

Fluid Mechanics & Machine

L T P
3 2

Total Theory Hrs 42
PF M 100 Th, 50 Pr

Contents:Theory

- 1. Properties of fluid** **2**
 - 1.1 Introduction, types of fluid, fluid mechanics, classification of fluid mechanics.
 - 1.2 Properties of fluid- Density, Specific gravity, Specific Weight, Specific Volume, Dynamic Viscosity, Kinematic Viscosity, Surface tension, Capillarity, Vapour Pressure, Compressibility.
- 2.Fluid Pressure & PressureMeasurement** **6**
 - 2.1 Fluid pressure, Pressure head, Pressure intensity, buoyancy, metacenter and metacentric height.
 - 2.2 Concept of absolute vacuum, gauge pressure, atmospheric pressure, absolute pressure.
 - 2.3 Simple and differential manometers, Burden pressure gauge.
 - 2.4 Total pressure, center of pressure of plane, regular surfaces immersed in liquid. Horizontally, vertically and inclined.
(Numerical on Manometers, Total Pressure & Centre of pressure)
- 3. Fluid Flow** **8**
 - 3.1 Types of fluid flows
 - 3.2 Continuity equation
 - 3.3 Bernoulli's theorem
 - 3.4 Venturimeter–Construction, principle of working, Coefficient of discharge, Derivation for discharge through venturimeter.
 - 3.5 Orifice meter– Construction, Principle of working, hydraulic coefficients, Derivation for discharge through Orifice meter
 - 3.6 Weir and Notch- Define & Differentiate between weir and notch. (No numerical)
 - 3.7 Pitot tube–Construction, Principle of Working.
(Simple Numerical only)
- 4. Flow Through Pipes** **4**
 - 4.1 Laws of fluid friction (Laminar and turbulent)
 - 4.2 Darcy's equation and Chezy's equation for frictional losses.
 - 4.3 Minor losses in pipes
 - 4.4 Hydraulic gradient and total gradient line.
 - 4.5 Hydraulic power transmission through pipe.
(Numerical to estimate major and minor losses)
- 5. Impact of jet** **3**
 - 5.1 Impact of jet on fixed vertical, moving vertical flat plates.

5.2 Impact of jet on curved vanes with special reference to turbines & pumps
(Simple Numericals on work done and efficiency)

6. Hydraulic Turbines

7

- 6.1 Layout and classification of hydro electric power plant.
- 6.2 Selection of turbine on the basis of head and discharge available
- 6.3 Construction and working principle of Pelton wheel, Francis and Kaplan turbine.
- 6.4 Calculation of Work done, Power, efficiency of turbine.
- 6.5 Draft tubes– types and construction, Concept of cavitations in turbines

7. Centrifugal Pumps

6

- 7.1 Construction, principle of working and applications.
- 7.2 Types of casing and impellers.
- 7.3 Concept of multistage
- 7.4 Priming and its methods, Cavitation
- 7.5 Manometric head, Work done, Manometric efficiency, Overall efficiency, NPS H.
- 7.6 Performance Characteristics of Centrifugal pumps.
- 7.7 Trouble Shooting.
- 7.8 Introduction to submersible and jet pump
(simple numericals on centrifugal pumps)

8. Reciprocating Pump

6

- 8.1 Construction, working principle and applications of single and double acting reciprocating pumps.
- 8.2 Concept of Slip, Negative slip, Cavitation and separation.
- 8.3 Use of Air Vessel.
- 8.4 Indicator diagram with effect of acceleration head & frictional head.
- 8.5 Trouble shooting

(No Derivations and Numerical on reciprocating pumps.)

RECOMMENDED BOOKS:

- 1. Hydraulics & Hydraulic Machines by Modi & Seth, Standard Book
- 2. Hydraulics & Hydraulic Machines by R.K. Bansal
- 3. Fluid Mechanics by A.K. Jain, Khanna Publishers
- 4. Hydraulic and Fluid Mechanics by Jagadish Lal, Metropolitan Book
- 5. Hydraulics, Fluid Mechanics and Fluid Machines by R.S. Khurmi
- 6. Fluid Mechanics & Hydraulics Machines by R K Rajput
- 7. Fluid Mechanics & Hydraulics Machines by Domkundwar
- 8. Hydraulics Machines by Jagadish Lal

List of Practical:

1. Calibration of Bourden pressure gauge with the help of Dead Weight Pressure gauge.
2. Determine the meta centric height of a floating body.
3. Verification of Bernoulli's Theorem.
4. Determination of Coefficient of Discharge of Venturi-meter
5. Determination of Coefficient of discharge, coefficient of Contraction and coefficient of velocity of orifice meter.
6. Determination of coefficient of discharge through rectangular notch.
7. Determination of coefficient of discharge through triangular notch.
8. To determine minor losses for flow through pipes.
9. Determination of coefficient of friction of flow through pipes.
10. Trial on Pelton/Francis wheel to determine overall efficiency.
11. Trial on centrifugal pump to determine overall efficiency.
12. Trial on reciprocating pump to determine overall efficiency.



1. Density (ρ) = $\frac{M}{V} \left(\frac{\text{kg}}{\text{m}^3} \right)$

Dimensional formula = $[ML^{-3}]$

ρ_{H_2O} at $4^\circ C = 1000 \text{ kg/m}^3$

Ques:- Why density of water is maximum at $4^\circ C$?

For the gas
ideal gas equation

$PV = nRT$

$P = \left(\frac{n}{V} \right) RT$

$P = \rho RT$

$P \uparrow$	$\rho \uparrow$
$T \uparrow$	$\rho \downarrow$

→ For the liquids no change of density w.r.t pressure and temperature.

2. Specific weight (weight density) = $\frac{\text{weight}}{\text{volume}} \left(\frac{\text{KN}}{\text{m}^3} \right)$

denoted by γ (gamma)

$\gamma = \frac{\text{weight}}{\text{volume}} = \frac{mg}{V} = \rho g$

$\therefore \gamma = \rho g$, $\gamma_{H_2O} = 9810 \text{ N/m}^3$

Dimensional formula = $[ML^{-2}T^{-2}]$

3. Specific gravity (s)

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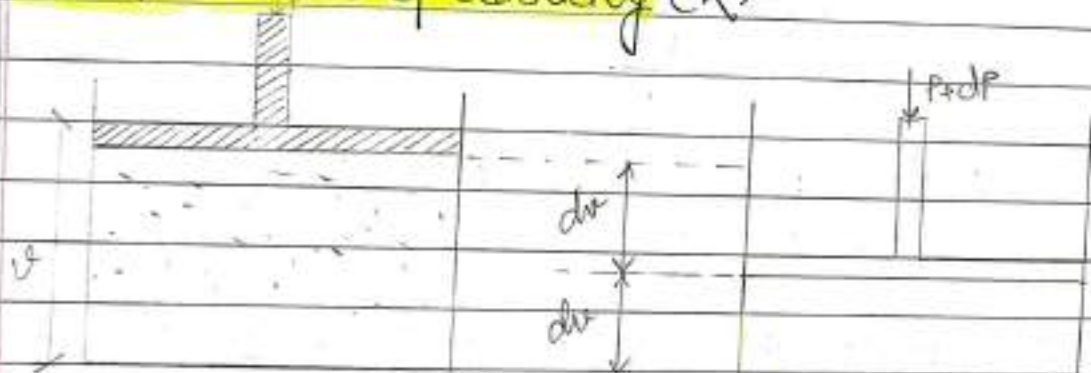
$$S_{\text{fluid}} = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

$$= \frac{\text{wt. density of fluid}}{\text{wt. density of standard fluid}}$$

Standard fluid for liquid $\rightarrow$ water
 Standard fluid for gas $\rightarrow$ air
 $S_{\text{H}_2\text{O}} = 1$, $S_{\text{Hg}} = 13.6$

NOTE:- The term Relative density is also sometimes used when the comparison is not made w.r.t to the standard fluid.

5. Bulk modulus of Elasticity (K)



$$V_i = V$$

$$P_i = P$$

$$V_f = V - dv$$

$$P_f = P + dP$$

$$K = \frac{\text{Change in pressure}}{\text{Volumetric strain}}$$

$$= \frac{\text{Change in pressure}}{\frac{\text{Change in volume}}{\text{Original volume}}}$$

$$= \frac{P_f - P_i}{\frac{V_f - V_i}{V_i}} = \frac{(P + dP) - (P)}{\frac{(V - dv) - (V)}{V}} = \frac{dP}{\frac{-dv}{V}}$$

$$\therefore K = -v \cdot \frac{dp}{dv}$$

$\therefore$ The -ve sign appears in the above equation as with increase of pressure, volume decreases.

6. Compressibility (β)

It is defined as the property of virtual of which fluid undergo a change of volume under the action of external pressure forces.

$$\therefore \beta = \frac{-dv/v}{dp} = \frac{1}{K}$$

$$\rho = m/v$$

$$\therefore m = \rho v$$

Differentiating both sides.

$$\Rightarrow dm = d(\rho v)$$

$$\Rightarrow 0 = \rho dv + v \cdot d(\rho) \quad (\because \text{mass is constant})$$

$$\Rightarrow -\rho dv = v \cdot d\rho$$

$$\therefore \frac{-dv}{v} = \frac{d\rho}{\rho}$$

$$K = -v \cdot \frac{dp}{dv} = \frac{dp}{\frac{d\rho}{\rho}} = \frac{\rho \cdot dp}{d\rho}$$

For constant pressure process

$$dp = 0$$

$$\therefore K = 0$$

For constant volume process

$$dv = 0$$

$$\therefore K = \infty$$

For, Isothermal Bulk modulus of elasticity

$$PV = \text{constant}$$

differentiating on both sides.

$$Pdv + vdp = 0$$

$$-\frac{dv}{v} = \frac{dp}{P}$$

$$\therefore K = \frac{dp}{-\frac{dv}{v}} = \frac{dp}{\frac{dp}{P}} = P$$

$$\boxed{K_{\text{iso}} = P}$$

Adiabatic bulk modulus of elasticity.

$$pv^\gamma = c \quad (\text{Adiabatic process})$$

$$\Rightarrow p \cdot \gamma \cdot v^{\gamma-1} dv + v^\gamma \cdot dp = 0$$

$$\Rightarrow \frac{p \cdot \gamma \cdot v^\gamma}{v} \cdot dv + v^\gamma \cdot dp = 0$$

$$\Rightarrow \frac{p \cdot \gamma \cdot v^\gamma}{v} \cdot dv = -v^\gamma \cdot dp$$

$$\therefore p \gamma \cdot \frac{dv}{v} = -dp$$

$$\therefore -\frac{dv}{v} = +\frac{dp}{p} \times \frac{1}{\gamma}$$

$$\therefore K = \frac{dp}{-\frac{dv}{v}} = \frac{dp}{\frac{dp}{p} \times \frac{1}{\gamma}} = P \times \gamma$$

$$\boxed{K_{\text{adibatic}} = P\gamma}$$

Here,

$\gamma \rightarrow$ Adiabatic index

$$\gamma = C_p / C_v$$

C_p — Specific heat of constant pressure.

C_v — Specific heat of constant volume.

7. Viscosity

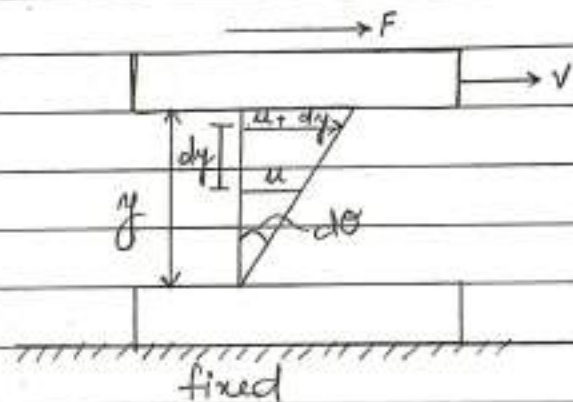
It is the internal resistance offered by the one fluid layer for the movement of adjacent fluid layer. It is due to the intermolecular exchange and the cohesive force between the molecules.

$$v = d/t$$

$$\therefore d = v \times t$$

$\rightarrow$ For the fluid particles which is on the top plate, travelled a distance = $v \times dt$

$$\tan \theta = \frac{v \times dt}{y}$$



For small angles $\tan \theta \approx \theta$

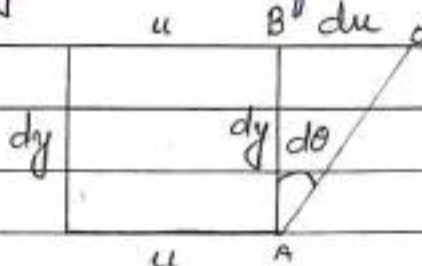
$$\theta = \frac{v \times dt}{y}$$

$$\therefore \frac{d\theta}{dt} = \frac{v}{y}$$

$\therefore$ The term $\frac{d\theta}{dt}$ is called 'Rate of angular deformation'.

From $\triangle ABO$

$$\tan \theta = \frac{du \cdot dt}{dy}$$



For small angle $\tan \theta \approx \theta$

$$\therefore d\theta = \frac{du \cdot dt}{dy}$$

$$\therefore \frac{d\theta}{dt} = \frac{du}{dy}$$

$\frac{d\theta}{dt}$ → Rate of shear strain or Rate of angular

deformation as, shear force, $F \uparrow$, Rate of angular deformation $\frac{d\theta}{dt} \uparrow$

$$F \propto \frac{d\theta}{dt}$$

$$\tau = F/A$$

$$\tau \propto F \quad (\because A \rightarrow \text{area of plate is constant})$$

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt}$$

where, μ is the proportionality constant and is known as coefficient of dynamic viscosity.

$$\mu = \frac{\tau}{\frac{d\theta}{dt}}$$

$$\text{If } \frac{d\theta}{dt} = 1, \mu = \tau$$

$\therefore$ Coefficient of dynamic viscosity is defined as the amount of shear stress required to produce unit

rate of angular deformation or, rate of shear strain.

$$\tau = \frac{\mu \cdot v}{y} = \frac{\mu \cdot dv}{dy}$$

$$\boxed{\tau = \mu \cdot \frac{dv}{dy}} \rightarrow \text{Newton's law of viscosity}$$

The term $\frac{dv}{dy}$ unit of $\frac{m/s}{m} = s^{-1}$

$\therefore$ The term $\frac{dv}{dy}$ is called 'velocity gradient'.

Unit of μ :

$$\tau = \mu \cdot \frac{dv}{dy}$$

$$\therefore \mu = \frac{\tau}{\frac{dv}{dy}} = \frac{N/m^2}{s^{-1}} = \frac{N \cdot s}{m^2} = Pa \cdot s$$

$$1 Pa \cdot s = \frac{N}{m^2} \times s = \frac{kg \cdot m \cdot s^{-2}}{m^2} \times s = \frac{kg}{m \cdot s} \text{ (S.I unit)}$$

$$\frac{1 kg}{m \cdot s} = \frac{1000 g}{100 cm \times s} = \frac{10 g}{cm \times s} \text{ angular velocity}$$

$$\frac{1 kg}{m \cdot s} = 10 \text{ poise}$$

$$v = \omega \times r$$

$$\omega = \frac{2\pi N}{60} \frac{\text{rad}}{\text{sec}}$$

$$\boxed{\therefore 1 \text{ poise} = 0.1 \frac{kg}{m \cdot s} = 0.1 \frac{N \cdot s}{m^2}} \quad \omega = \frac{1}{\text{sec}}$$

* If $\mu = 0$, then $\tau = 0$, the fluid is called an 'Ideal fluid'.
The fluid which satisfies Newton's law of

viscosity is called 'Newtonian Fluid'.

For Newtonian fluid $\tau = \mu \frac{du}{dy}$

e.g. Petrol, diesel, water etc.

The fluid which does not obey Newton's law of viscosity is called Non-newtonian fluid.

The study of Non-Newtonian fluid is known as "Rheology".

E.g: Blood, Milk, Clay etc.

The Relationship between shear stress and velocity gradient for Non-newtonian fluid.

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

If $n=1$, $A=\mu$, $B=0$, then the model is identified as Newtonian model

Fluids

Newtonian fluid

Non-newtonian fluid

Time Independent

Time dependent

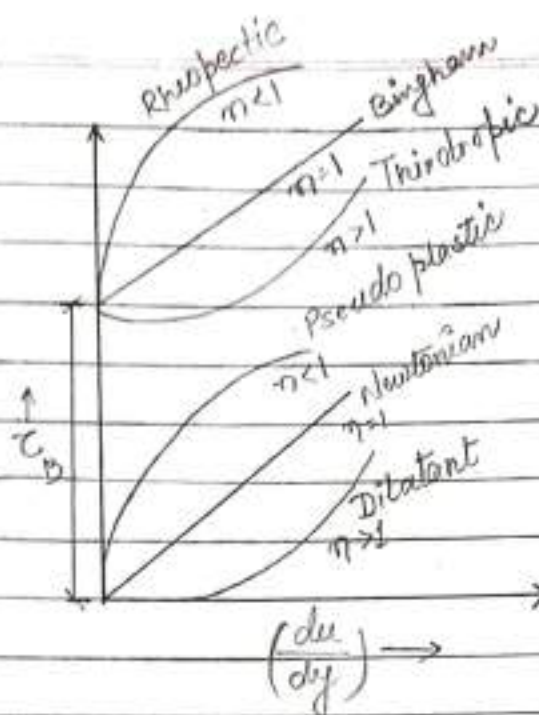
Dilatant
(milk)

Pseudo
(Blood)

Bingham
(Toothpaste)

Thixotropic
(Lipstick)

Rheopexic
(Gypsum
suspension in water)



Kinematic viscosity (ν)

$$\nu = \frac{\text{dynamic viscosity}}{\text{density}}$$

Unit of kinematic viscosity

$$\nu = \frac{\mu}{\rho} = \frac{\frac{\text{kg}}{\text{m} \cdot \text{s}}}{\frac{\text{kg}}{\text{m}^3}} = \frac{\text{m}^2}{\text{s}} \quad \text{1 cStoke} = 10^{-4} \text{ m}^2/\text{s}$$

$$= (100)^2 \frac{\text{cm}^2}{\text{s}}$$

$$= 10^4 \text{ stroke}$$

$$\therefore 1 \text{ m}^2/\text{s} = 10^4 \text{ stroke}$$

Vapour pressure

Consider a closed container, in which a liquid is filled by leaving an empty space above the liquid surface. If the temperature is increased the vapour bubble will start forming and leaves from the

liquid surface and get collected in empty space. The size will reach where the rate of collapsing of vapour bubble into the liquid surface. Under the condition vapour phase is said to be in phase equilibrium with the liquid.

The pressure exerted by the vapour upon the liquid surface is called vapour pressure.

If the partial pressure of water vapour is less than the vapour pressure, then no water is present in the lake.

If the partial pressure of water vapour is equal to the vapour pressure then water is present in the lake.

~~If the pressure falls below~~

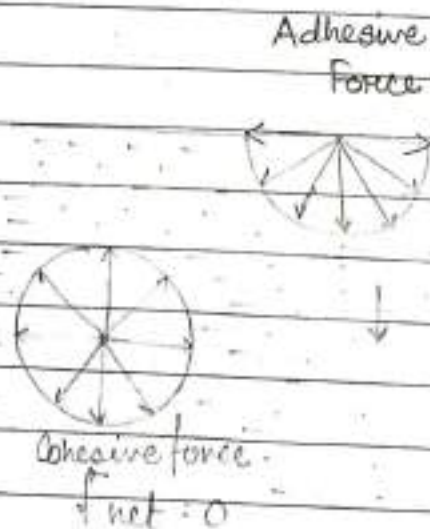
If any fluid flow problems, at any section the pressure should not fall less than or equal to vapour pressure, to avoid the cavitation.

If the pressure falls below the vapour pressure then vapour bubble will form and these will be carried along with flowing fluid into the high pressure region where they will burst and it may damage the surfaces and the action is called the pitting action.

Surface Tension

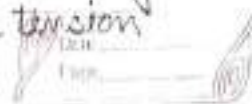
Tension exerted by fluid at the surface is called Surface Tension.

Consider a molecule which is completely inside the liquid and



Unit (N/m).
of Surface Tension

Force per unit length
= Surface tension



it is surrounded by equal no. of molecules.

∴ The net force acting upon such molecules is zero.

If we consider another molecule which is at the surface, it is under the influence of attractive force by its own molecules and the gas molecules attractive force.

The cohesive force are greater than the adhesive force and it is acted by a net downward vertical force as well as the surface molecules are also subjected to cohesive force by the molecules which lies to the left and right of the surface of the liquid is under tension. The tension per unit length is called surface tension.

Units - N/m or J/m²

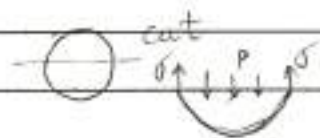
Eg. - (a) A small needle can float

(b) Collection of dust particles on the liquid surface

(c) Small birds can sit in the ponds for drinking water.

(d) formation of liquid droplets.

Case (1) Liquid droplets



d = diameter of droplet

σ = surface tension = surface tension $\times$ length

p = pressure in excess of atmosphere under equilibrium.

Pressure force = surface tension

$$p \times \pi/4 \times d^2 = \sigma \times (\pi \times d)$$

$$\therefore p = \frac{4\sigma}{d}$$

Case (2)

soap bubble

Pressure force = Surface tension

$$p \times \frac{\pi}{4} \times d^2 = \sigma \times (2 \times \pi \times d)$$

$$\therefore p = \frac{8\sigma}{d}$$



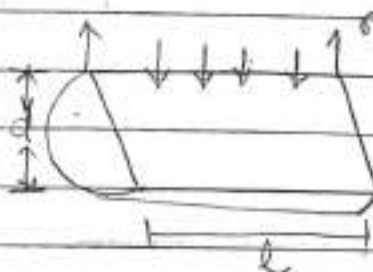
Case (3)

Liquid jet

Pressure force = surface tension

$$p \times l \times d = \sigma \times 2l$$

$$\therefore p = \frac{2\sigma}{d}$$



Surface tension is due to cohesive forces, surface tension decrease with increase in temperature.

$$P_{atm} = \rho_{water} g h$$

$$1 = 1000 \times 9.81$$

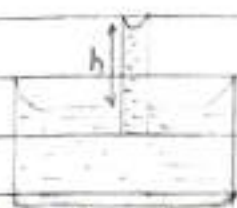
$$h = \frac{1}{1000 \times 9.81}$$

$$h = 10^{-5} \text{ m}$$

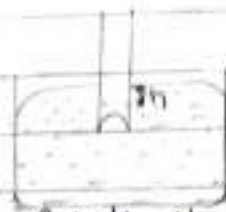
Effect of pressure in viscosity

For liquid, viscosity is practically independent of pressure, except at extremely high pressure for gases, dynamic viscosity is generally independent of pressure particularly (at low to moderate pressure) but kinematic pressure viscosity decreases as density is proportional to pressure.

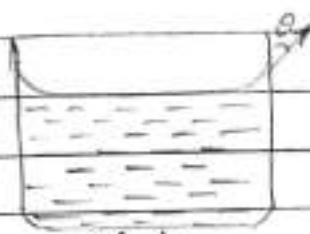
Capillarity



→ Capillarity rise
→ Adhesive > Cohesive

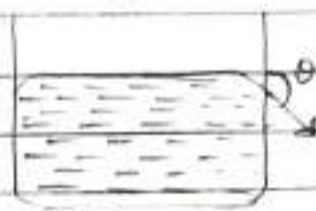


→ Capillarity fall
→ Cohesive > Adhesive



Water
Concave up
 $\theta < 90^\circ$

wetted case
Adhesive > Cohesive



Mercury
Concave down
 $\theta > 90^\circ$

not wetted case
Cohesive > Adhesive

The Rise and Fall of the liquid level in a narrow vertical tube when it is inserted in a liquid from the level of the liquid in a container under equilibrium.

Surface tension force = Weight of liquid in the tube

$$\pi d \times \sigma \cos \theta = \rho g \times \frac{\pi}{4} \times d^2 \times h$$

$$\therefore h = \frac{4\sigma \cos \theta}{\rho g d}$$

$h = \frac{4\sigma \cos \theta}{\rho g d}$
--

If $\theta > 90^\circ$, $\cos \theta = -ve$

$\therefore \eta$ becomes -ve (capillarity fall)

A $d \uparrow, h \downarrow$

$\therefore$ For the tubes of manometer to avoid the correction for capillarity, the $d > 6\text{mm}$ is preferred.

For water,

$$\theta = 0^\circ$$

$$h = \frac{4\sigma}{\rho g d}$$

NOTE : Capillarity is due to both cohesion and adhesion

Numericals :-

1. A plate with surface area of 0.4m^2 and weight of 500N slides down on an inclined plane at 30° to the horizontal at a constant speed of 4m/s . If the inclined plane is lubricated with an oil of dynamic viscosity 2 poise , find the thickness of lubricant film.

Sol:-

Surface area of plate ' A ' = 0.4m^2

Weight of plate ' W ' = 500N

Velocity of sliding plate ' V ' = 4m/sec

Dynamic viscosity ' μ ' = 2 poise

= 0.2 poise

Let,

The thickness of lubricant film - ' y '

As the plate slides down the shear stress ' τ ' generates in a upward direction.

$\therefore$ For equilibrium at terminal velocity component of weight of plate in the Direction of motion =
Shear friction in opposite direction

$$\therefore W \sin 30^\circ = \tau \cdot A$$

According to Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\therefore W \sin 30^\circ = \frac{\mu \cdot v}{y} \cdot A$$

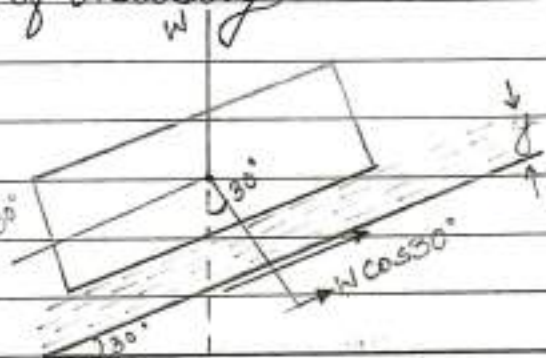
$$500 \times \frac{1}{2} \times 0.2 \times 4 \times 0.4$$

$$\text{Solving, } y = 1.20 \times 10^{-3} \text{ m} = 1.20 \text{ mm}$$

$\therefore$ The thickness of lubricant film = 1.20 mm.

2. A thin square plate $1\text{ m} \times 1\text{ m}$ is placed horizontally in a gap of height 2 cm. Filled with oil of viscosity 10 poise and pulled at a constant velocity of 0.10 m/sec. Find the force on the plate.

The gap is now filled with another oil, when the plate is placed at a distance of 0.5 cm, from one of the surfaces of the gap and pulled with the same velocity, the force on the plate remains same as



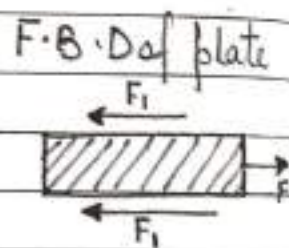
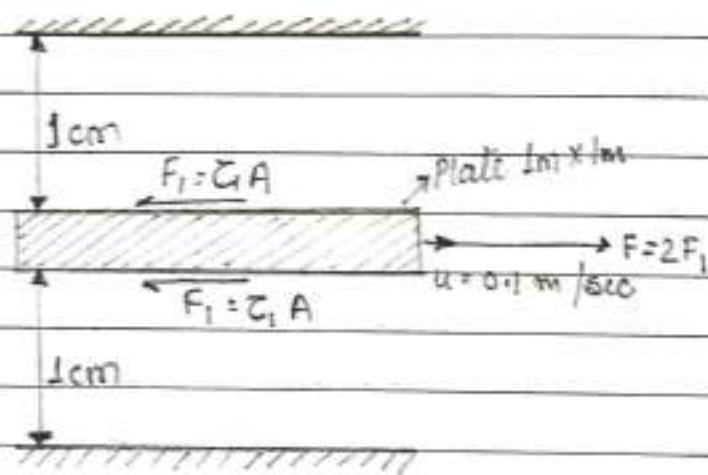
Refer. Find the viscosity of new oil.

Sol:- Data given,

$$\mu = 10 \text{ Poise} = 1.0 \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$u = 0.1 \text{ m/sec.}$$

Case 1:-



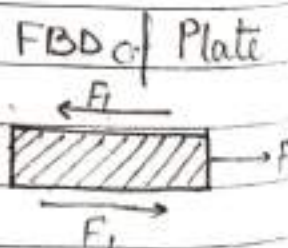
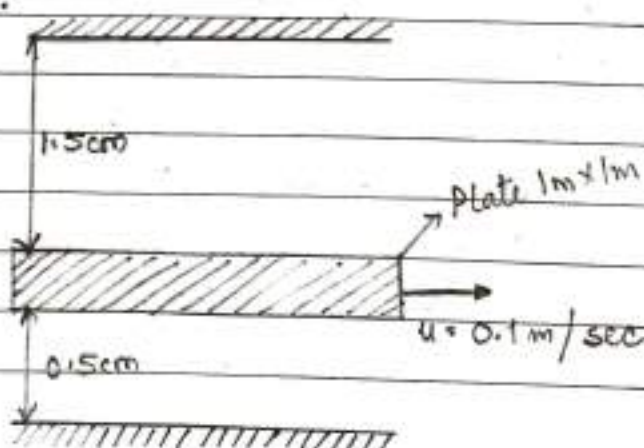
Force on one side of plate

$$\Sigma F = m\vec{a} \quad (\vec{a} = 0)$$

$$F_1 = \mu \left(\frac{du}{dy} \right) \times A = 1 \times \frac{0.1}{(1 \times 10^{-2})} \times 1 = 10 \text{ N}$$

$$F = 2F_1 = 2 \times 10 = 20 \text{ N}$$

Case 2:-



$$\begin{aligned}\text{Force on top of plate} &= \mu \left(\frac{du}{dy} \right)_{\text{top}} \times \text{Area} \\ &= \mu \cdot \frac{0.1}{1.5 \times 10^{-2}} \times 1 = \frac{20}{3} \mu\end{aligned}$$

Force on bottom of plate

$$\begin{aligned}&= \mu \left(\frac{du}{dy} \right)_{\text{bottom}} \times \text{Area} \\ &= \mu \cdot \frac{0.1}{(0.5 \times 10^{-2})} \times 1 = 20 \mu\end{aligned}$$

$$\text{Total force in Case (2)} = \left(\frac{20}{3} \mu + 20 \mu \right) = \left(\frac{80}{3} \mu \right)$$

From (1) and (3)

$$\frac{80}{3} \mu = 20$$

$$\mu = \left(\frac{3}{4} \right) = 0.75 \left(\frac{\text{Ns}}{\text{m}^2} \right) \quad \text{Ans:-}$$

Q. Velocity = $5 \sin(2\pi y)$

$\mu = 5 \text{ poise}$

τ (stress) at $y = 0.05 \text{ m} = ?$

Sol:- $\tau = \mu \frac{du}{dy}$

$$\tau = \mu \cdot \frac{5 \sin 2\pi y}{dy}$$

$$\tau = \mu \times 5 \times \cos 2\pi y \times 2\pi$$

$$\tau = 0.5 \times 5 \times (\cos 2\pi \times 0.05) \times 2\pi$$

$$\tau = 15.70 \text{ N/m}^2 \quad \underline{\underline{\text{Ans:-}}}$$

- Q. A thrust bearing consists of a 10cm diameter pad rotating on another pad separating by an oil film $\mu = 80 \text{ cP}$ by 1.50mm. Compare the power dissipated in the bearing if it rotates at 100 revolutions per minute.

Sol:- Consider a small element of width S_r at a radius r subtending a small angle $\delta\theta$ at the centre of the pad as shown in the plan view. The area of the element is $S_r \times r\delta\theta$

It experiences a shear force SF due to shearing action of the fluid contained between the pads.

The rotational speed of the pad is given by

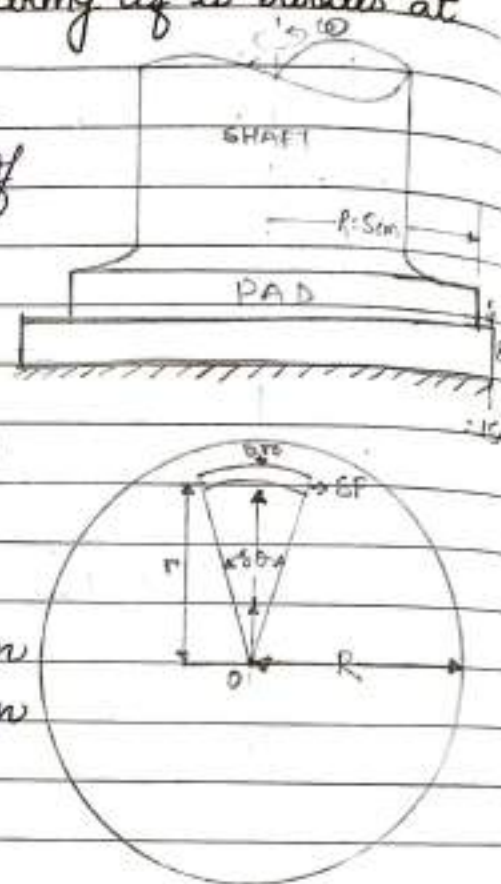
$$\omega = 2\pi \times \frac{100}{60} = 10.47 \text{ rad/s}$$

The linear velocity u at a radial distance r must be

$$u = r\omega = 10.47r \text{ m/s}$$

Since the lower pad is at rest, the velocity gradient

$$\frac{du}{dy} = \frac{10.47r - 0}{0.0015} = 6980r \text{ rad/s}$$



From the data,

$$\mu = 80 \text{ cP} = 0.08 \text{ P} = 0.08 \text{ Ns/m}^2$$

The shearing stress at the location of the element must be

$$\tau = \mu \frac{du}{dy} = 0.08 \times 6980 r = 558.4 r \text{ N/m}^2$$

and the shearing force δF is given by

$$\delta F = \tau \cdot \delta r \cdot r \delta \theta = 558.4 r^2 \delta r \delta \theta \text{ N}$$

The torque δT required to provide the shearing force over the element is

$$\delta T = r \delta F = 558.4 r^3 \delta r \delta \theta \text{ Nm}$$

and total torque on the shaft must be

$$\begin{aligned} T &= \int_0^R \int_0^{2\pi} 558.4 r^3 dr d\theta = 2\pi \times 558.4 \int_0^R r^3 dr \\ &= 2\pi \times 558.4 \times \frac{R^4}{4} = 2\pi \times 558.4 \times \frac{0.05^4}{4} \\ &= 0.00548 \text{ Nm} \end{aligned}$$

The power dissipated i.e. the rate of energy lost in the bearing is given by

$$\begin{aligned} \text{Power} &= T \cdot \omega = 0.00548 \times 10.47 \\ &= 0.0574 \text{ W. Ans.} \end{aligned}$$

Q. Calculate the gauge pressure and the absolute pressure within

a) a droplet of water 0.4 cm in dia and

(b) a jet of water 0.4 cm in dia.

Sol:- $\sigma_{\text{water}} = 0.073 \text{ N/m}$

a) $P = P_{\text{gauge}} + 1 \text{ atm}$

$$P = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.073}{0.004} = 73 \text{ N/m}^2$$

b) $P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$

$$= 101300 + 73$$

$$= 101373 \text{ N/m}^2$$

$$= 101.373 \text{ kN/m}^2$$

for the jet

$$P_{\text{gauge}} = \frac{2\sigma}{d} = \frac{2 \times 0.073}{0.004}$$

$$= 36.5 \text{ N/m}^2$$

$$P_{\text{atm}} = 101300 + 36.5$$

$$= 101336.5 \text{ N/m}^2$$

$$= 101.34 \text{ kN/m}^2$$

Ans:-

PRESSURE MEASUREMENT

Intensity of Pressure

The pressure is defined as the normal force exerted by static fluid per unit area.

$$\text{Intensity of pressure (P)} = \frac{\text{Normal force}}{\text{Area}}$$

→ Pressure is compressive in nature.

Unit :- (i) N/m^2 (Pa)

(ii) KPa, MPa, GPa

(iii) mm of column of liquid

(iv) $1 \text{ bar} = 10^5 \text{ Pa}$

Types of pressure

1. Atmospheric Pressure (P_{atm})

It is defined as the pressure exerted by the atmosphere air upon the surface.

$$\begin{aligned} P_{\text{atm}} &= 760 \text{ mm of Hg} \\ &= 10.33 \text{ m of H}_2\text{O} \end{aligned}$$

$$\begin{aligned} P_{\text{atm}} &= \rho_{\text{Hg}} \times g \times h \\ &= 13.6 \times 1000 \times 9.81 \times 760 \times 10^{-3} \text{ Pa} \\ &= 101.396 \text{ KPa} = 1.01396 \text{ bar} \approx 1 \text{ bar} \end{aligned}$$

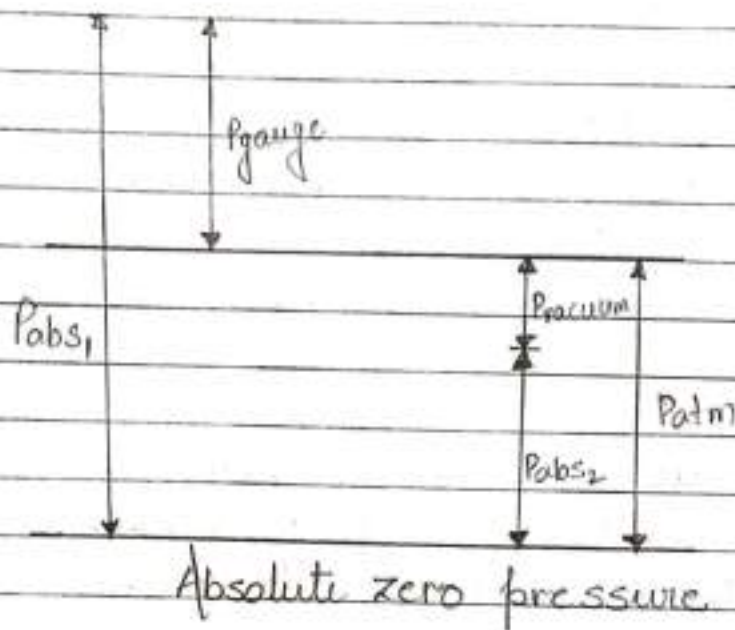
2. Gauge Pressure (P_{gauge})

Date _____
Page _____

It is defined as the pressure measured by the gauge (instruments) in which atmospheric pressure is taken as datum. If the measure pressure is lower than atmospheric pressure, then it is called negative gauge pressure or vacuum pressure.

3. Absolute zero pressure -

It is the pressure exerted by absolute vacuum.



$$P_{abs_1} = P_{atm} + P_{gauge}$$

$$P_{abs_2} = P_{atm} - P_{vacuum}$$

Hydrostatic Law

It states that rate of increase of pressure in the vertically downward direction in a static fluid is equal to the specific wt. of the fluid.

$$\boxed{\frac{\partial P}{\partial z} = \gamma}$$

under equilibrium condition

$$\sum F_y = 0$$

$$-P \times dA + \left(P + \frac{\partial P}{\partial z} \cdot dz\right) \times dA - W = 0$$

$$\frac{\partial P}{\partial z} \cdot dz \cdot dA = \gamma \times dA \times dz$$

$$\boxed{\therefore \frac{\partial P}{\partial z} = \gamma = \rho g}$$



$\partial P = \rho g \cdot dz$
both side integration,

$$P = \rho g z + C$$

At $z = 0$, $P = P_{atm}$

$$P_{atm} = \rho g \times 0 + C$$

$$\therefore C = P_{atm}$$

$$\therefore P = \rho g z + P_{atm}$$

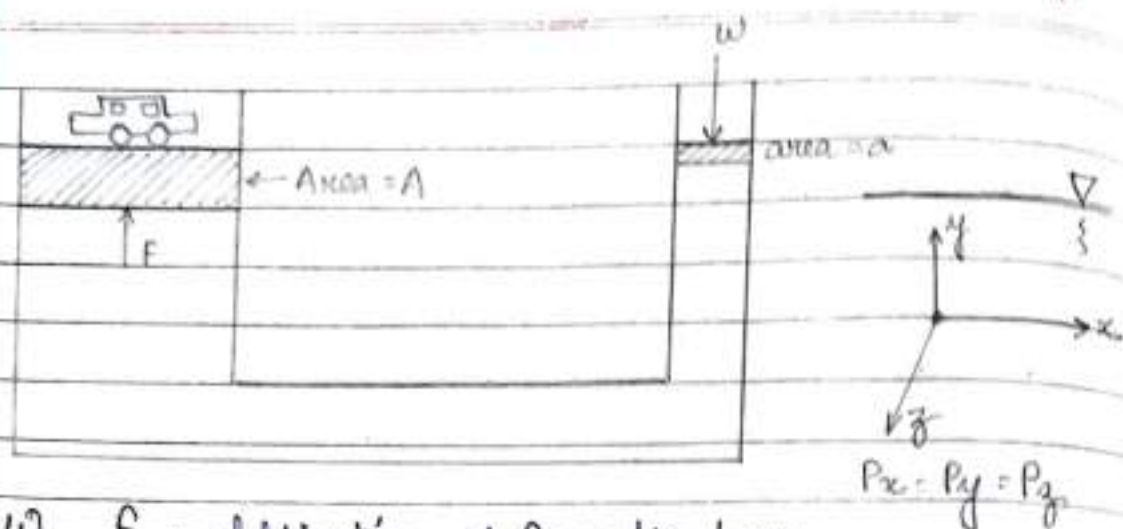
$$P - P_{atm} = \rho g z$$

$$\boxed{P_{gauge} = \rho g z}$$

At distance z from free surface, $P = \rho g z$

Pascal's law:

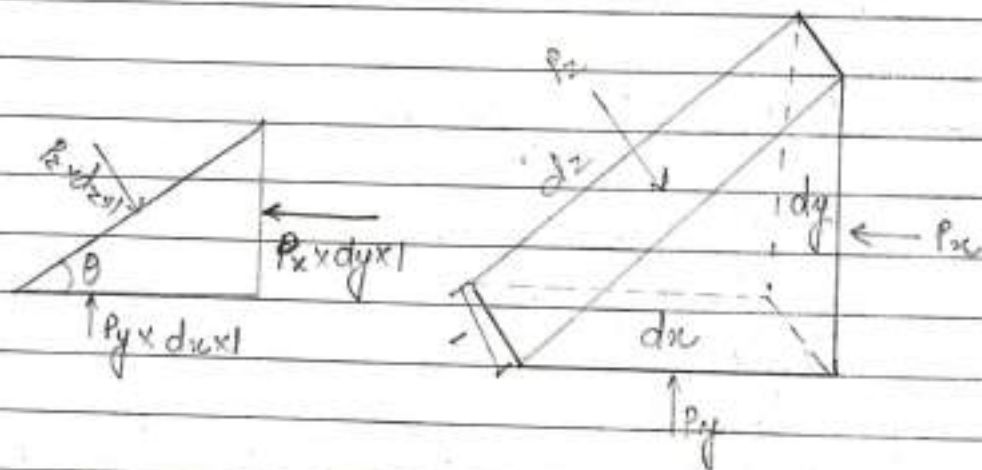
It states that in a static fluid, the pressure acts equally in all the directions.



$\frac{w}{a} = \frac{F}{A}$; application of Pascal's law
(Hydraulic lift)

$$\therefore F = \left(\frac{A}{a}\right) w$$

Consider a wedge shape fluid elements
Consider unit elements,
Resolve the forces along x -direction



$$\Rightarrow P_x \times dy \times 1 - P_z \times dz \times 1 \times \sin \theta$$

$$P_x dy = P_z dz \sin \theta \quad \text{--- (1)}$$

Resolve forces along y -direction

$$P_y dx = P_z dz \cos \theta \quad \text{--- (2)}$$

$$\sin \theta = \frac{dy}{dz}$$

$$\therefore dy = dz \sin \theta$$

$$\textcircled{1} \Rightarrow P_x \cdot dy = dy P_z$$

$$\therefore P_x = P_z \quad \text{--- (3)}$$

$$\cos \theta = \frac{dx}{dz}$$

$$\therefore dx = dz \cos \theta$$

$$\textcircled{2} \Rightarrow P_y dx = P_z dx$$

$$\therefore P_y = P_z \quad \text{--- (4)}$$

From eqn (3) & (4)

$$P_x = P_y = P_z$$

Pressure measurement device

Manometer

Simple

Differential

1. Piezometer

2. U-tube manometer

3. Single column manometer

i) Vertical column

ii) Inclined column

Manometer :

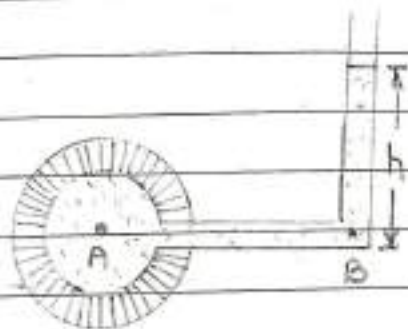
These are the devices which are used to measure the pressure at any point in the pipe by balancing the column of fluid by the different kind of the fluid.

a) Simple manometer

These are the device which contains a narrow tube whose one end is connected to the point where the pressure is to be measured and the other end is open to the atmosphere.

a) Piezometer

$$P_A = P_B = \rho g H + P_{atm}$$



$$\therefore P_A = \rho g H \text{ (gauge pressure)}$$

Piezometer is used for measurement of moderate pressure.

Piezometer is not useful for measurement of gas pressure since gases never form a surface with the atmosphere.

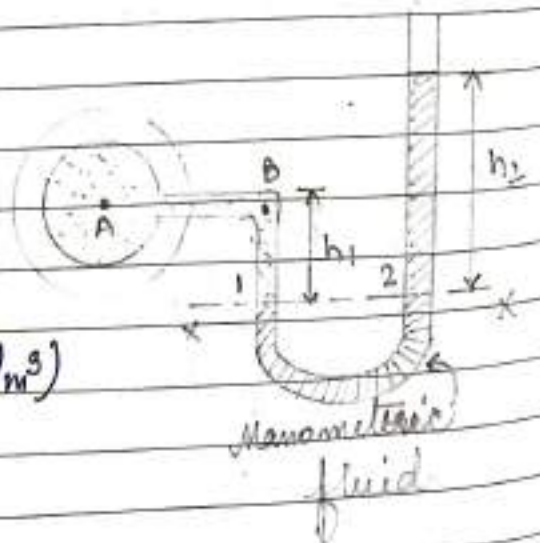
b) U-tube manometer

Manometer has manometric fluid.

→ Generally Hg as it has

i) low vapour pressure

ii) Highest density (13600 kg/m^3)



At datum line X-X

$$P_1 = P_2$$

$$\Rightarrow P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}$$

$$P_A + \rho_1 g h_1 = \rho_2 g h_2 + P_{atm}$$

$$\therefore P_A = \rho_2 g h_2 - \rho_1 g h_1 + P_{atm}$$

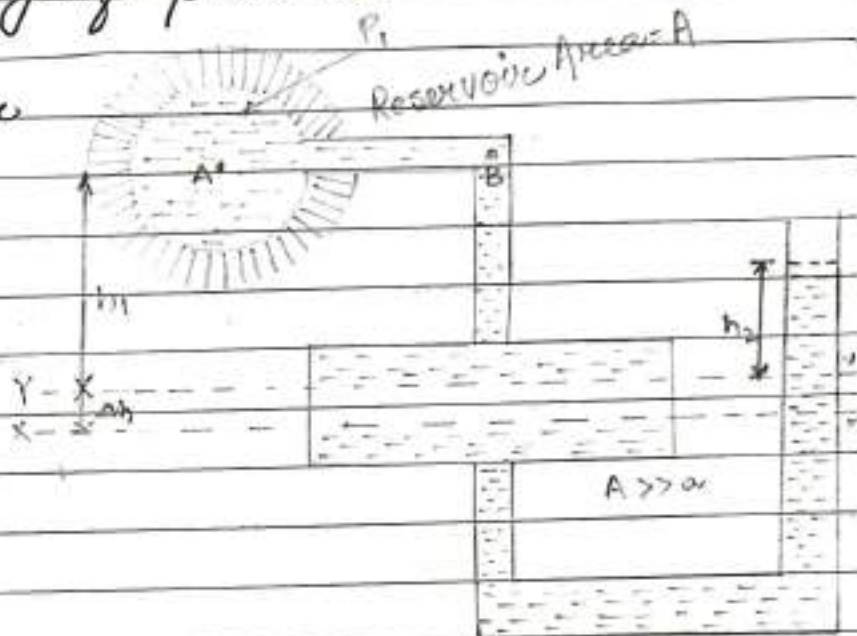
$$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1} \rightarrow \text{gauge pressure}$$

(c) Single column manometer

At datum line

$$P_A + \rho_1 g (h_1 + \Delta h) = P_{atm} + \rho_2 g (h_2 + \Delta h)$$

$$\therefore P_A = (\rho_2 g h_2 - \rho_1 g h_1) + (\rho_2 - \rho_1) g \Delta h$$



For volume equal

$$A \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a h_2}{A}$$

Since $a \ll A$, then Δh can be neglected

$$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1} \text{ (gauge pressure)}$$

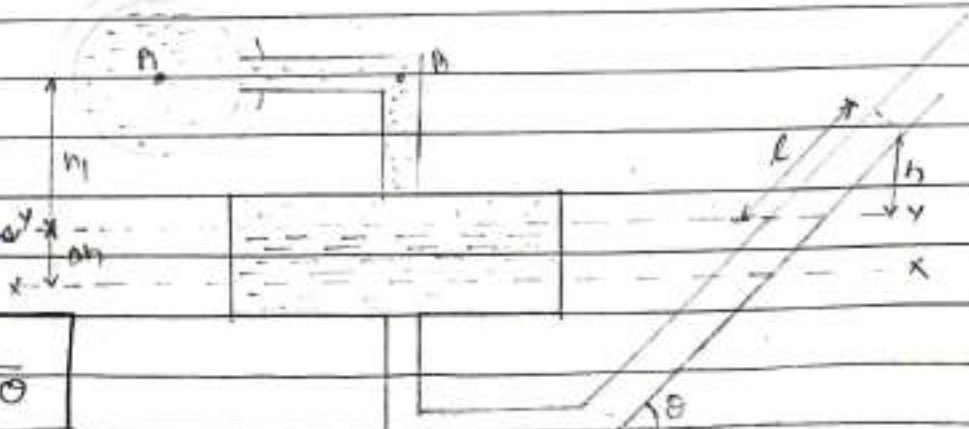
(d) Single column Inclined manometer

$$P = \rho g h$$

$$h_2 = L \sin \theta$$

As θ increases

Sensitivity decreases



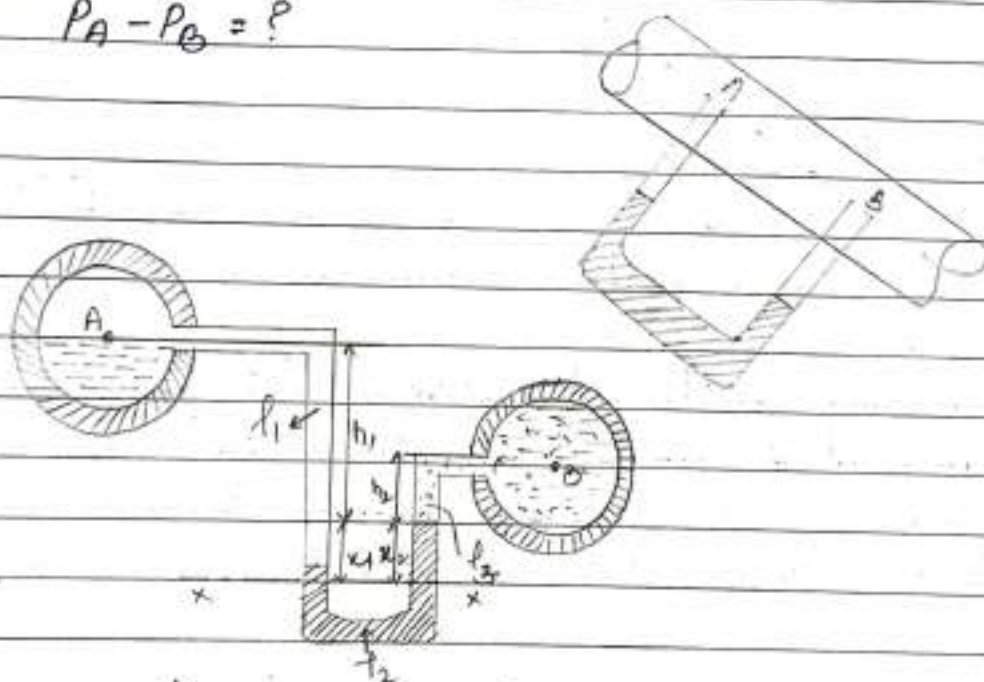
$$\therefore \text{Sensitivity} = \frac{1}{\sin \theta}$$

θ°	Sensitivity
0°	∞
30°	2
40°	1.15
90°	1

Differential Manometer:

It is a device used to measure the difference of pressure between any two points that may lie in the single pipe or in different pipe lines.

$$P_A - P_B = ?$$



At datum ^{line} X-X
 $P_1 = P_2$

$$P_A + \rho_1 g (h_1 + x) = P_B + \rho_2 g h_2 + \rho_3 g x$$

$$P_A - P_B = \rho_3 g h_2 - \rho_1 g h_1 - \rho_2 g x - \rho_1 g x$$

If the points A & B lie in the same horizontal line and having same fluid.

$$h_1 = h_2$$

$$P_A - P_B = (p_2 - p_1) g x$$

$$\therefore h_A - h_B = \frac{(p_2 - p_1) g x}{p_1 g}$$

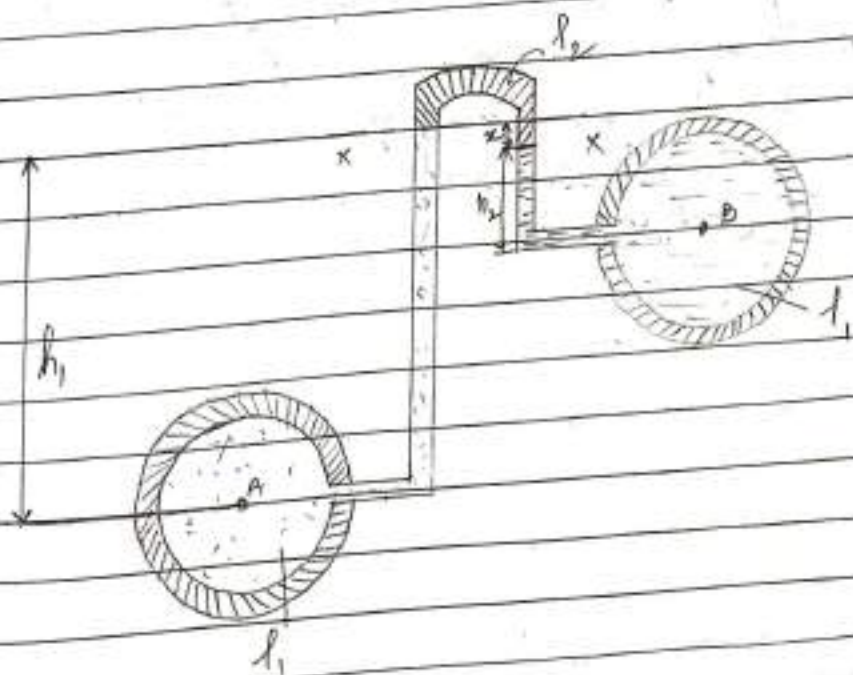
$$h_A - h_B = \left(\frac{p_2}{p_1} - 1 \right) x$$

$$= \left(\frac{p_2 / \rho_{H_2O}}{p_1 / \rho_{H_2O}} - 1 \right) x$$

$$\therefore h_A - h_B = \left(\frac{S_2 - S_1}{S_1} \right) \cdot x$$

$$\therefore h_A - h_B = \left(\frac{S_m}{S_L} - 1 \right) \cdot x$$

Inverted U-Tube Differential Manometer

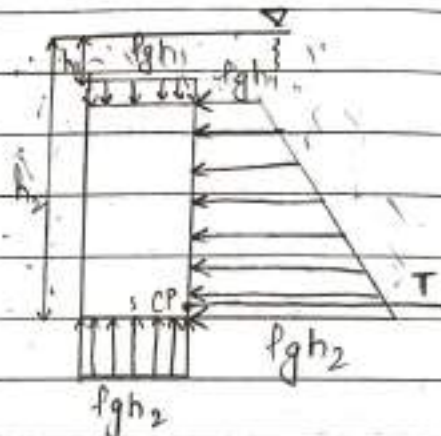
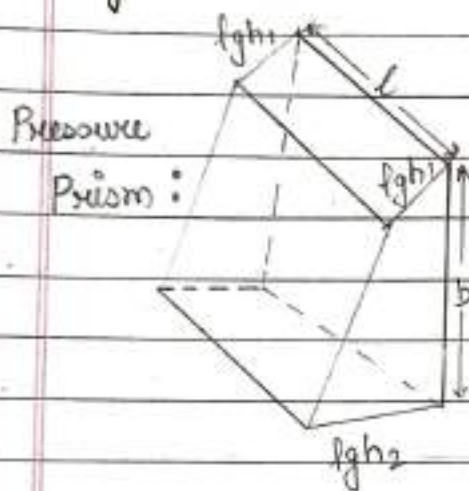


At the datum line $x-x$

$$P_A - \rho_1 g h_1 = P_B - \rho_1 g h_2 - \rho_2 g x$$

$$\therefore \boxed{P_A - P_B = \rho_1 g h_1 - \rho_1 g h_2 - \rho_2 g x}$$

Hydrostatic Forces on Submerged Surfaces.



Total Pressure (TP):

Whenever a surface either a plane or curve is completely submerged in the static fluid, the pressure force variations will take place across the surface. The resultant of all the pressure force variations is called 'Total Pressure'. It has units of 'Force' (N).

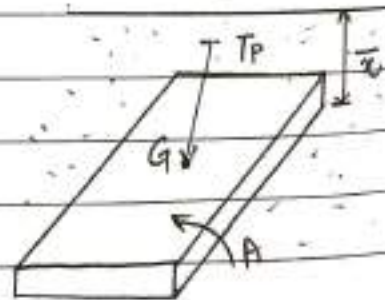
Centre of Pressure (CP):

It is the point at which total pressure will act.

a) Horizontal Surface.

Let $A \rightarrow$ area of surface

$w \rightarrow$ sp. wt. of liquid



$\bar{x}$ = distance of C.G from free surface.

$$\begin{aligned} T.P &= \text{pressure} \times \text{area} \\ &= \rho g \bar{x} \times A = \omega A \bar{x} \end{aligned}$$

$$TP = \omega A \bar{x} \text{ \& } CP = \bar{x}$$

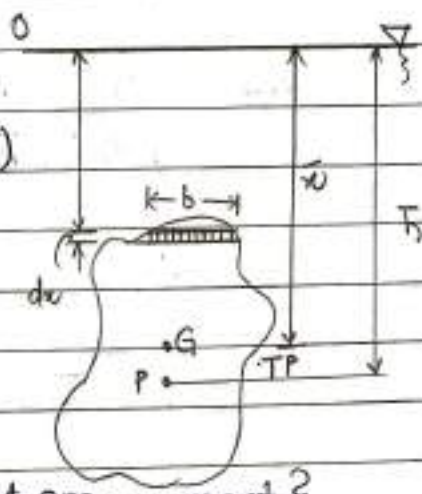
b) Vertical Surface

Pressure force acting on elemental area = $\rho g x \cdot (b dx)$

$$\begin{aligned} \text{Sum of the pressure forces} &= \int \rho g (b dx) \times x \\ &= \rho g \int (b dx) x \end{aligned}$$

$$\left\{ (b dx) x = \text{first area moment} \right\}$$

$$TP = \rho g A \bar{x} = \omega A \bar{x}$$



"Sum of the moments of individual forces is equal to the moment caused by the resultant force -"
Varignon's Theorem.

$$\begin{aligned} \text{Moment caused by pressure force acting on} \\ \text{elemental area about free surface} &= \text{Force} \times x \\ &= (P \times A) \times x \\ &= \rho g x \cdot b dx \cdot x \\ &= \rho g (b dx) x^2 \end{aligned}$$

$$\begin{aligned} \text{Sum of moments} &= \int \rho g (b dx) x^2 \\ &= \rho g \int (b dx) x^2 \quad [(b dx) x^2 = \text{second area moment} = I] \\ &= \rho g I_0 \end{aligned}$$

Parallel Axis Theorem:

$$I_G I_0 = I_G (I_G + A\bar{x}^2) \quad \text{--- (i)}$$

Moment caused by TP about free surface

$$= TP \times \bar{h}$$

$$= \omega A \bar{x} \times \bar{h} \quad \text{--- (ii)}$$

From (i) and (ii)

$$I_G A \bar{x} \times \bar{h} = I_G (I_G + A\bar{x}^2) \quad \left\{ \begin{array}{l} I_G A \bar{x} \times \bar{h} = I_G I_0 \\ \Rightarrow \bar{h} = \frac{I_0}{A\bar{x}} \end{array} \right\}$$

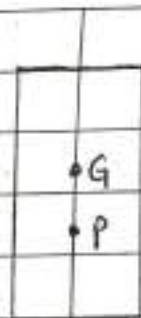
$$\therefore \bar{h} = \frac{I_G}{A\bar{x}} + \frac{A\bar{x}^2}{A\bar{x}}$$

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}} \quad ; \quad I_G \rightarrow \text{MI of the surface about and axis passing through C.G.}$$

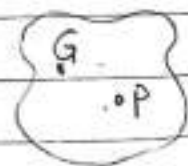
$\therefore \bar{h} > \bar{x} \Rightarrow \text{C.P. always lies below C.G.}$

$$\bar{h} - \bar{x} = \frac{I_G}{A\bar{x}}$$

* For symmetrical surfaces, C.G. & C.P. lie along the same line.



Symmetrical



Unsymmetrical

$\begin{aligned} TP &= \omega A \bar{x} \\ CP &= \bar{x} + \frac{I_G}{A\bar{x}} \end{aligned}$
--

c) Inclined Surface

Pressure force acting on elemental area = $\rho g x \cdot (b dx)$

$$\sin \theta = \frac{x}{a} = \frac{\bar{x}}{c} = \frac{h}{d}$$

$$= \rho g (b dx) a \sin \theta$$

Sum of pressure forces

$$= \int \rho g (b dx) a \sin \theta$$

$$= \rho g (\sin \theta) \int (b dx) a$$

$$= \rho g \sin \theta \times A \times c$$

$$= \rho g \sin \theta \times A \times \frac{\bar{x}}{\sin \theta} = \rho g A \bar{x}$$

$$TP = A \omega \bar{x}$$

Moment of pressure force acting on elemental area

$$= \rho g x \cdot (b dx) a$$

$$= \rho g (a \sin \theta) (b dx) a$$

Sum of the moments = $\int \rho g (b dx) a^2 \sin \theta$

$$= \rho g \sin \theta \int (b dx) a^2$$

$$= \rho g \sin \theta \cdot I_x$$

$$= \rho g \sin \theta (I_G + A c^2) \quad c = \frac{\bar{x}}{\sin \theta}$$

$$= \rho g \sin \theta \left[I_G + A \left(\frac{\bar{x}}{\sin \theta} \right)^2 \right] \quad \text{--- (i)}$$

Moment caused by total pressure about $x-x$, = $TP \times d$

$$= A \omega \bar{x} \times d \quad \text{--- (ii)}$$

From (i) & (ii)

$$\omega A \bar{x} d = \omega A \bar{x} \frac{h}{\sin \theta} = \rho g \sin \theta \left(I_G + A \frac{\bar{x}^2}{\sin^2 \theta} \right)$$

$$\bar{h} = \frac{I_G \sin^2 \theta}{A \bar{x}} + \bar{x}$$

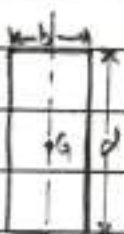
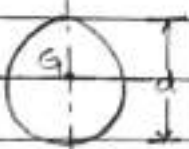
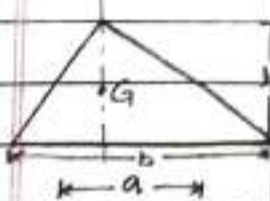
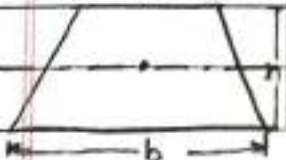
$$\begin{aligned} TP &= \omega A \bar{x} \\ CP &= \frac{I_G \sin^2 \theta}{A \bar{x}} + \bar{x} \end{aligned}$$

• For horizontal surface, $\theta = 0^\circ$,

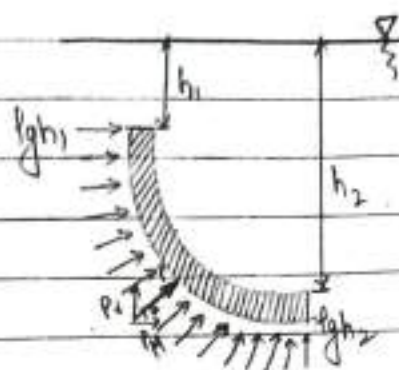
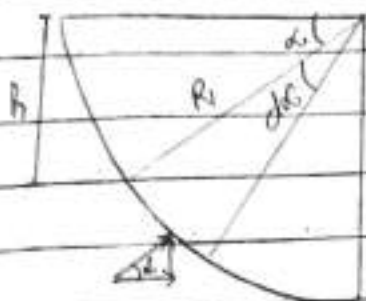
$$\bar{h} = \bar{x}$$

• For vertical surface, $\theta = 90^\circ$,

$$\bar{h} = \bar{x} + \frac{I_G}{A \bar{x}}$$

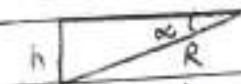
Shape	C.G from base	Area	I_G parallel to base
	$d/2$	$b \times d$	$\frac{bd^3}{12}$
	$d/2$	$\frac{\pi}{4} d^2$	$\frac{\pi}{64} d^4$
	$h/3$	$\frac{1}{2} bh$	$\frac{bh^3}{36}$
	$\left(\frac{2a+b}{a+b}\right) \frac{h}{3}$	$\frac{1}{2} (a+b) h$	$\frac{a^2 + 4ab + b^2}{36(a+b)} \times h^3$

d) Curved Surface



$$dA = R \times \alpha \times 1$$

$$\text{Pressure force acting on } dA = \rho g h \times dA \\ = \rho g h \times R d\alpha$$



$$\sin \alpha = \frac{h}{R}$$

$$h = R \sin \alpha$$

$$\therefore \text{TP acting on } dA = \rho g (R \sin \alpha) R d\alpha = \rho g R^2 \sin \alpha d\alpha$$

$$\text{Horizontal component of TP} = \rho g R^2 \sin \alpha d\alpha \times \cos \alpha$$

$$\text{Sum of horizontal components} = \int_{\alpha=0}^{\pi/2} \rho g R^2 \sin \alpha \cos \alpha d\alpha$$

$$= \rho g R^2 \int_{\alpha=0}^{\pi/2} \sin \alpha \cos \alpha d\alpha$$

$$= \rho g R^2 \left(\frac{-\cos 2\alpha}{2} \right)_{\alpha=0}^{\pi/2}$$

$$= \frac{\rho g R^2}{2}$$

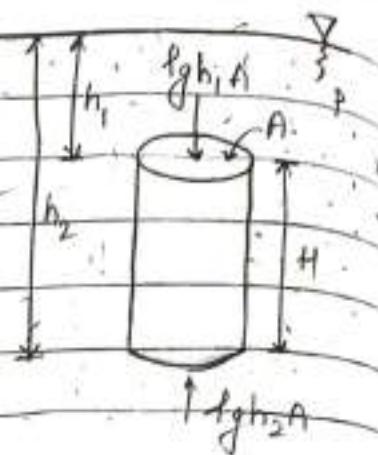
$$= \rho g (R \times 1) \frac{R}{2} = W A \bar{x}$$

$$P_H = W A \bar{x}$$

$P_V = W$ of liquid supported by curved surface

Buoyancy

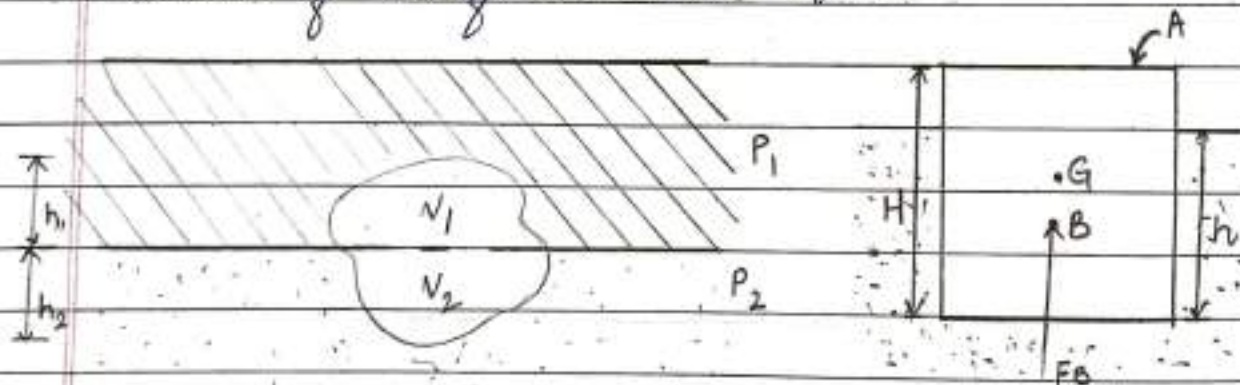
$$\begin{aligned}
 \text{Buoyancy Force} &= \rho g h_2 A - \rho g h_1 A \\
 &= \rho g A (h_2 - h_1) \\
 &= \rho g A H = \rho g V_{\text{body}} \\
 &= \rho g V_{\text{fluid displaced}} \\
 &= \text{Weight of fluid displaced}
 \end{aligned}$$



Whenever an object submerged partially or completely, it will be subjected to the net resultant vertical upward force and is known as 'Buoyancy force' or 'Force of Buoyancy'.

The point at which the force of buoyancy acts is known as 'Centre of Buoyancy'. The Centre of Buoyancy is the centroid of the volume of fluid displaced (volume of body immersed).

$$\text{Volume of body immersed} = A \times h$$



$$\text{Buoyancy force} = \rho_1 g V_1 + \rho_2 g V_2$$

At $p = 1 \text{ bar}$, $T = 20 + 273 = 293 \text{ K}$
 $p = \rho R T$

$$\therefore \rho = \frac{P}{RT} = \frac{100}{0.287 \times 293} = 1.18 \text{ kg/m}^3$$



$F_B = \text{weight of liquid displaced} + \text{weight of air displaced.}$

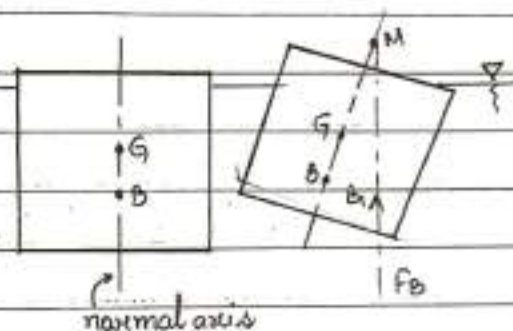
Since P of air is very small in magnitude, weight of air displaced will be neglected.

$\therefore$ For floating bodies,

$$F_B = \text{weight of liquid displaced}$$

Meta centre (M)

It is defined as the point about which the body will try to oscillate.

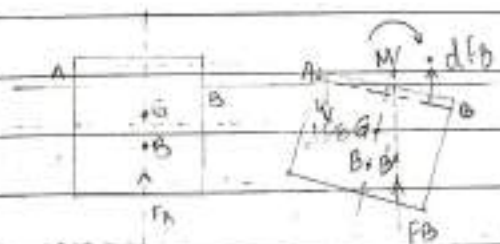


* **Metacentric Height (GM)**

It is the distance from metacentre to centre of gravity.

① **Analytical Method**

$$GM = \frac{I_{yy}}{V} - BG$$



$I_{yy} \rightarrow$ moment of inertia about $y-y$ axis

$V \rightarrow$ volume of liquid body immersed.

Due to change of centre of buoyancy, F_B will induce a moment.

This moment will ^{be} exactly equal in magnitude to the moment caused by the forces, F_B .

② Experimental Method

Let w be the weight of object and w_1 be the weight of small object.

$$F_B \times GG_1 = w_1 \times x$$

$$w \times GG_1 = w_1 \times x$$

$$w \times Gm \times \tan \theta = w_1 \times x$$

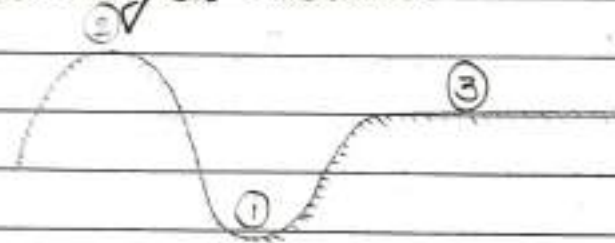
$$\therefore Gm = \frac{w_1 \times x}{w \tan \theta}$$



$$\tan \theta = \frac{GG_1}{Gm}$$

Conditions of Equilibrium

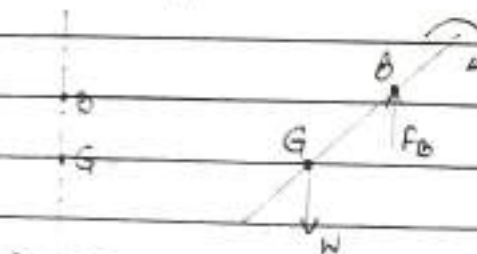
a) Submerged Bodies



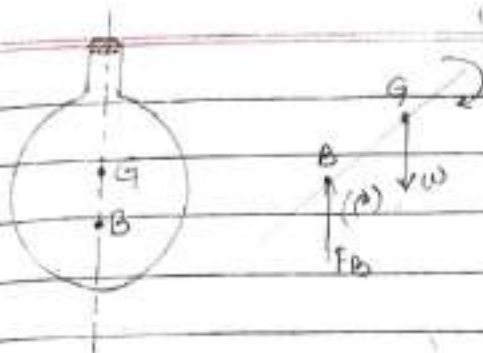
① → Stable Equilibrium

② → Unstable Equilibrium

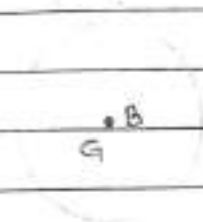
③ → Neutral Equilibrium



Stable Equilibrium
(B is above G)

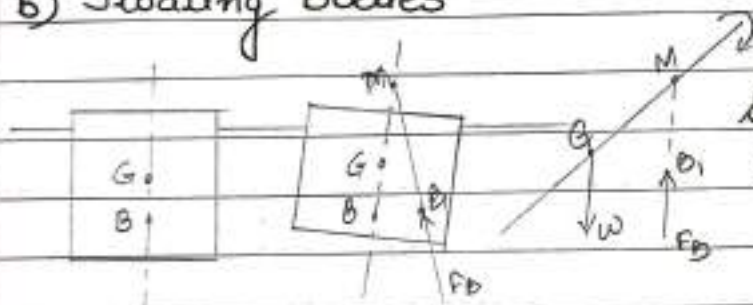


Unstable Equilibrium
(B is below G)

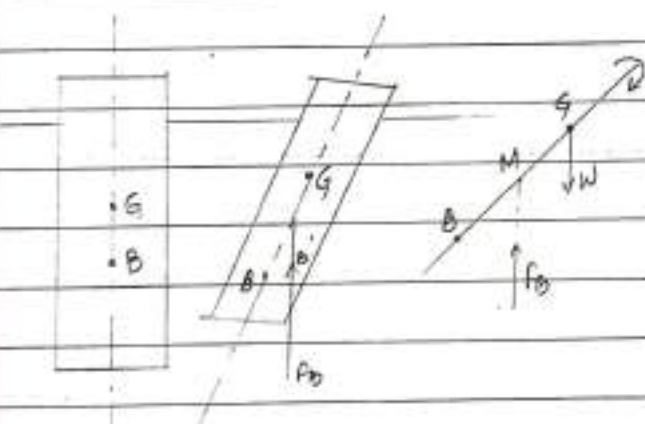


Neutral Equilibrium
(B coincides with G)

b) Floating Bodies



Stable Equilibrium
(M is above G)



Unstable Equilibrium
(M is below G)

Neutral Equilibrium (M coincides with G)

Time period of oscillation, $T = 2\pi \sqrt{\frac{I}{GM \times g}}$

GM $\rightarrow$ Metacentric height

When ship is loaded vertically as a stack, CG shifts upward and chances of M coming

below G are more. So, for the safer side, GM is increased, but it cannot be increased randomly considering the comfort factors. ($T \propto \frac{1}{\sqrt{GM}}$)

Summary of Annotations

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