing of stirrup

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

IS 456 Pg-73
@ point vertical stirrup

$$V_{us} = V_u - \tau_c b d$$

$$= 235.125 \times 10^3 - 0.66 \times 300 \times 700$$

$$= 96525 \text{ N}$$

$$= 96.5 \text{ kN}$$

using 8mm ϕ 2 legged stirrup

$$A_{sv} = \frac{\pi \times 8^2 \times 2}{4} = 100.53 \text{ mm}^2$$

$$A_{sv} = 100.53 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 100.53 \times 700}{91.5 \times 10^3}$$

$$= 263.28 \text{ mm} \approx 250 \text{ mm}$$

Assuming 250mm spacing.

- (i) Max spacing = 300mm
- (ii) $0.75 d = 0.75 \times 700 = 525 \text{ mm}$
- (iii) 250mm

Take minimum spacing from above the following.

So, provide spacing of 250mm at support & gradually increase to 300mm centre.

Design of Singly Reinforced Beam

Introduction

The aim of design is to decide the size or dimension of the beam and provide suitable reinforcement. The section designed should perform safely during its life time. Beam is a flexural member which resists loads mainly by bending. Singly reinforced beam have steel reinforcement in tension zone but in doubly reinforced beam steel reinforcement is provided in the tension as well as compression zone.

Type-1. Determining area of tensile steel if b , d and M_u are given

1. Determining $x_{u\max}$ [Pg-70 IS 456]

2. Determining the limiting moment of resistance $M_{u\lim}$.

$$M_{u\lim} = 0.36 f_{ck} \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d} \right) b d^2$$

[Pg-96]

IS 456

③ Compare M_u and $M_{u\lim}$

(i) $M_u = M_{u\lim}$ the section is balanced and the area of steel can be found as

$$A_{st} = \frac{M_u}{0.87 f_y \left(d - 0.42 x_{u\max} \right)}$$

or

$$A_{st} = \frac{0.36 f_{ck} b x_{u\max}}{0.87 f_y}$$

(ii) $M_u < M_{u\lim}$, the section is under Reinforced and the area of steel is calculated by

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right) \quad \left[\begin{array}{l} \text{IS 456} \\ \text{Pg-56} \end{array} \right]$$

(iii) $M_u > M_{u\lim}$ the section is over reinforced and ~~hence~~ hence should be redesigned.

Q1: Determine the area of tensile reinforcement required for singly reinforced simply supported beam of size $300\text{mm} \times 600\text{mm}$ effective to resist a factored bending moment of 220 kNm . use M20 concrete and Fe 500 grade steel.

Soln:- Given data, $b = 300\text{mm}$
 $d = 600\text{mm}$
 $M_u = 220\text{ kNm}$
 $f_{ck} = 20\text{ N/mm}^2$, $f_y = 500\text{ N/mm}^2$

* ii Depth required for balance section

$$d = \sqrt{\frac{M_u}{R_u \cdot b}}$$

$$R_u = 0.36 f_{ck} \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d} \right)$$

$$= 0.36 \times 20 \times 0.46 \left(1 - 0.42 \times 0.46 \right)$$

$$R_u = 2.67$$

$$d = \sqrt{\frac{M_u}{R_u \cdot b}}$$

$$= \sqrt{\frac{220 \times 10^6}{2.67 \times 300}}$$

$$d = 524.07 \approx 525 \text{ mm}$$

$$d = 525 < 600 \text{ mm}$$

Hence, it is under reinforced section

(ii) Area of tensile reinforcement (A_{st})

$$M_u = 0.87 f_y A_{st} \cdot d \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right)$$

$$220 \times 10^6 = 0.87 \times 500 A_{st} \times 600 \left(1 - \frac{500 \times A_{st}}{20 \times 300 \times 600} \right)$$

$$A_{st} = 975 \text{ mm}^2$$

Q2. Design a balanced section for a reinforced concrete beam subjected to a bending moment of 150 kNm. Use M20 concrete and Fe 415 steel.

Solⁿ:

$$\text{Factored B.M} = 1.5 \times 150 = 225 \text{ kNm}$$

$$\text{Depth required for balanced section} = d = \sqrt{\frac{M_u}{R_u \cdot b}}$$

$$\text{Assuming } b = 300 \text{ mm}, R_u = 2.76$$

$$d = \sqrt{\frac{225 \times 10^6}{2.76 \times 300}}$$

$$= 521 \approx 525 \text{ mm}$$

$$\text{Total depth} = 525 + 25 + 10 = 560 \text{ mm}$$

(i) Area of steel required

$$\left[\begin{array}{l} \text{Cover} = 25 \text{ mm} \\ \text{dia bar} = \frac{20}{2} = 10 \end{array} \right]$$

$$M_u = 0.87 f_y A_{st} \cdot d \left(1 - \frac{A_{st} \cdot f_y}{f_{ck} b d} \right)$$

$$225 \times 10^6 = 0.87 \times 415 \times A_{st} \times 525 \left(1 - \frac{415 A_{st}}{20 \times 300 \times 525} \right)$$

$$A_{st} = 1480 \text{ mm}^2$$

using 20mm dia bars

$$\text{No. of bars} = \frac{1480}{\frac{\pi}{4} \times 20^2} = 5 \text{ No.}$$

Q3. An RCC beam is required to resist a bending moment of 70 kNm. Design the beam for flexure, taking $\frac{b}{d} = 0.5$. Use M20 concrete and TMT (Fe 415 bars)

Soln.

Given data,

$$M = 70 \text{ kNm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\frac{b}{d} = 0.5 \Rightarrow b = 0.5d$$

(i) Design or factored moment

$$M_u = M \times \gamma_f = 70 \times 1.5$$

$$= 105 \text{ kNm}$$

$$= 105 \times 10^6 \text{ Nmm}$$

(ii) Effective depth required

$$R_u = 0.36 f_{ck} \frac{x_{u \max}}{d} \left(1 - 0.42 \frac{x_{u \max}}{d} \right)$$

$$\frac{x_{u \max}}{d} = 0.48$$

$$R_u = 0.36 \times 20 \times 0.48 \left(1 - 0.42 \times 0.48 \right)$$

$$R_u = 2.76$$

$$M_u = R_u \cdot b \cdot d^2$$

$$105 \times 10^6 = 2.76 \times 0.5d \cdot d^2$$

$$105 \times 10^6 = 2.76 \times 0.5d^3$$

$$d = \sqrt[3]{\frac{105 \times 10^6}{2.76 \times 0.5}}$$

$$d = 425 \text{ mm}$$

$$b = \frac{425}{2}$$

$$b = 0.5 \times 425$$

$$b = 212.5 \approx 215 \text{ mm}$$

$$\boxed{d = 425 \text{ mm}} \quad \boxed{b = 215 \text{ mm}}$$

(iii) Area of steel (A_{st})

$$A_{st} = \frac{M_u}{0.87 f_y (d - 0.42 x_{u\max})}$$

$$A_{st} = \frac{105 \times 10^6}{0.87 \times 415 (425 - 0.42 \times 204)}$$

$$\begin{aligned} x_{u\max} &= 0.48d \\ &= 0.48 \times 425 \\ &= 204 \text{ mm} \end{aligned}$$

$$A_{st} = 857.06 \text{ mm}^2$$

using 20 mm ϕ bar

$$\text{No. of bar} = \frac{857}{\frac{\pi \times 20^2}{4}}$$

$$= 3$$

∴ provide 3-20 mm ϕ bars

Assuming effective cover = 45 mm

$$\text{Overall depth of beam} = 425 + 45 = 470 \text{ mm}$$

∴ Beam dimension = 215 mm $\times$ 470 mm

Q4. A rectangular reinforced concrete beam is simply supported on two masonry walls 230 mm thick and 6 m apart (c/c). The beam is carrying an imposed load of 15 kN/m. Design the beam with all necessary checks. Use M25 concrete and Fe415 steel.

Soln Given, $l_{ec} = 6 \text{ m}$

Imposed load = 15 kN/m

$f_{ck} = 25 \text{ N/mm}^2$ [For M25 concrete]

$f_y = 415 \text{ N/mm}^2$ [For Fe 415 steel]

* (i) Assuming a trial total depth of 500 mm | $\frac{l}{12} = \frac{6000}{12}$

$\Rightarrow 500 \text{ mm}$

$$b = 250 \text{ mm}$$

$$\text{Effective depth, } d = 500 - 50 = 450 \text{ mm}$$

[Assuming effective cover = 50 mm]

(iii) Effective span (l):- The effective span is least of the following.

(a) Centre to centre of support = 6.0 m

(b) Clear span + d = $5.77 + 0.45$ [Clear span = $6 - 0.23$ = 5.77 m]
= 6.22 m

So, $l = 6$ m

(iii) Design load (w_u) and factored moment (M_u)

@ Self weight of beam = $D \times b \times \text{unit wt. of RCC}$
= $0.500 \times 0.25 \times 25$
= 3.125 kN/m

Imposed load = 15 kN/m

Total load, $w = 18.125$ kN/m

Design load = $w \times 1.5$
= 18.125×1.5

$w_u = 27.18$ kN/m

$$M_u = \frac{w_u \cdot l^2}{8} = \frac{27.18 \times 6^2}{8}$$

$$M_u = 122.3 \text{ kNm}$$
$$= 122.3 \times 10^6 \text{ Nmm}$$

(iv) Minimum effective depth required

$$\frac{x_{u\max}}{d} = 0.48$$

$$R_u = 0.36 f_{ck} \frac{x_{u\max}}{d} \left(1 - 0.42 \frac{x_{u\max}}{d} \right)$$

$$R_u = 0.36 \times 25 \times 0.48 (1 - 0.42 \times 0.48)$$

$$R_u = 3.45$$

$$d_{req} = \sqrt{\frac{M_u}{R_u \cdot b}}$$

$$= \sqrt{\frac{122.3 \times 10^6}{3.45 \times 250}}$$

$$d_{req} = 376 \text{ mm}$$

376 mm < 450 mm. Hence ok,

Since the depth of the section is more than the depth required for balance section so, the section is designed as under reinforced section.

Adopt $D = 500 \text{ mm}$ and $b = 250 \text{ mm}$

$$d = 500 - 20 - 8 - \frac{20}{2} = 462 \text{ mm}$$

[Assuming ~~clear~~ clear cover = 20 mm, 8 mm ϕ bar stirrup and 20 mm ϕ bar main bar.]

(iv) Area of steel required

for under R/F section area of steel required is calculated as follows:-

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right)$$

$$M_u = 0.87 \times 415 \times A_{st} \times 462 \left(1 - \frac{415 \times A_{st}}{250 \times 462 \times 25} \right)$$

$$122.3 \times 10^6 = 0.87 \times 415 \times A_{st} \times 462 \left(1 - \frac{415 \times A_{st}}{250 \times 462 \times 25} \right)$$

$$A_{st} = 833 \text{ mm}^2$$

(vi) Minimum area of steel (A_s)

$$A_s = \frac{0.85 b d}{f_y} = \frac{0.85 \times 250 \times 462}{415}$$

$$A_s = 236 \text{ mm}^2$$

using 20 mm dia bar

$$\text{No. of bars} = \frac{A_{st}}{A_{\phi}}$$

$$= \frac{833}{\frac{\pi}{4} \times 20^2} = 2.65 \approx 3$$

∴ provide 3-20 mm dia bars,

$$A_{st} \text{ provided} = 3 \times 314 \\ = 942 \text{ mm}^2$$

(vii) Check for deflection

$$pt = \frac{100 \times 942}{250 \times 462} = 0.8\%$$

IS 456: 2000

$$\left[pt = \frac{100 A_{st}}{bd} \right]$$

Pg-73

$$f_s = 0.58 f_y \left[\frac{A_{st \text{ reqd}}}{A_{st \text{ provided}}} \right]$$

$$\left[\begin{array}{l} \text{IS 456: 2000} \\ \text{Pg-38} \end{array} \right]$$

$$= 0.58 \times 415 \left[\frac{833}{942} \right]$$

$$= 212 \text{ N/mm}^2$$

Interpolating for $f_s = 212 \text{ N/mm}^2$ and $pt = 0.8\%$

$$\left[\begin{array}{l} \text{for } f_s = 190, k_t = 1.35 \\ f_s = 240, k_t = 1.2 \end{array} \right]$$

$$k_t = 1.35 + \left(\frac{1.35 - 1.2}{190 - 240} \right) \times (212 - 190)$$

$$= 1.29$$

$$\left(\frac{l}{d} \right)_{\max} = 20 \times 1.29 = 25.8$$

[Pg-37]

$$\frac{(l/d)}{d}_{provided} = \frac{6000}{462} = 12.9$$

$$\left(\frac{l}{d}\right)_{max} > \left(\frac{l}{d}\right)_{provided} \text{ hence ok}$$

viii) Design for shear

$$V_u = \frac{W_u \cdot L}{2} = \frac{27.2 \times 5.77}{2} \quad \left[L = \text{clear span} = 5.77 \text{ m} \right]$$

$$= 78.5 \text{ kN} = 78500 \text{ N}$$

Nominal shear stress (τ_v)

$$\tau_v = \frac{V_u}{bd} = \frac{78500}{250 \times 462}$$

$$\tau_v = 0.68 \text{ N/mm}^2$$

Design shear strength of concrete (τ_c)

$P_t = 0.8\%$ and M25 concrete

$$\tau_c = 0.57 + \left(\frac{0.64 - 0.57}{1.00 - 0.75} \right) \times (0.8 - 0.75)$$

$$\tau_c = 0.58 \text{ N/mm}^2$$

[IS 456:2000]
Pg-73
Table 19

$$\tau_{cmax} = 3.1 \text{ N/mm}^2$$

[IS 456:2000]
Pg-73
Table 20

$$\tau_{\max} > \tau_c$$

$\therefore \tau_v > \tau_c$, hence shear reinforcement is required

Shear to be carried by reinforcement = V_{us}

$$V_{us} = V_u - \tau_c b d$$

$$= 78500 - 0.58 \times 250 \times 462$$

$$V_{us} = 11510 \text{ N}$$

Using 8 mm ϕ 2 legged stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$\text{Spacing of stirrup, } S_v = \frac{0.87 f_y A_{sv} \cdot d}{V_{us}}$$

$$S_v = \frac{0.87 \times 415 \times 100.53 \times 462}{11510}$$

$$S_v = 1456 \text{ mm}$$

Maximum spacing as per minimum reinforcement

$$S_v = \frac{0.87 A_{sv} \cdot f_y}{0.46}$$

$$= \frac{0.87 \times 100.53 \times 415}{0.4 \times 250} = 362 \text{ mm}$$

The spacing should be least of the following

(i) $0.75d = 0.75 \times 462 = 346 \text{ mm}$

(ii) 300 mm

(iii)

$\therefore \text{Spacing } s_v = 300 \text{ mm}$

provide $8 \text{ mm } \phi 2 \text{ lgd @ } 300 \text{ mm c/c}$ throughout the length of the beam.

provide $2-10 \text{ mm } \phi$ anchor bar in the compression zone.

(ix) Check for development length

$$M_1 = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$M_1 = 0.87 \times 415 \times 942 \times 462 \left(1 - \frac{942 \times 415}{250 \times 462 \times 25} \right)$$

$$M_1 = 135856988 \text{ N}$$

$$V_u = 78500 \text{ N}$$

using no bend or hook, $l_o \approx 0$

$$\frac{M_1}{V_u} + l_o = \frac{135856988}{78500} + 0$$
$$= 1730 \text{ mm}$$

$$L_d = \frac{0.87 f_y}{4 \tau_{bd}} = \frac{20 \times 0.87 \times 415}{4 \times 1.6 \times 2.4}$$

$$= 0.87 \times 20 \quad L_d = 805.9 \text{ mm}$$

$$L_d = 805.9 \text{ mm}$$

$\frac{M_1}{V} + l_o > L_d$. Hence, code requirement are satisfied.

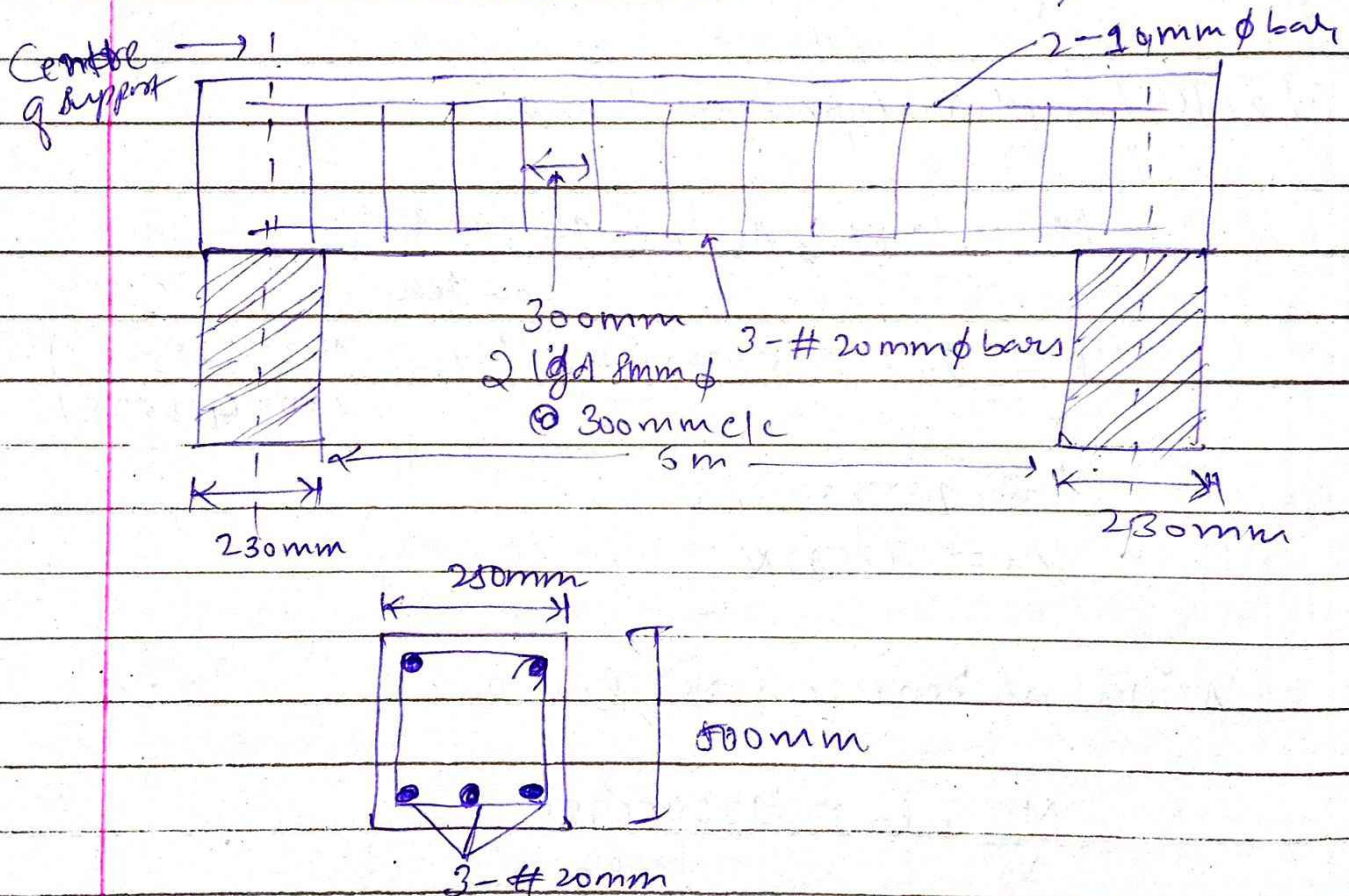
(xi) Design Summary.

Beam Size = 250mm X 500mm

Main tensile bars = 3-20mm ϕ bars of Fe 415

Shear stirrup = 8mm ϕ 2 lgd @ 300mm c/c

Anchor bars = 2-10mm ϕ



Introduction

Slab is a two-dimensional or planar element, used in all types of structure such as floors and roof coverings. The thickness of slab is very small as compared to its length and width. Slabs are classified on the basis of l_y/l_x ratio i.e., length of the longer span (l_y) / length of the shorter span (l_x) into following two types:-

- (i) One way slab.
- (ii) Two way slab.

One way slab.

One way slab are those ~~which~~ slabs in which l_y/l_x ratio is greater than 2.

This is also called slab spanning in one direction as the bending takes place only ~~one~~ at shorter span. The main reinforcement is provided along the shorter span.

One way slab is analysed by assuming it to be a beam of 1m width.

Two way slab

The slab which is supported on all four edges and having l_y/l_x ratio less than or equal to 2. The slab spanning in two direction and bending takes place in both the directions. The main reinforcement is provided along both the direction in two way slab.

* Difference between one way slab and two way slab.

one way slab	Two way slab
1) $l_y/l_x \geq 2$	1) $l_y/l_x \leq 2$
2) The bending takes place in one direction only i.e. shorter span	2) The bending takes place in both the direction.
3) Depth required is more.	3) Depth required is less.
4) Main steel reinforcement is provided along shorter span.	4) Main steel reinforcement is provided along both span.
5) Less economical as thickness is more and the amount of steel is also more.	5) More economical as the thickness of slab is less and the amount of steel required is less.

* Design Steps of One way slab.

- ① Cross sectional dimension.
- ② Determine the effective span.
- ③ Design load and shear force & moment determination.
- ④ Effective depth required.

$$d_{req} = \sqrt{\frac{M_u}{R_u \cdot b}}$$

- ⑤ Area of tensile reinforcement (A_{st})
[IS 456:2000 Pg 96 Cr-1.1]

- ⑥ Determination of bent-up bars.
- ⑦ Distribution steel determination.
- ⑧ check for shear.
- ⑨ check for deflection.
- ⑩ Check for development length.

Date: _____
Page: _____

Q1.) Design a simply supported slab for a Room $7.5\text{m} \times 3.5\text{m}$ clear in size and is supported on beams 200mm width. The slab is carrying an imposed load 5KN/m^2 . Use M20 mix and Fe415 steel.

Soln:
Given,
Dimension of slab

$$7.5\text{m} \times 3.5\text{m}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$f_y = 415\text{N/mm}^2$$

$$\text{Imposed load} = 5\text{KN/m}^2$$

$$\text{width of beam} = 200\text{mm}$$

(i) (a) Check it is one way or two way slab

$$= \frac{L_y}{L_x} = \frac{7.5}{3.5} = 2.14\text{m} > 2$$

hence, it is one way slab.

(b) Assuming total depth of slab

$$= 150\text{mm}$$

} Assume clear cover $= 20\text{mm}$
 ϕ bar $= 10\text{mm}$

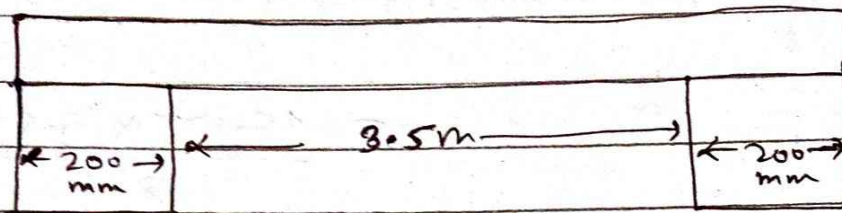
$$d = \frac{150 - 20 - 10}{2}$$

$$d = 125\text{mm} (d)$$

(ii)

Eff. span

- ② centre to centre distance
(clear span + Bearing wall)



$$3.5 + 0.2 = 3.7 \text{ m}$$

$$= 3.62$$

- ⑤ clear span + eff depth

$$= (3.5 + 0.125) \text{ m}$$

$$= 3.625 \text{ m} < 3.7$$

Take least value

$$\text{So, eff. span} = 3.625 \text{ m}$$

(iii) Design load shear force & moment :-

① self weight of slab (w_u) $= (0.15 \times 25)$
 $= 3.75 \text{ kN/m}$

② Imposed load $= 5 \text{ kN/m}$

③ Total load $= 3.75 + 5 \text{ kN}$
 $= 8.75 \text{ kN/m}$

④ Design load $= (8.75 \times 1.5)$
 $[w_u] = 13.125 \text{ kN/m}$

⑤ Shear force (V_u) $= \frac{w_u \cdot l}{2}$

$$V_u = \frac{13.125 \times 3.5}{2}$$

$$= 23.789 \text{ N} \quad 22.968 \text{ kN}$$

$$= 23789 \text{ N} \quad 22970 \text{ N}$$

③ Factored Moment (M_u) = $\frac{W_u \cdot l^2}{8}$

$$= \frac{13.125 \times (3.625)^2}{8}$$

$$= 21.56 \text{ kNm}$$

$$= 21.56 \times 10^6 \text{ Nmm}$$

④ Eff. depth required —

$$\text{Depth required (} d_{\text{req}} \text{)} = \sqrt{\frac{M_u}{R_u \cdot b}}$$

$$R_u = 0.36 f_{ck} \frac{m_{\text{max}}}{d} \left(1 - 0.42 \frac{m_{\text{max}}}{d} \right)$$

$$\frac{m_{\text{max}}}{d} = 0.48$$

[For M20 Concrete]
Pg - 70 IS 456]

$$R_u = 0.36 \times 20 \times 0.48 \cdot (1 - 0.42 \times 0.48)$$

$$= 2.76$$

$$(d_{\text{req}}) = \sqrt{\frac{21.56 \times 10^6}{2.76 \times 1000}}$$

$$d_{\text{req}} = 88 \text{ mm}$$

Hence $\phi > d_{req}$

(v) Area of tensile steel (A_{st})

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b d} \right)$$

$$21.6 \times 10^6 = 0.87 \times 415 \times A_{st} \times 125 \left(1 - \frac{A_{st} \times 415}{20 \times 1000 \times 125} \right)$$

$$A_{st} = 524.4 \text{ mm}^2$$

Using 10 mm dia bars

$$A_{\phi} = \frac{\pi}{4} \times 10^2 = 78.5 \text{ mm}^2$$

$$\text{Spacing of 10mm dia bar} = \frac{100 \times A_{\phi}}{A_{st}}$$

$$= \frac{1000 \times 78.5}{523.4}$$

$$= 150 \text{ mm}$$

$\therefore$ provide 10mm ϕ @ 150 mm c/c

This spacing is less than

(i) $3d = 3 \times 125 = 375 \text{ mm}$

(ii) 300 mm

Bending alternate bars at 510mm $\left(\frac{l}{7} = \frac{3625}{7} = 517 \text{ mm} \right)$

from centre of support or 410mm from the face of support.

(vi) Distribution steel

Distribution & reinforcement is provided in the longer direction i.e 7.5 m

$$\geq 0.12\% \text{ of } \times \text{ sectional area}$$

$$\geq \frac{0.12}{100} \times 1000 \times 150 = 180 \text{ mm}^2$$

Using 6mm ϕ bar

$$A_{\phi} = \frac{\pi}{4} \times 6^2 = 28.3 \text{ mm}^2$$

$$\therefore \text{Spacing of 6mm } \phi \text{ bar} = \frac{1000 \times A_{\phi}}{A_{st}}$$

$$= \frac{1000 \times 28.3}{180} = 157.2 \text{ mm}$$

$\therefore$ provide 6mm ϕ @ 150mm c/c in the longer direction.

(vii) Check for shear

$$\text{Factored Shear force} = V_u = \frac{w_u \cdot L}{2}$$

$$= \frac{13.125 \times 3.5}{2}$$

$$= 22.97 \text{ kN}$$

$$= 22.97 \text{ kN}$$

(viii) Nominal shear stress, τ_v

[IS 456:2000]
Pg-83

$$\tau_v = \frac{V_u}{bd} = \frac{22970}{1000 \times 125} = 0.18 \text{ N/mm}^2$$

(ix) Design shear strength of concrete (τ_c)

[IS 456:2000]
Pg-73

$$P_t = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 262.15}{1000 \times 125}$$

$$= 0.21 \%$$

$$A_{st} = \frac{524.3}{2}$$

$$= 262.15$$

~~For~~ [from IS 456:2000 Pg-73 Table 19]

$$\tau_c = 0.28 + \left(\frac{0.36 - 0.28}{0.25 - 0.15} \right) (0.21 - 0.15)$$

$$\tau_c = 0.328 \text{ N/mm}^2$$

$\therefore \tau_v < \tau_c$. Hence OK

(x) Check for deflection

$$P_f = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 524}{1000 \times 125} = 0.4 \%$$

$$f_s = 0.58 f_y \left[\frac{A_{st \text{ reqd}}}{A_{st \text{ provide}}} \right]$$

$$= 0.58 \times 425 \left[\frac{524}{524} \right]$$

$$f_s = 240 \text{ N/mm}^2$$

For $P_t = 0.4\%$, $f_s = 240 \text{ N/mm}^2$, $K_f = 1.55$
from [IS 456:2000 Pg-38 fig 4].

$$\left(\frac{l}{d} \right)_{\max} = 20 \times K_f = 20 \times 1.55 = 31$$

$$\left(\frac{l}{d} \right)_{\text{provided}} = \frac{3625}{125} = 29$$

$$\left(\frac{l}{d} \right)_{\max} > \left(\frac{l}{d} \right)_{\text{provided}} \text{ hence OK}$$

(X1) check for development length.

Moment of resistance at support by 20 mm ϕ bars @ 300 mm c/c = M_1

$$A_{st} = \frac{524}{2} = 262 \text{ mm}^2$$

$$M_1 = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} b d} \right) \quad \left[\text{IS 456:2000 Pg-96} \right]$$

$$= 0.87 \times 415 \times 262 \times 125 \left(1 - \frac{415 \times 262}{20 \times 1000 \times 125} \right)$$

$$M_1 = 11.31 \times 10^6 \text{ Nmm}$$

$$V_u = 22970 \text{ N}$$

providing no hooks

$$l_0 = 0$$

$$\left[\begin{array}{l} \text{IS 456:2000} \\ \text{Pg-44} \\ \text{Clause 26.2.3.3} \end{array} \right] \frac{M_1}{V} + l_0 = \frac{11.31 \times 10^6}{22970} + 0 = 492.38 \approx 500 \text{ mm}$$

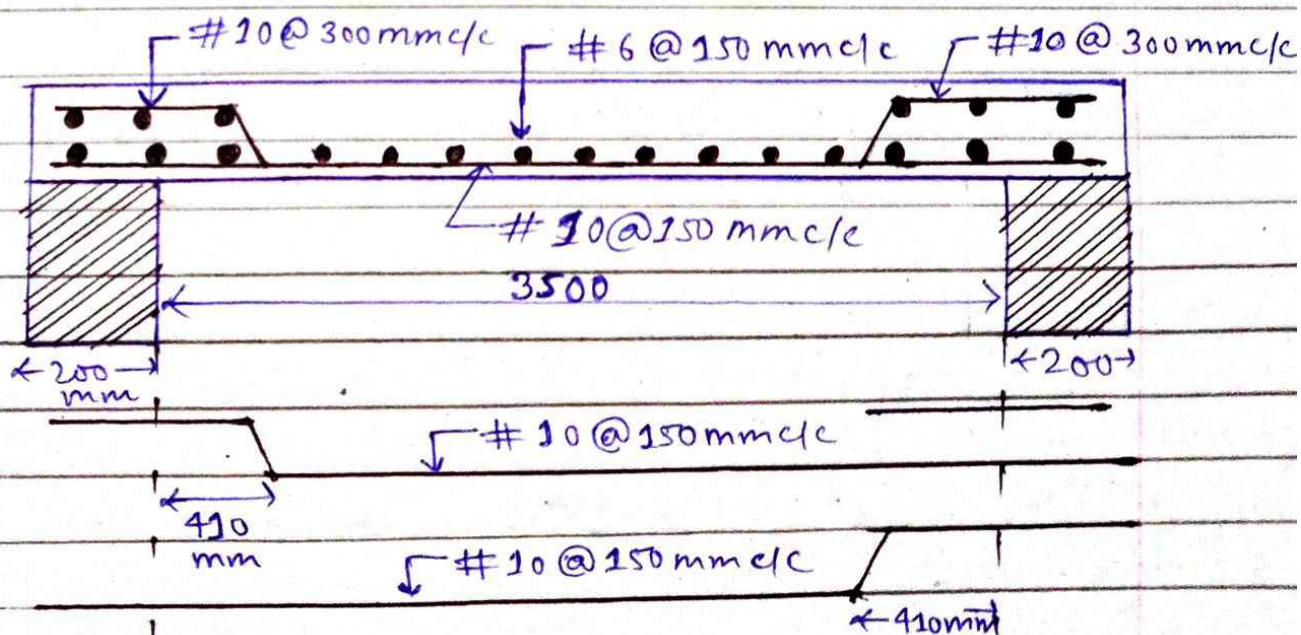
$$\left(\begin{array}{l} (6s \geq 0.87 f_y) \\ \text{Pg-42} \end{array} \right) L_d = \frac{(0.87 f_y) \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$$

Pg-43

$$\frac{M_1}{V} + l_0 > L_d$$

$$\left[\begin{array}{l} \tau_{bd} = \text{IS 456:2000} \\ \text{Pg-43 Table -} \\ 26.2.1.1 \end{array} \right]$$

Hence, codal requirements are satisfied. The arrangements of reinforcement is shown in fig.



$$M_1 = 11.31 \times 10^6 \text{ Nmm}$$

$$V_u = 22970 \text{ N}$$

providing no hooks

$$l_0 = 0$$

$$\left[\begin{array}{l} \text{IS 456:2000} \\ \text{Pg-44} \\ \text{clause 26.2.3.3} \end{array} \right] \frac{M_1}{V} + l_0 = \frac{11.31 \times 10^6}{22970} + 0 = 492.38 \leq 500 \text{ mm}$$

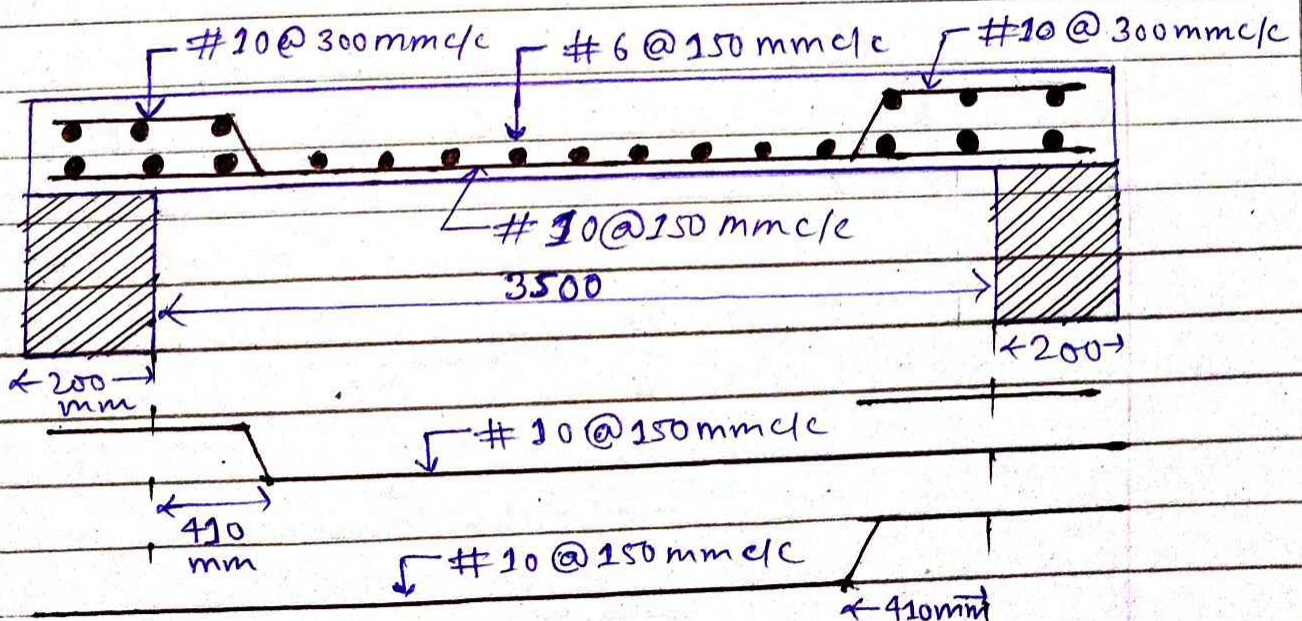
$$\left(\begin{array}{l} \text{IS 456:2000} \\ \text{Pg-42} \end{array} \right) L_d = \frac{(0.87 f_y) \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 10}{4 \times 1.2 \times 1.6} = 470 \text{ mm}$$

Pg-43

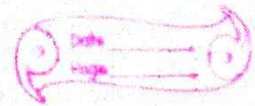
$$\frac{M_1}{V} + l_0 > L_d$$

$$\left[\begin{array}{l} \tau_{bd} = \text{IS 456:2000} \\ \text{Pg-43 Table -} \\ \text{26.2.1.1} \end{array} \right]$$

Hence, codal requirements are satisfied. The arrangements of reinforcement is shown in fig.



Two way slab



Introduction

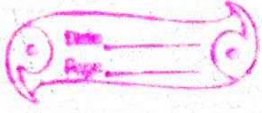
Two way slab are those slabs which are supported on all the four edges and having $\frac{L_y}{L_x} \leq 2.0$. In this type of slab bending occurs in both the direction i.e longer direction as well as shorter direction.

There are two types of two way slab.

- ① **Restrained Slab:-** When a two way slab is loaded, the corners get lifted up. These corners can be prevented from lifting by providing fixidity at the support by beam or walls. Such type of slab in which the corners are prevented from lifting are called as restrained slabs. In this slabs special torsion reinforcement is provided at the edges to prevent cracking of the corners. These are also called as slabs with corners held down.

The Indian standard specifies two ways of providing fixidity in the case of restrained slabs:-

- ① The slab is simply supported on four edges by beams and the beams and slabs are cast monolithic (at the same time).
- ② Slab is supported on the edges by the four walls and there are superstructure walls also above the slabs.



② Unrestrained slabs:- The slabs in which corners are not prevented from lifting are called as unrestrained slabs or slabs with corners not held down.

* IS CODE Method of Design of two way slab (Annex D, IS 456:2000) (Pg-90)

Q1. Design a reinforced concrete slab for a room of clear dimensions $4\text{m} \times 5\text{m}$. The slab is supported on walls of width 300mm . The slab is carrying a line load of 4kN/m^2 and floor finish 1kN/m^2 . Use M20 concrete and Fe415 steel. The corners of slab are held down.

Soln Given data,

Dimensions of slab = $4\text{m} \times 5\text{m}$

width of walls = 300mm

Line load (Imposed) = 4kN/m^2

Floor finish = 1kN/m^2

$f_{ck} = 20\text{N/mm}^2$

$f_y = 415\text{N/mm}^2$

① Check whether one way slab or two way slab.

$$\frac{l_y}{l_x} = \frac{5}{4} = 1.25\text{m} < 2$$

Hence, two way slab.

Assuming total depth of slab $D = 180 \text{ mm}$

Assuming ^{clear} cover = ~~45~~ 20 mm
main bar = 8 mm ϕ

$$d = 180 - 20 - \frac{8}{2} \quad \left\{ \frac{l}{d} = \frac{4000}{25} = 160 \text{ mm} \right\}$$
$$= 156 \text{ mm} \leq 160 \text{ mm}$$

② Effective span
Effective span in x-direction

① Centre to centre = $4 + 0.3 = 4.3 \text{ m}$ 300 mm = 0.3 m

② Clear span + Effective depth = $4 + 0.16 = 4.16 \text{ m}$

$$l_x = 4.16 \text{ m}, \quad l_y = 5 + 0.16 = 5.16 \text{ m}$$

③ Design load (w_u)

$$\text{Self wt. of slab} = \frac{d}{1000} \times \frac{b}{1000} \times 25 = 0.18 \times 1 \times 25 = 4.5 \text{ kN/m}$$

$$\text{Finishing load} = 1 \times 1 = 1 \text{ kN/m}$$

$$\text{Live load} = 4 \times 1 = 4 \text{ kN/m}$$

$$\text{Total load} = 4.5 + 1 + 4 = 9.5 \text{ kN/m}$$

$$\text{Factored or designed load} = 9.5 \times 1.5$$
$$= 14.25 \text{ kN/m}$$

[load factor = 1.5]

④ Design moment and shear

(from IS 456:2000 pg-90 Table 26.)

$$\text{for } \frac{l_y}{l_n} = 1.25$$

$$\alpha_x = ?$$

$$1.2 = 0.072$$

$$1.3 = 0.079$$

$$\alpha_x = 0.072 + \frac{0.079 - 0.072}{1.3 - 1.2} \times (1.25 - 1.2)$$

$$\alpha_x = 0.075$$

$$\alpha_y = 0.056$$

{ IS 456:2000
Table 26 pg-90 }

$$M_x = \alpha_x w_u \cdot l_n^2$$

[IS 456:2000 pg-90
D.1.1]

$$= 0.075 \times 14.25 \times 4.16^2 = 18.49 \text{ kNm}$$

$$= 18.5 \text{ kNm}$$

$$= 18.5 \times 10^6 \text{ Nmm}$$

$$M_y = \alpha_y w_u \cdot l_n^2$$

$$= 0.056 \times 14.25 \times 4.16^2 = 13.81 \text{ kNm}$$

$$= 13.81$$

$$M_y = \alpha_y \cdot w_u \cdot l_n^2$$

[pg-90]

$$= 0.056 \times 14.25 \times 4.16^2$$

$$= 13.81 \text{ kNm}$$

$$= 13.81 \times 10^6 \text{ Nmm}$$

Maximum shear force = V_u

$$V_u = w_u \cdot l_x \cdot \left(\frac{r}{2+r} \right)$$

$$= 14.25 \times 4.16 \left(\frac{1.25}{2+1.25} \right) =$$

$$V_u = 22.8 \text{ kN}$$

$$= 22800 \text{ N}$$

⑤ Minimum depth required (d_{reqd})

$$d_{req} = \sqrt{\frac{M_u}{R_u \cdot b}} = \left[R_u = 2.76 \text{ for } M20 \text{ and } Fe415 \right. \\ \left. \text{Table 4.3} \right]$$

$$R_u = 0.36 \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) f_{ck}$$

$$= 0.36 \times 0.48 (1 - 0.42 \times 0.48) \times 20 \\ = 2.76$$

$$d_{req} = \sqrt{\frac{18.5 \times 10^6}{2.76 \times 1000}}$$

$$d_{req} = 81.87 \approx 82 \text{ mm}$$

Hence OK,

⑥ Design of main reinforcement

① along shorter span in x-direction (middle strip):

$$\text{width of middle strip} = \frac{3}{4} l_y$$

$$= \frac{3}{4} \times 5.16$$

$$= 3.9 \text{ m}$$

$$d = 160 \text{ mm}$$

[IS 456:2000]
Pg-90
Fig 25A

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} d_y}{b d f_{yk}} \right]$$

$$18.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 160 \left[1 - \frac{A_{st} \times 415}{1000 \times 160 \times 20} \right]$$

$$A_{st} = 334.7 \text{ mm}^2$$

Using 8mm ϕ bars $A_{\phi} = \frac{\pi}{4} \times 8^2 = 50.3 \text{ mm}^2$

$$\text{Spacing} = \frac{1000 \times A_{\phi}}{A_{st}}$$

$$= \frac{1000 \times 50.3}{334.7}$$

$$= 150 \text{ mm}$$

(i) $3d = 3 \times 160 = 480 \text{ mm}$

(ii) $300 = 300 \text{ mm}$

(iii) 150 mm

} take less Value.

Using HYSD bars.

$$A_{st \text{ min}} = \frac{0.12 b D}{100}$$

$$= \frac{0.12 \times 1000 \times 180}{100} = 216 \text{ mm}^2$$

$$A_{st \text{ provided}} = \frac{1000 \times 30.3}{150} = 335.33 \text{ mm}^2$$

$$335. > 216$$

$$A_{st \text{ provided}} > A_{st \text{ min}}$$

Hence ok

∴ provide 8mm ϕ @ 150mm c/c in the middle strip of width 3.9 m

(ii) Along longer span in y-direction (middle strip)

$$\text{Width of middle strip} = \frac{3}{4} l_x$$

$$= \frac{3}{4} \times 4.16 = 3.12 \text{ m}$$

Effective depth along y direction

$$d = 160 - 4 - 4 = 152 \text{ mm}$$

$$M_u = 0.87 f_y A_{st} \cdot d \left(1 - \frac{f_y A_{st}}{b d f_{ck}} \right)$$

$$13.81 \times 10^6 = 0.87 \times 415 \times A_{st} \times 152 \left(1 - \frac{415 A_{st}}{1000 \times 152 \times 20} \right)$$

$$A_{st} = 261 \text{ mm}^2$$

$$A_{st} > A_{st \text{ min}}$$

$$\text{Spacing of 8mm } \phi \text{ bars} = \frac{1000 \times 50.3}{261}$$

$$= 192 \text{ mm}$$

- | | |
|--------------|-------------------|
| (i) 192 mm | } take less value |
| (ii) 3d | |
| (iii) 300 mm | |

∴ provide 8mm dia bars @ 190mm c/c in middle strip of width 3.12m

(iii) Reinforcement in edge strip

$$A_{s \min} = 216 \text{ mm}^2$$

Using 8mm ϕ bars

$$\text{Spacing} = \frac{1000 \times 50.3}{216} = 232 \text{ mm}$$

(Spacing is less than 3d or 450mm)

Using 8mm ϕ bars @ 230 mm c/c in the edge strip of width $\frac{1}{2} (5.16 - 3.9)$

i.e 1.04m along X-direction and edge of width $\frac{1}{2} (4.16 - 3.12)$ i.e 0.52m along Y-direction.

(7) Check for shear

$$\text{Nominal shear stress} = \tau_v = \frac{V_u}{bd}$$

$$\tau_v = \frac{22800}{1000 \times 160} = 0.14 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 269}{1000 \times 160} = 0.16 \%$$

Taking min^m value of τ_c from code book
[IS 456: 2000 Pg-73]
Table-19

$$\tau_c = 0.28 \text{ N/mm}^2$$

For 180mm thickness of slab $K = 1.24$

$$\tau_c = 0.28 \times 1.24 = 0.34 \text{ N/mm}^2$$

[IS 456: 2000]
Pg-72
Clause 40.2.1.1

$\therefore$ Beam is safe in shear

⑧ Check for deflection

$$P_t = 0.21\%$$

$$f_s = 0.58 f_y \left[\frac{A_{st \text{ reqd}}}{A_{st \text{ provided}}} \right] = 0.58 \times 415 \left[\frac{334.7}{335} \right]$$
$$= 240 \text{ N/mm}^2$$

For $P_t = 0.21\%$

$$f_s = 240 \text{ N/mm}^2$$

$$K_t = 1.6$$

[IS 456: 2000 Pg-38]
Fig-4

$$\left(\frac{l}{d} \right)_{\max} = 20 \times 1.6 = 32$$

$$\left(\frac{l}{d} \right)_{\text{provided}} = \frac{41.60}{160} = 26$$

$$\left(\frac{l}{d} \right)_{\max} > \left(\frac{l}{d} \right)_{\text{provided}} \quad \text{Hence ok}$$

⑨ Torsional Reinforcement at Corners.

$$\text{Mesh Size} = \frac{l_x}{5} = \frac{4.16}{5} = 0.832 \text{ m} \text{ or } 840 \text{ mm}$$

$$\text{Area of torsional reinforcement} = \frac{3 \times 335}{4} = 251 \text{ mm}^2$$

Using 8 mm ϕ bars

$$A_{\phi} = \frac{\pi}{4} \times 8^2 = 50.3$$

$$\text{Spacing} = \frac{1000 \times 50.3}{251} = 201 \text{ mm or } 200 \text{ mm}$$

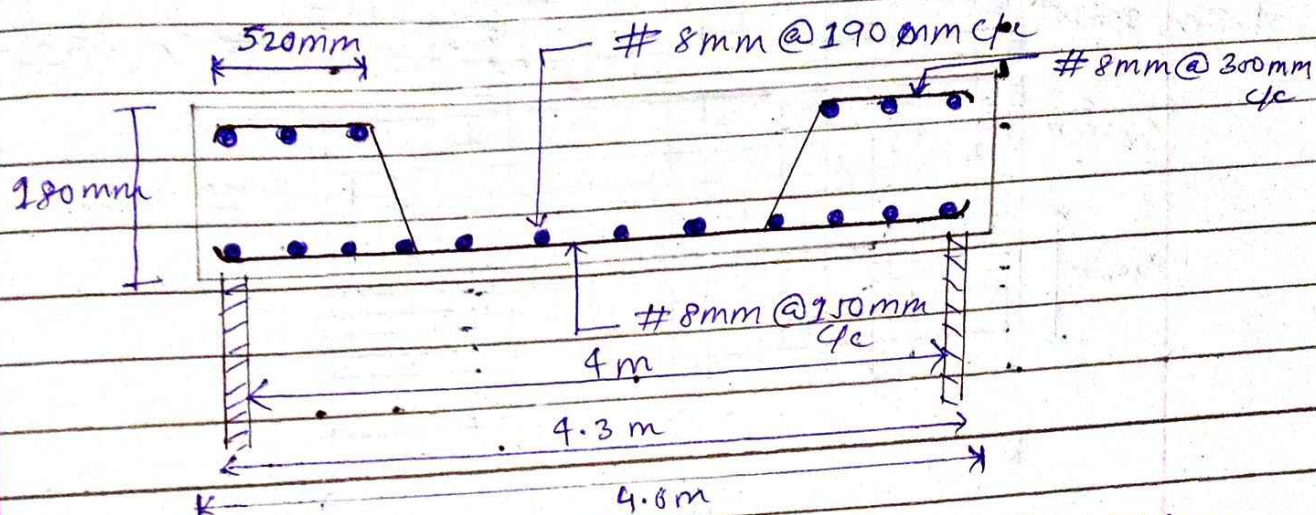
∴ provide 8 mm mesh of bars @ 200 mm/c/c in a mesh.

⑩ Arrangement of reinforcement

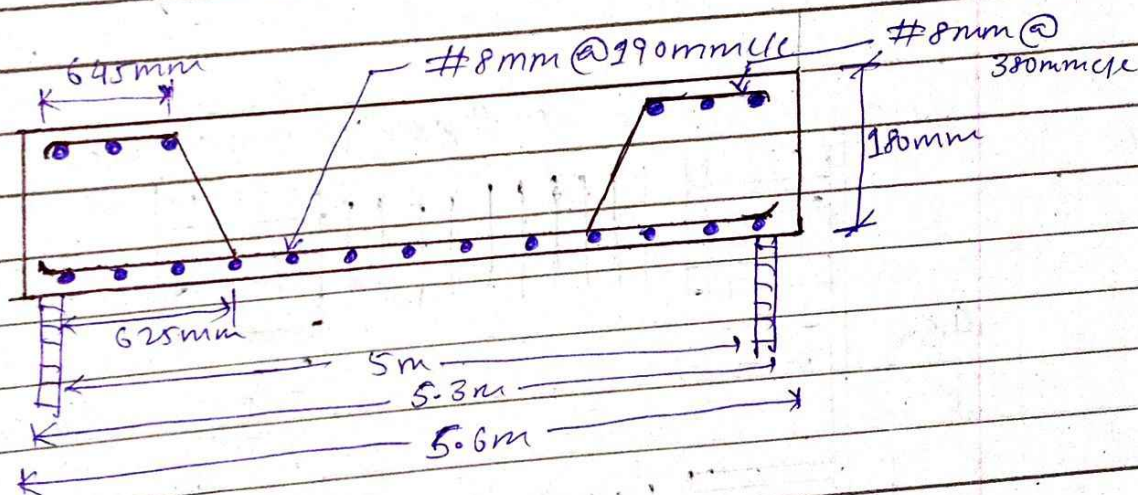
(i) Bending half of the main bars at a distance of $0.15 l_x$ i.e. 0.65 m from the centre of support or $(650 - 130) = 520 \text{ mm}$ from edge of support along x-direction available length of bars at top = $650 - 130$ i.e. 520 mm from centre of support.

(ii) Bending half of the main bars at a distance of $0.15 l_y$ i.e. 775 mm from centre of support or $775 - 150 = 625 \text{ mm}$ from edge of support along y-direction, available length of

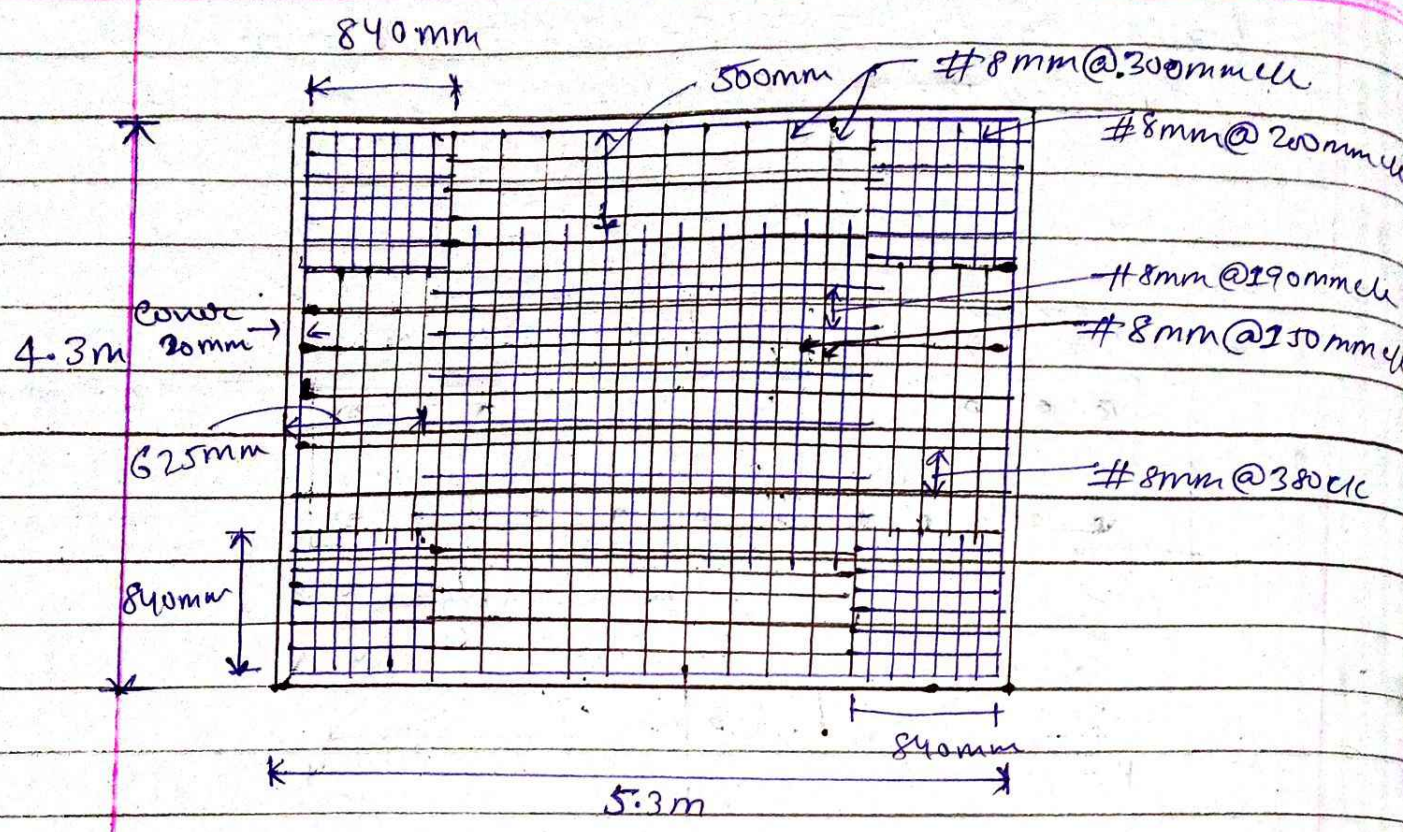
basis at top = $775 - 130 = 645 \text{ mm}$
from centre of support.



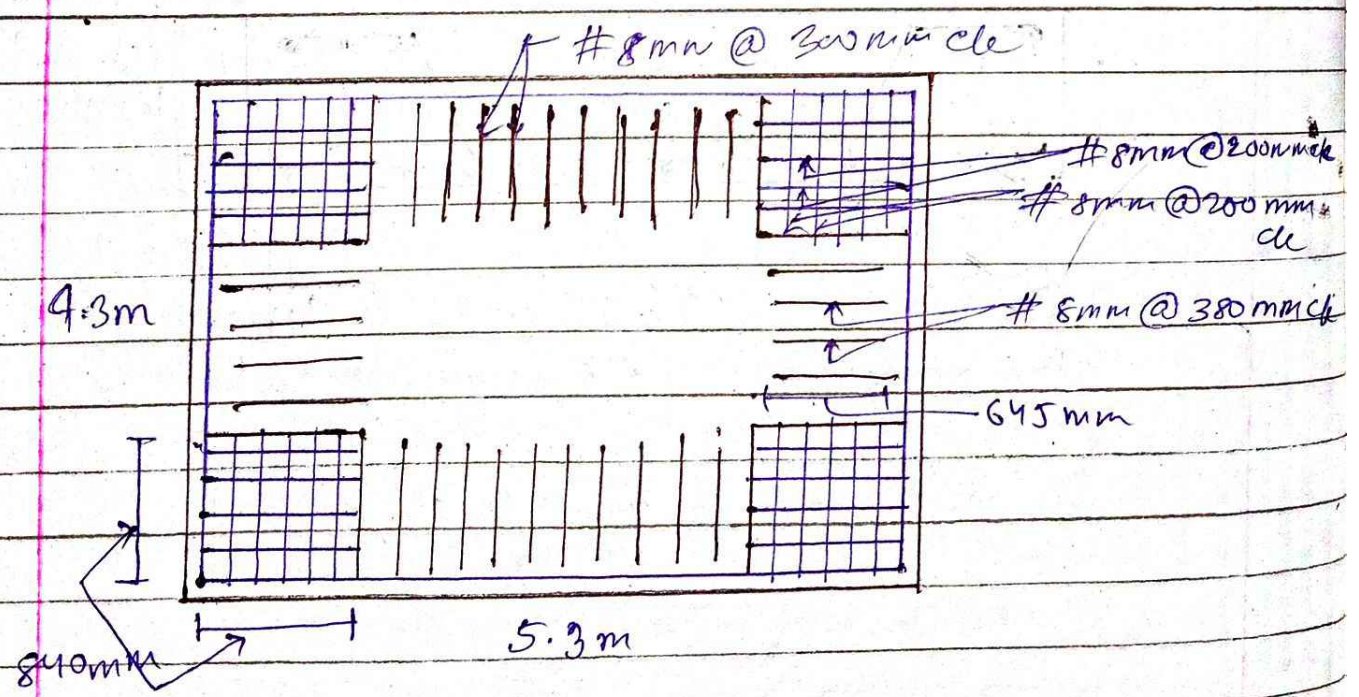
(a) Section along short span at mid span



(b) Section along long span

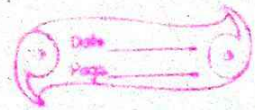


(c) plan of bottom reinforcement



(d) plan of top reinforcement

Design of Stair

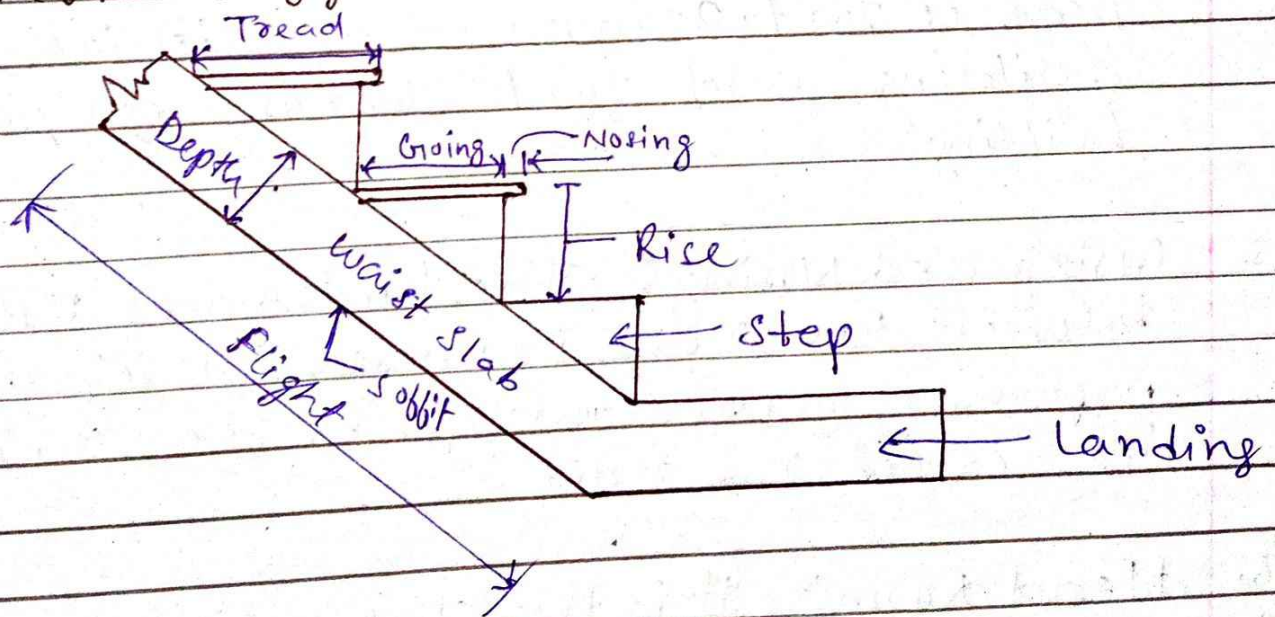


A stair case is a means of giving access to different floors or levels of a building. Stair cases are used in almost all buildings. It consists of a number of steps arranged in a way that a person can move from one level to another.

* Types of staircase

- 1) Straight stairs.
- 2) Quarter turn stair.
- 3) open well type (Half turn stair)
- 4) Dog legged stair.
- 5) Circular or spiral stairs

* Terminology



1. **Flight:-** Flight is the length of the stair case between two landings. It is the sloping and portion (slab) of the stairs. The numbers of steps in a flight varies from 3 to 12.
2. **Landing:-** Landing is the intermediate, horizontal portion provided in a staircase. It is provided for relaxing while climbing and entering or exiting a staircase.
3. **Rise:-** The vertical height of a step is called rise or riser. It varies from 150mm to 180mm for residential building and 120 to 150 mm for public building.
4. **Tread:-** The horizontal distance between two riser is called as tread. The width of tread is 200 to 250 mm for residential building and 200 to 300 mm for public building.
5. **Going and Nosing:-** The horizontal distance between two risers is known as going and portion projecting out from riser surface is called as nosing.
6. **Head Room:-** It is the clear height available between one flight and other above it.
7. **Soffit:-** It is the bottom surface of the waist slab.

* proportioning of staircase

Some thumb rules for making staircase

1. $2 \times \text{Riser} + \text{Tread} = 600 \text{ to } 640 \text{ mm}$
2. $\text{Riser} \times \text{Tread} = 40,000 \text{ to } 42,000 \text{ mm}^2$
3. Width of private staircase is about 900 mm and that of public building is about 1800 mm to 2400 mm.
4. Clear height between a flight and the other vertically above it (head room) should not be less than 2.10 m.
5. The angle of flight with the horizontal should ~~be~~ be 25° to 40° .
6. For free flow of users the width of landing should be equal to the width of stairs.

Q1. Design a dog legged stair case for an office building in a room measuring $(3 \times 6) \text{ m}$ clear dimension. Floor to floor height is 3.5 m . The building is a public building - liable to overcrowding - stairs are supported on brick walls 230 mm thick at the end of landings. Use M20 concrete Fe 415 steel.

Soln:- (i) proportioning of various dimensions of stair case

Available width of stair case $= 3 \text{ m}$

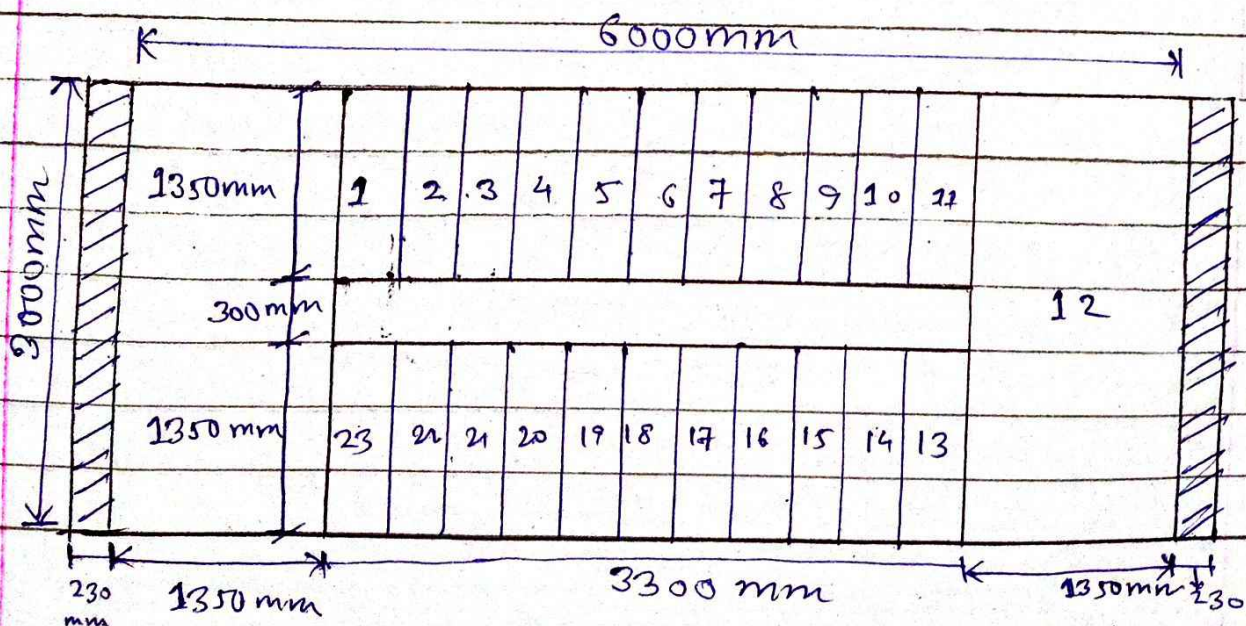
Considering 2 flights of dog legged stair case,
Assume width of each flight as 1.35 m

$$\text{Space between the two flights} = 3 - 2 \times 1.35 = 0.30 \text{ m}$$

Floor to floor height $= 3.5 \text{ m}$

As there will be two flights, each flight will have a height of

$$\frac{3.50}{2} = 1.75 \text{ m}$$



Date _____
Page _____

Assuming height of risers as 150mm as it is public building.

$$\text{No. of riser} = \frac{1750}{150} = 11.66 \approx 12$$

$$\text{No. of Step (Tread)} = 12 - 1 = 11$$

Let width of each tread = 300mm

$$\begin{aligned}\text{Total going} &= 300 \times 11 = 3300\text{mm} \\ &= \underline{\underline{3.3\text{m}}}\end{aligned}$$

Total length available = 6m

$$\begin{aligned}\therefore \text{width of each landing} &= \frac{6 - 3.3}{2} \\ &= 1.35\text{m} = 1350\text{mm}\end{aligned}$$

ii) Design of Staircase

a) Effective span of flight = Centre to centre distance of walls

$$= 6 + \frac{0.23}{2} + \frac{0.23}{2}$$

$$= 6.23\text{m} = 6230\text{mm}$$

b) Thickness of waist slab = $\frac{1}{20}$ of span (approx)

$$= \frac{6.23}{20} \times 1000 = 311.5\text{mm}$$

Let us take $d = 300\text{mm}$ and $D = 325\text{mm}$.

(iii) Loads

(a) weight of waist slab in plan (per m width of flight)

$$= D \sqrt{1 + \frac{R^2}{T^2}} \times 25$$

$$= 0.325 \sqrt{1 + \frac{150^2}{300^2}} \times 25 = 9.1 \text{ kN/m}$$

(b) weight of steps (per m width of flight)

$$= \frac{25RT}{2T}$$

$$= \frac{1}{2} \times \frac{0.15 \times 0.30}{0.30} \times 25 = 1.875 \text{ kN/m}$$

$$\text{Total dead load} = 9.1 + 1.875 = 10.975 \text{ kN/m}$$

$$\text{Live Load} = 5 \text{ kN/m}^2 = 5 \text{ kN/m per m width of stair case}$$

$$\text{Total DL + LL} = W = 10.975 + 5 = 15.975 \text{ kN/m}$$

say 16 kN/m

$$\text{factored load, } W_u = 16 \times 1.5 = 24 \text{ kN/m}$$

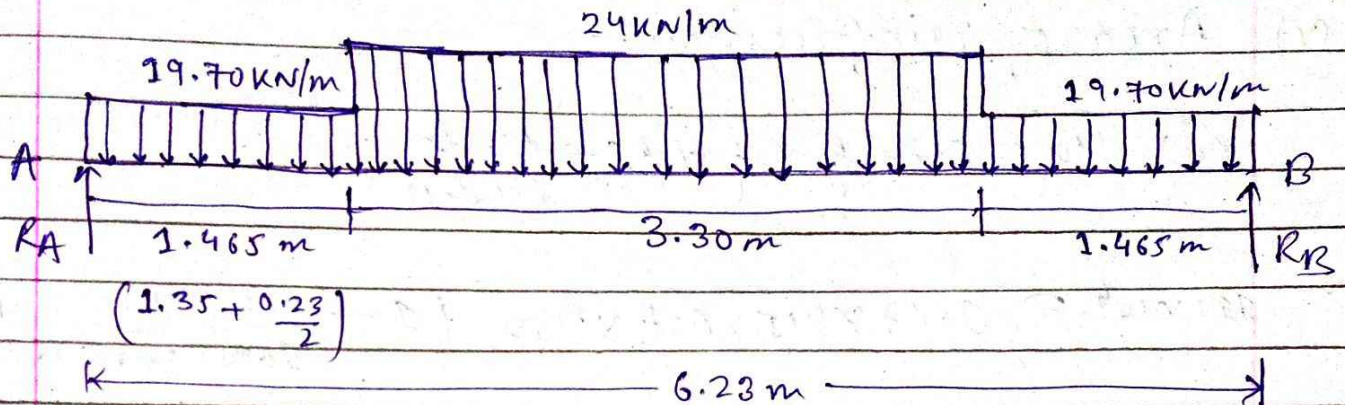
for landing. $DL = 0.325 \times 25 \times 1 = 8.125 \text{ kN/m}$

$$LL = 1 \times 5 \text{ kN/m}^2 = 5 \text{ kN/m per m width of landing.}$$

$$\text{Total DL + LL} = 8.125 + 5 = 13.125 \text{ kN/m}$$

$$\text{factored load} = 1.5 \times 13.125 \text{ kN/m} \\ = 19.70 \text{ kN/m}$$

Load diagram of the stair shall be as follows:



(iv) Design Moment

$$\text{Reaction at support, } R_A = R_B = \frac{(2 \times 19.70 \times 1.465) + (24 \times 3.3)}{2} \\ = 68.5 \text{ kN}$$

$$\text{BM at mid span, } M_u = \left(\frac{68.5 \times 6.23}{2} \right) - \left[\frac{19.70 \times 1.465 \times (1.465 + 3.30)}{2} \right] \\ - \left(\frac{24 \times \frac{3.30}{2} \times \frac{3.30}{4} \right)$$

$$M_u = 112 \text{ kNm}$$

Max. BM allowed for a singly reinforced section with Fe 415 bars (Table 5.2)

$$M_{u\text{lim}} = 0.138 f_y b d^2 \\ \leq 0.138 \times 20 \times 1000 \times 300^2 \times \frac{1}{10^6} \\ = 248.4 \text{ kNm}$$

$$248.4 > 112 \text{ kNm}$$

Hence section can be designed as singly reinforced.

(v) Area of reinforcement

$$① M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

$$112 \times 10^6 = 0.87 \times 415 \times A_{st} \times 300 \left(1 - \frac{A_{st} \times 415}{1000 \times 300 \times 20} \right)$$

$$A_{st} = 1120 \text{ mm}^2$$

Using 16 mm dia bars

$$A_{\phi} = \frac{\pi \times 16^2}{4} = 201 \text{ mm}^2$$

$$\text{Spacing} = \frac{201}{1120} \times 1000 = 179 \text{ mm}$$

Provide 16 mm ϕ @ 170 c/c

② Distribution steel = 0.12% of area

$$= \frac{0.12}{100} \times 1000 \times 325 = 390 \text{ mm}^2$$

$$\text{Spacing of } 10 \phi \text{ bars} = \frac{78.5}{390} \times 1000 = 201 \text{ mm}$$

Provide 10 ϕ @ 200 mm c/c

(vi) Development Length

$$\left(\frac{\tau_{bd}}{f_y} = 1.2 \right)$$

IS 456:2000 Pg-43

(Table 26.2.1.2)

$$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}} = \frac{16 (0.87 \times 415)}{4 \times 1.6 \times 1.2}$$

$$= 752 \text{ mm}$$

providing 800 mm length of bars at points where L_d is required as shown in Fig.

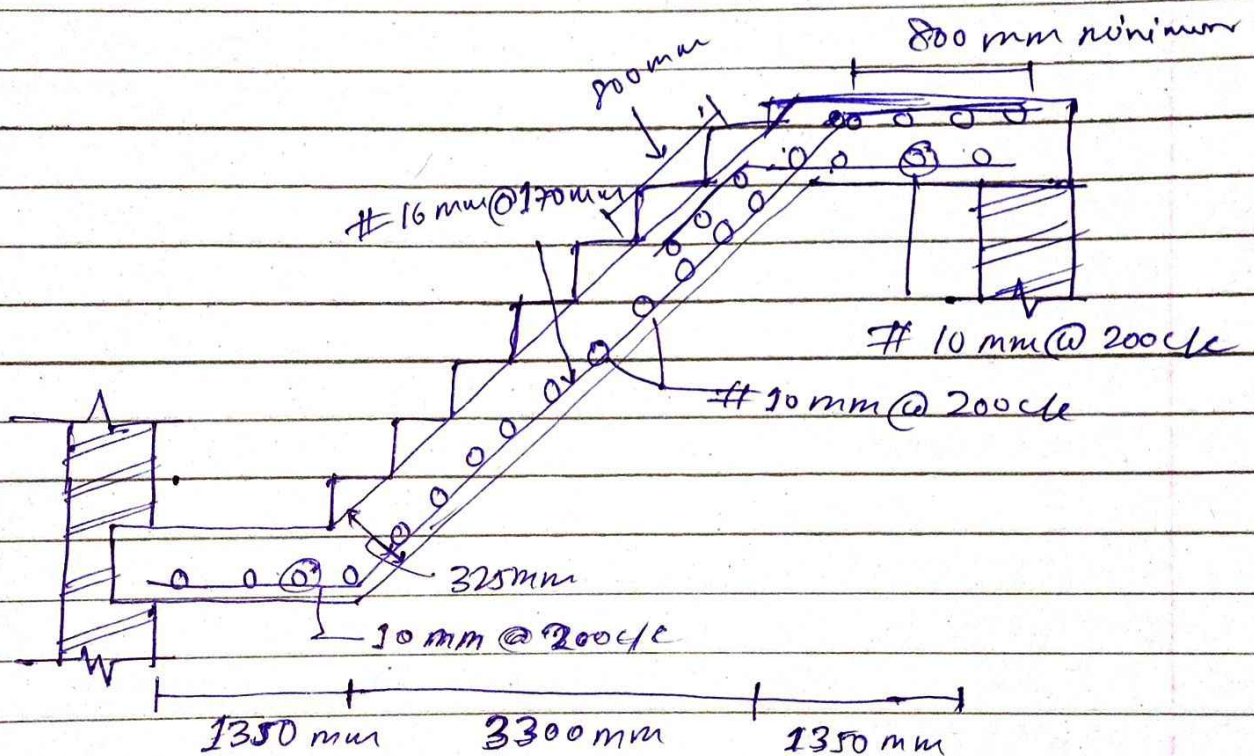


Fig. 3