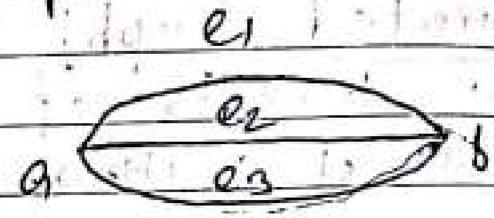
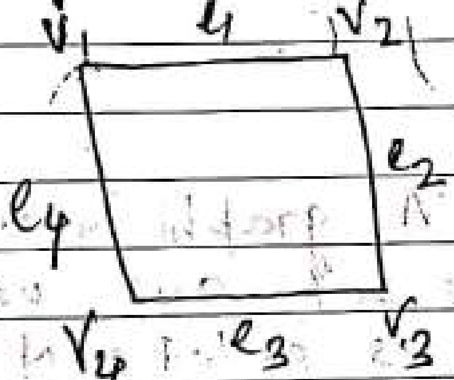
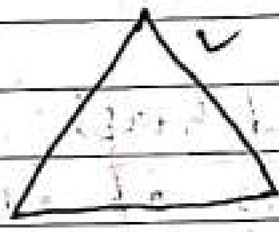
Parallel or Multiple edge: in a graph it may be possible to have more than one edge with a single pair of vertices such edges are called parallel edges.

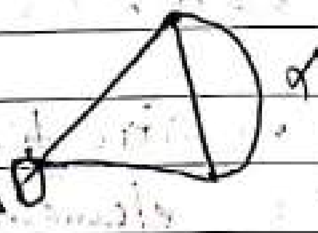
e_1, e_2, e_3 are parallel edges.



Simple graph: A graph which contain neither self loop nor parallel edge called simple graph.

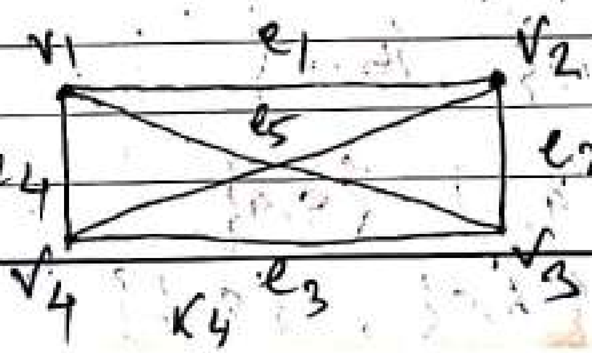


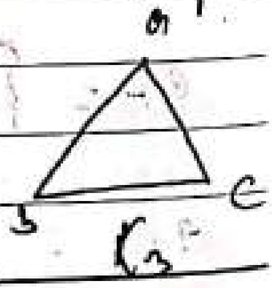




(K_n) complete graph: A simple graph in which there is exactly one edge between each pair of distinct vertices is called complete graph.

- every vertex is connected to every other vertex

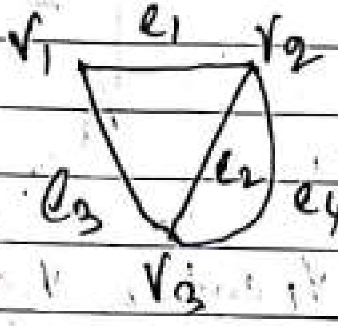




Multigraph: A graph containing parallel edge is called multigraph.

No loops are allowed.

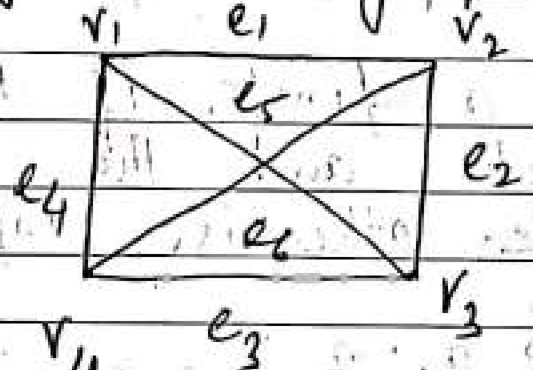
- More than one edge b/w two vertices



e_4, e_2 are parallel edges.

Order and size of Graph: The no. of vertices in a graph G is called order.

The no. edge in a graph G is called size.



Order = 4 , size = 6

Adjacent vertices and Adjacent edges:
Two vertices u and v are said to be adjacent if there exist an edge (AV).

If two edge have a common vertex then they are called adjacent edge.

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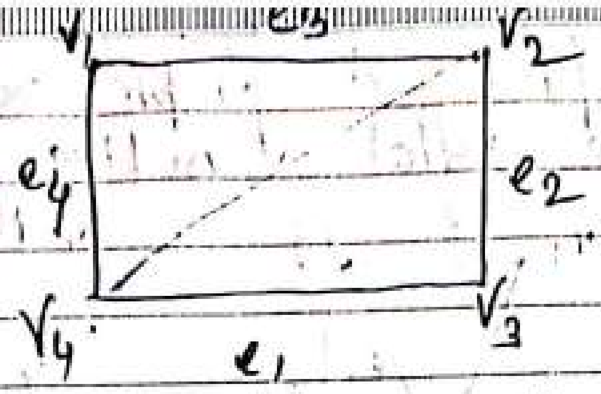


Adjacent Vertices = v_1 and v_2, v_2 and v_3, v_3 and v_4, v_4 and v_1

Adjacent Edges = e_1e_2, e_2e_3, e_3e_4, e_4e_1.

Finite Graph and Infinite Graph: A graph is ~~which~~ a finite if both its vertex set and the edge set are finite. otherwise infinite.

$v = \{0, 1, 2, 3, \ldots \infty\}$ infinite

$e = \{1, 2, 3 \ldots \infty\}$ infinite.

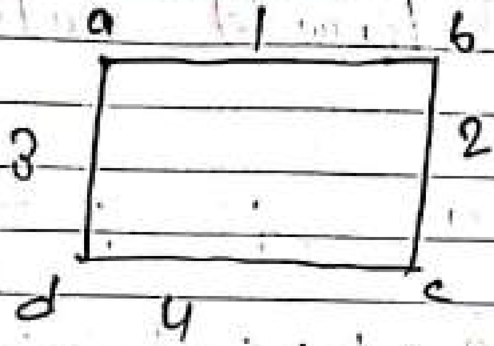
$v = \{a, b, c\}$

$e = \{1, 2, 3\}$ finite

Weighted Graph: A graph in which weights are assigned to every

non-negative real numbers

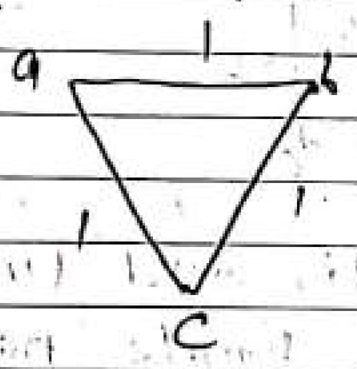
edge is called a weighted graph.



1, 2, 3, 4 are weigh graph.

W(e) = weight of the edge

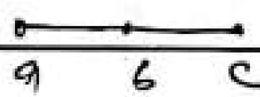
Path : In a path, vertices and edges may be repeated any number. The number of edge in a path is called length of the path. A path of length zero is called trivial path.



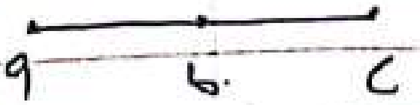
Path	length
a-b-c	3
a-b	1
b-c	= 1
a	0

Open Path: A path in which initial and terminal vertices are distinct is called open path. a→b→c→d —

a-b

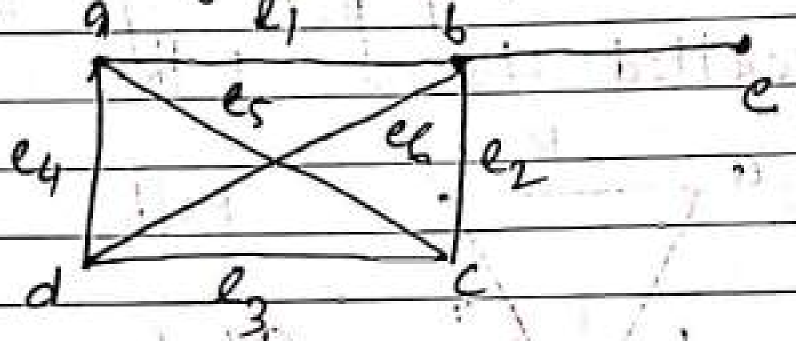


closed path: A path which initial and terminal vertices are same.

a-b-c-a 

a -> b -> c -> b -> a

Simple Path: A path is said to to be simple if all the edge and vertices on path are different except possibly at the end points



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i) a-b-c-a

ii) a-b-c-d-c-a

iii) a-b-c-d-a

iv) a-b-c-a-b

(i) and (iii) are simple path

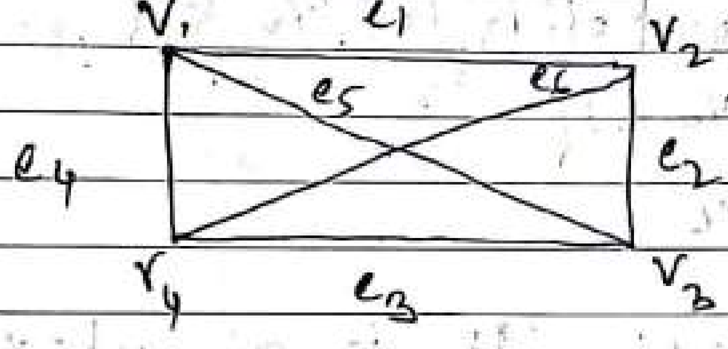
(ii) and (iv) are not simple path

Circuit and cycle: A path of length $l \geq 1$ with no repeated edges and whose end points are equal is called a circuit.

In a circuit, repitation of vertices is allowed.

A cycle is a circuit with no other repeated vertices except the end point.

every cycle is a circuit but a circuit need not by cycle.



$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_1$ X V

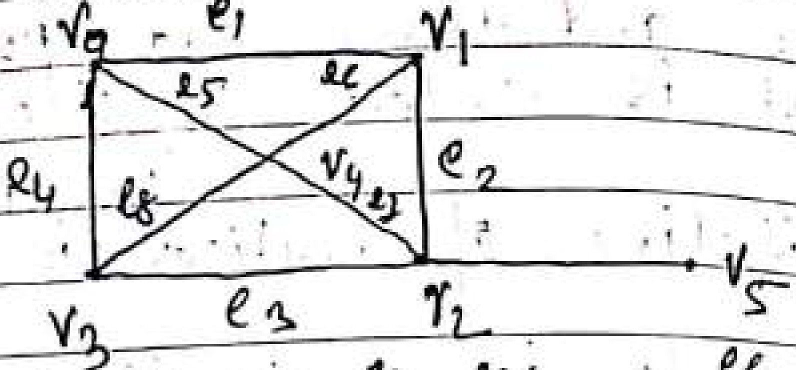
$e_1 e_2 e_5$ x E

Edge Disjoint Path and Vertex Disjoint Path:

Two paths in a graph are said to be edge disjoint if they have no common edges but they have a common vertices.

Two paths in a graph are

said to be vertex disjoint if they have no common vertices



e_5 e_8 e_6 e_7

edge disjoint = $(v_0, v_4), (v_4, v_3), (v_1, v_4), (v_4, v_2)$ are edge disjoint.

vertex disjoint = $\{(v_3, v_2), (v_2, v_5)\}$,
e_3 e_9
$\{(v_0, v_4)\ (v_4, v_1)\}$
e_5 e_6

Cut Vertex:- It is a vertex by which if we remove that vertex then the graph will be disconnected graph.

Cut Set: Cut set partition all the vertices into two disjoint sets. Cut set always contain only one branch and rest of edges are chord

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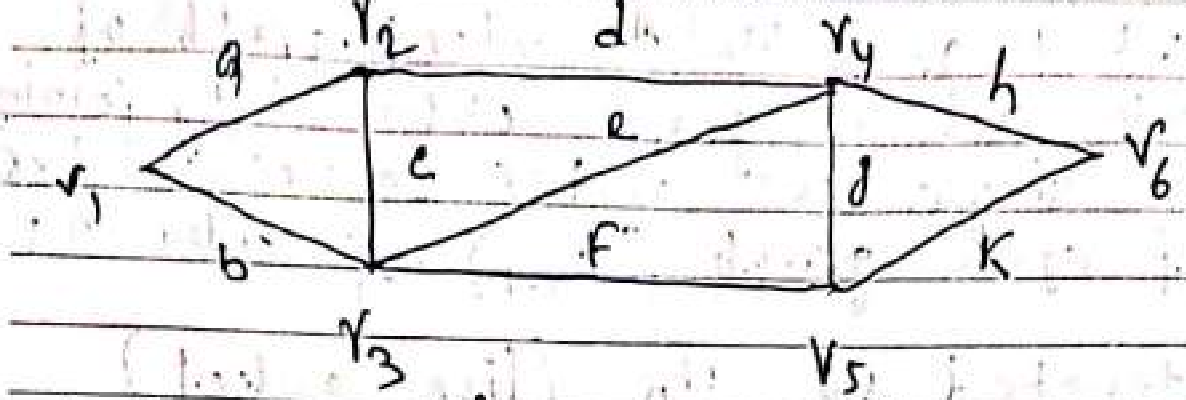
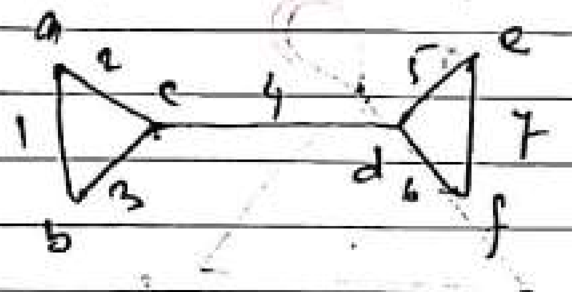


branches : {b, c, f, h, k, g}

Chord : {a, d, e, j}

Bridge :- It is an edge with connects the two vertices and removal of such edge disconnects the graph into two disjoint graph



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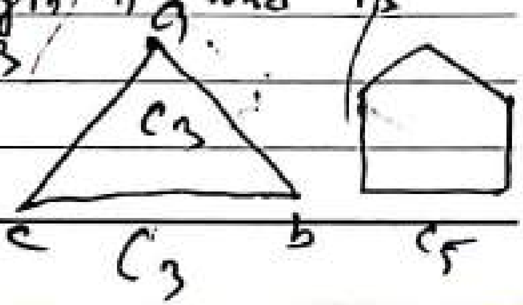
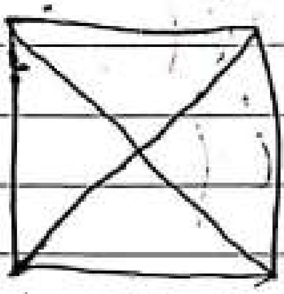
Bridge graph

Cycle graph (C_n) A cycle graph of order n is a connected graph whose edges form a cycle of length n and is denoted by C_n, n ≥ 3

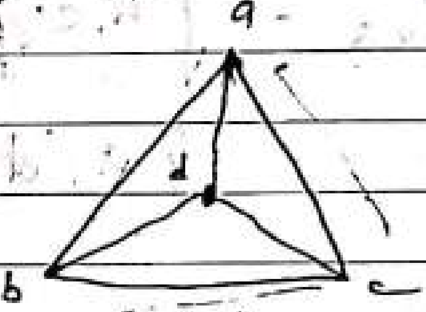


Wheel graph(W_n) A wheel graph of order n is a graph obtained by joining a single new vertex to each vertex of cycle graph (C_{n-1}) of order n-1

denoted by W_n (like wheel)

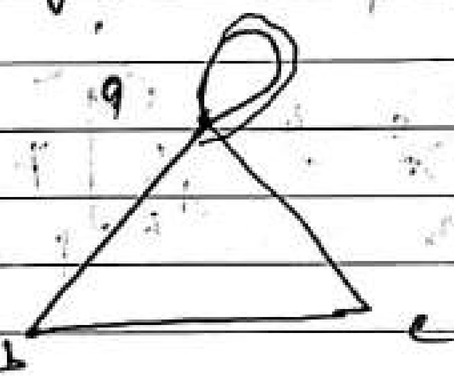


W_4

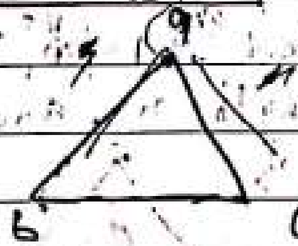


C_3/W_3

Pseudo Graph :- loop and multiple edges are allowed



Degree Vertex :- The no. of edges



d(a) = 2

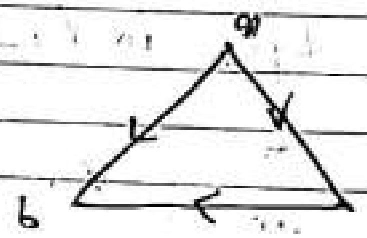
d(b) = 2

d(c) = 2

In degree in(a) = 0 out(a) = 2

(b) = 2 (b) = 0

out degree (c) = 1 c = 1



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Bipartite Graph:

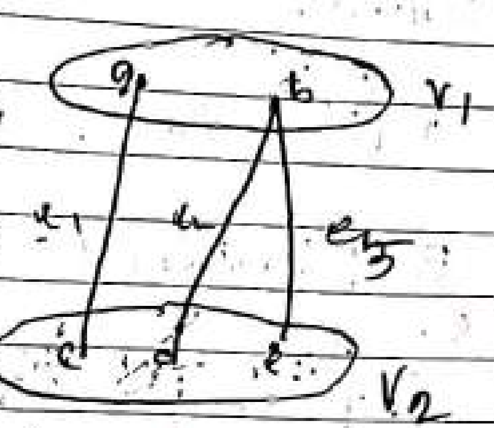
$G = \{V, E\}$

· If vertex set can be split in two parts V_1, V_2 such that every edge has its ends in two different classes

· It has no loop.

$V : \{a, b, c, d, e\}$

$V_1 : \{a, b\}$ $\quad$ $V_2 : \{c, d, e\}$



Complete Bipartite:

· each vertices of V_1 are connected to each vertices of V_2

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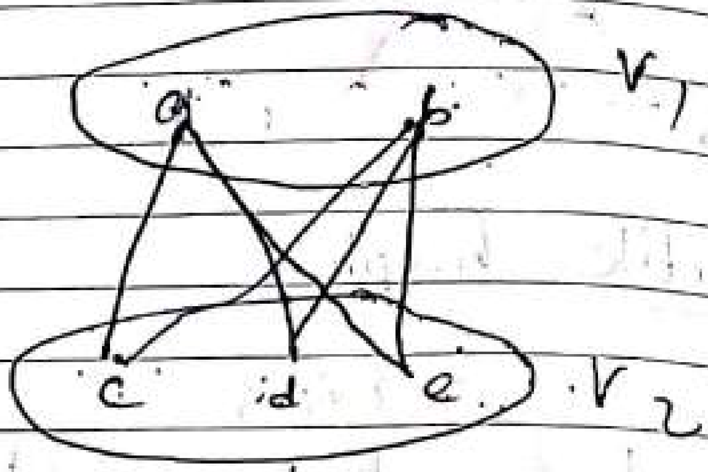
It is denoted by $K_{m,n}$

m = No. of edges in V_1

n = No of edges in V_2



$K_{2,3}$

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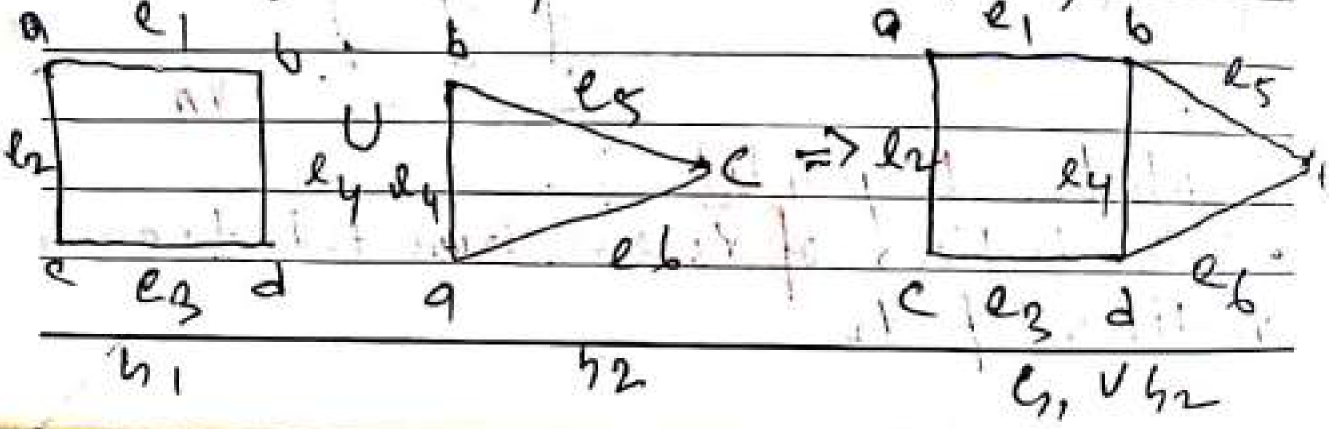
Operations of Graph

① **Union;** G_1 / G_2

$$V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$$

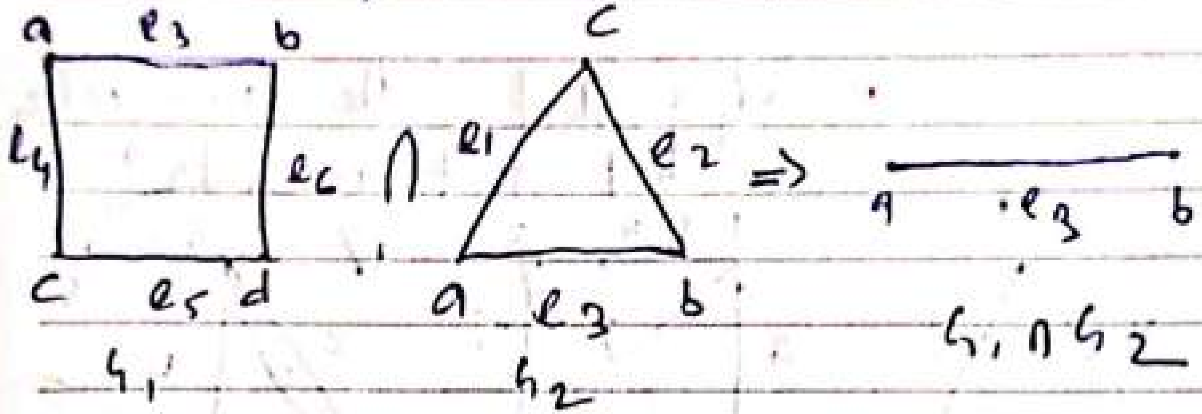
$$E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$$



(2) Intersection

$V(G_1 \cap G_2) = V(G_1) \cap V(G_2)$

$E(G_1 \cap G_2) = E(G_1) \cap E(G_2)$

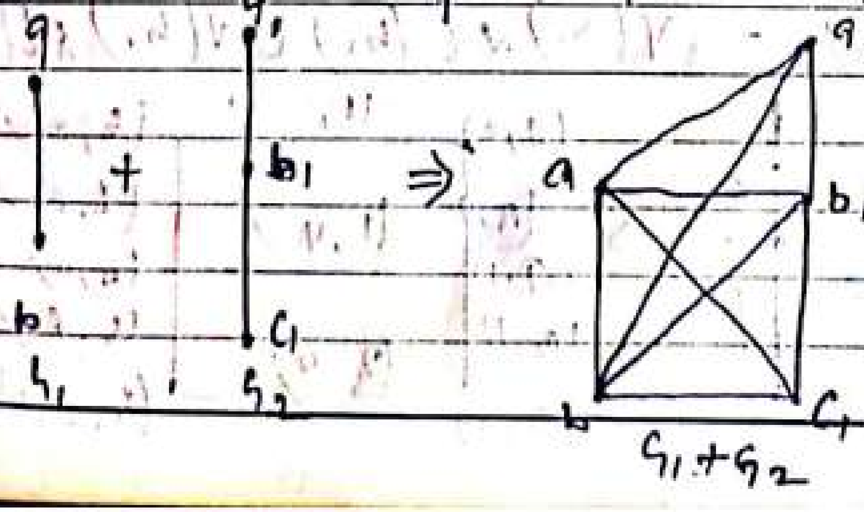


(3) Sum of two Graphs:

$G_1 + G_2$

$V(G_1 + G_2) = V(G_1) + V(G_2)$

$E(G_1 + G_2)$ = edges, which are in G_1 and in G_2 and edges obtained by joining each vertex of G_1 to each vertex of G_2.

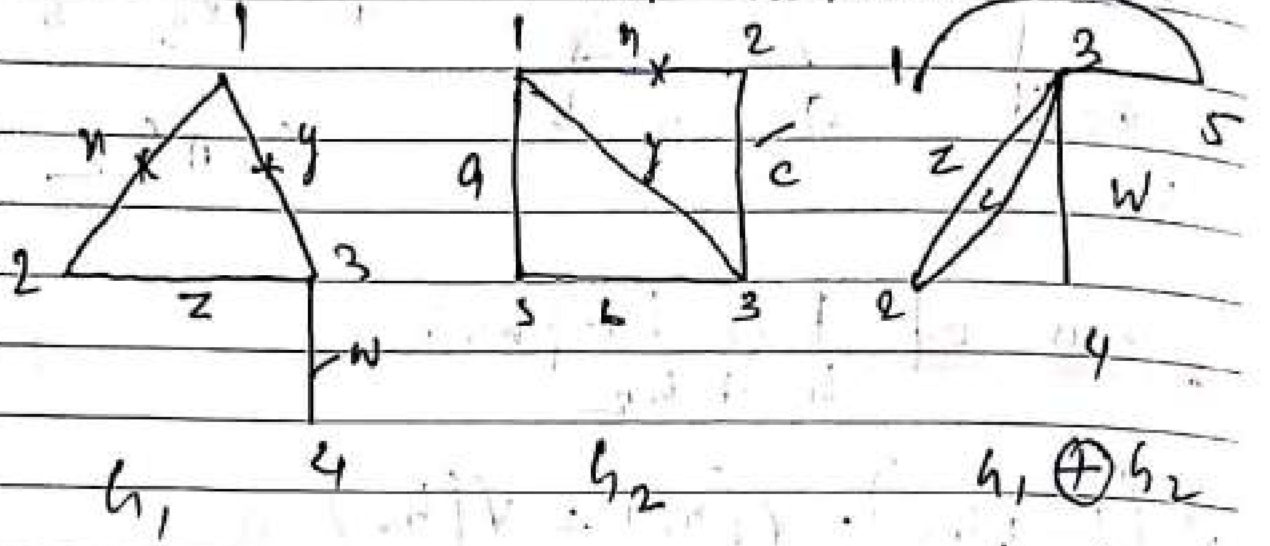


(4) Ring Sum: $G_1 \oplus G_2$

$V(G_1 \oplus G_2) = V(G_1) \cup V(G_2)$

$E(G_1 \oplus G_2) = E(G_1) \cup E(G_2) - E(G_1) \cap E(G_2)$

edges that either in G_1 or G_2 but not in both



(5) Product: $(G_1 \times G_2)$

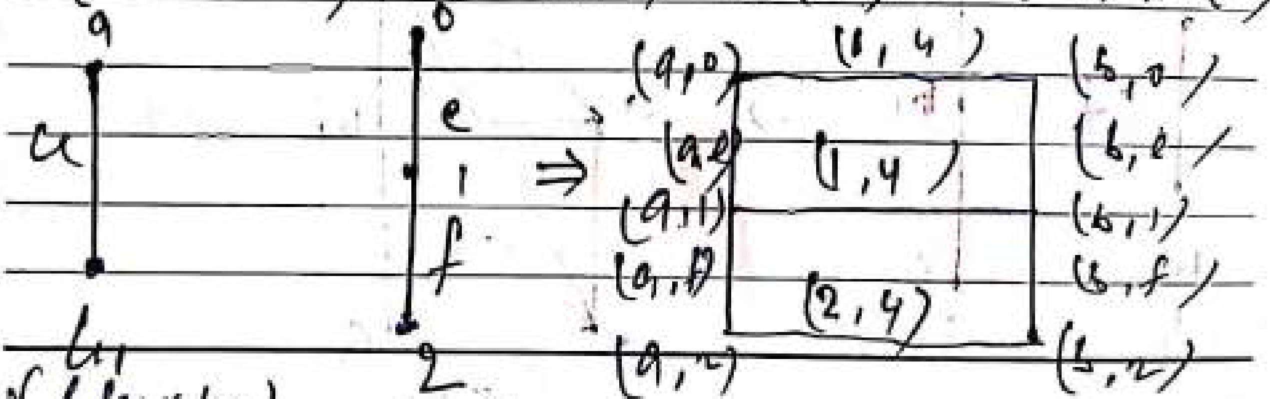
$V(G_1) = \{a, b\}$

$V(G_2) = \{0, 1, 2\}$

$E(G_1) = \{a\}$

$V(G_1 \times G_2) = V(G_1) \times V(G_2)$, $E(G_2) = \{e, f\}$

$E(G_1 \times G_2) = V(G_1) \times E(G_2) \cup V(G_1) \times E(G_2)$



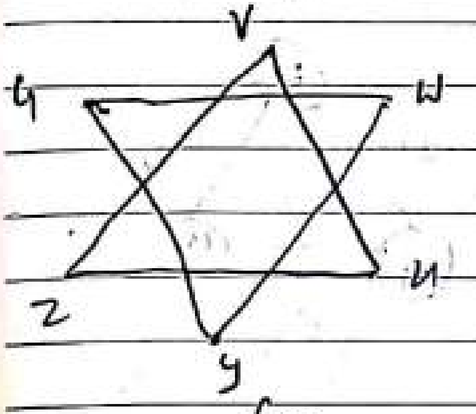
$V(G_1 \times G_2) =$

$V(G_1 \times G_2) = \{(a,0),(a,1),(a,2),(b,0),(b,1),(b,2)\}$

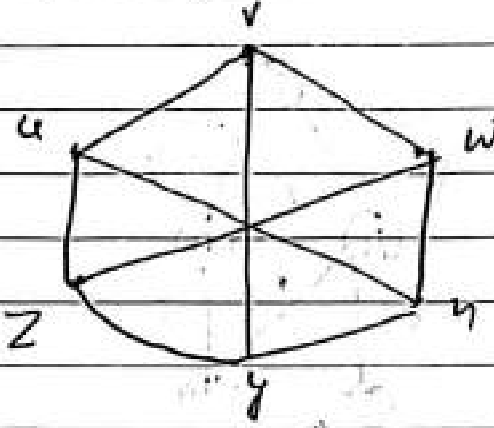
$E(G_1 \times G_2) = \{(a,e),(a,f),(b,e),(b,f),(0,u),(1,u),(2,u)\}$

Complement : $G \to G'$

Two vertices u and v adjacent only when they are not adjacent in G.



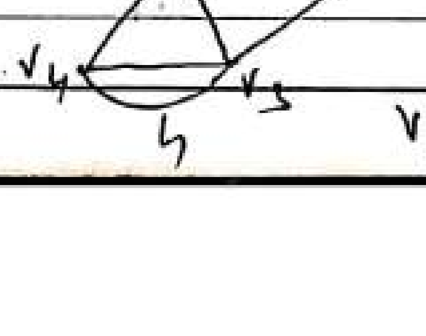
G :



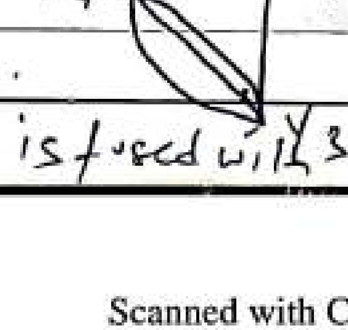
G'

Fusion / Merged / Identified :

v_1 and v_2 in graph G is said to be fused if these two vertices are replaced by a single new vertex v such that every edge that was adjacent to either v_1 or v_2 or both adjacent.



G



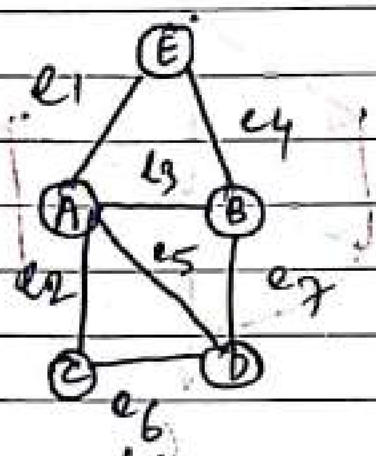
v_1 is fused with v_4 v_3

Subgraph : Suppose there are two graph

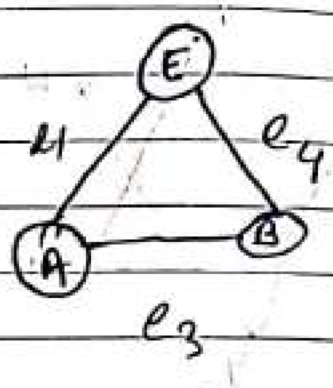
$$H = (V_2, G_2)$$

$$G = (V_1, G_1)$$

H is a subgraph of G if set of vertices of graph H is a subset of graph G set of edge of graph H is a subset of graph G.

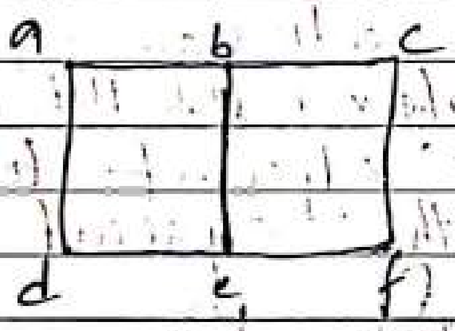


(G)

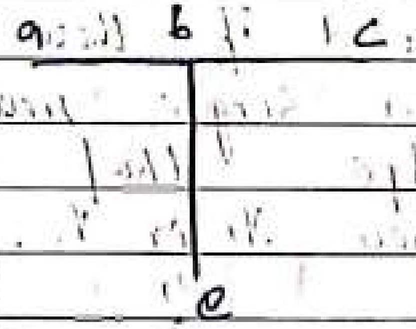


H

or, a graph having all of its vertices and edges in G.



G

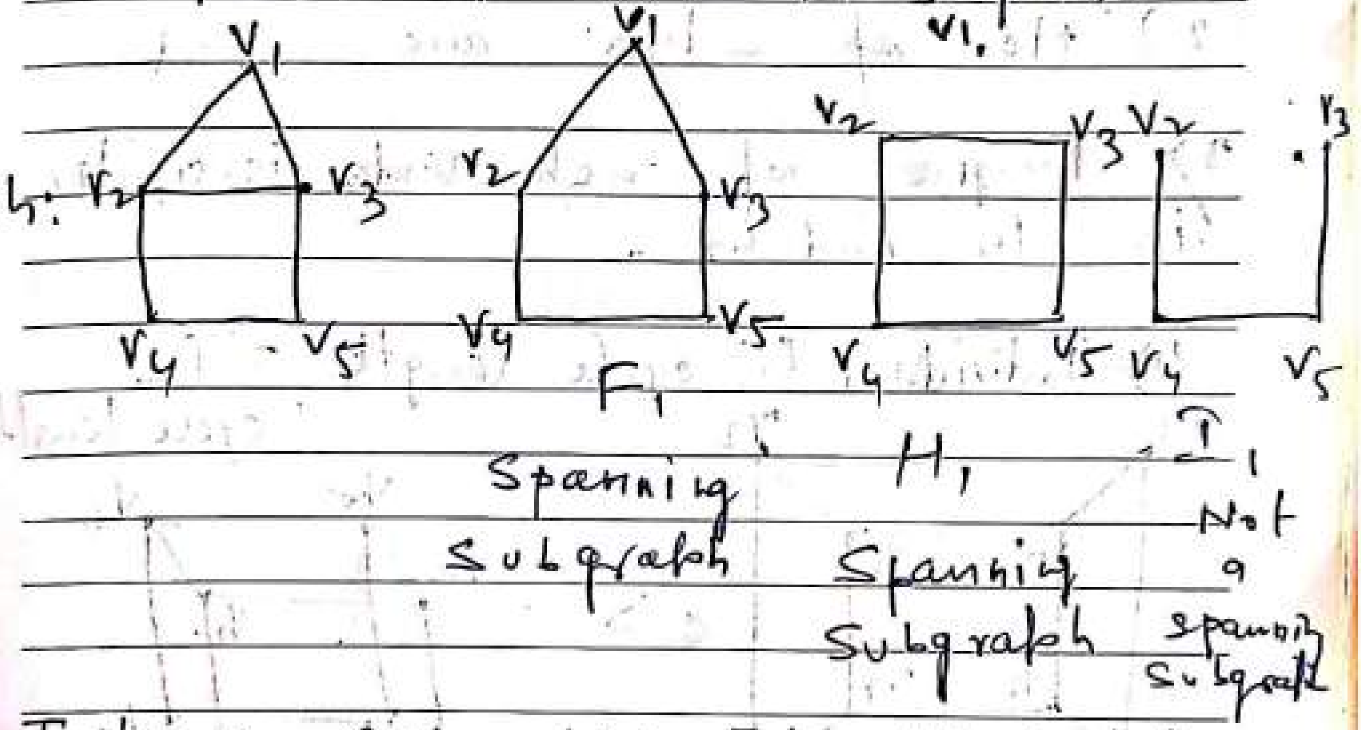


G_1

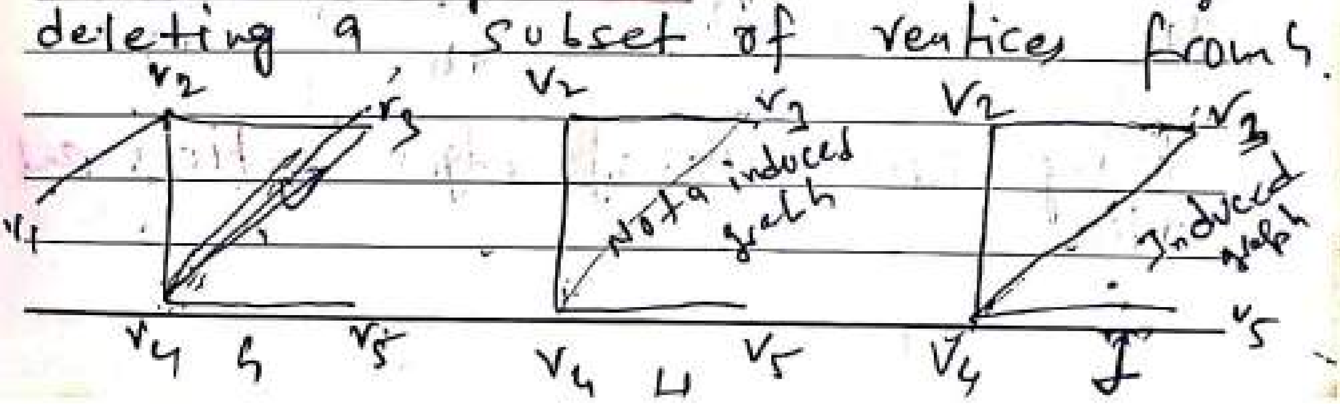
So, if G is a subgraph of G_1, then G_1 is a supergraph of G.

Subgraph
- Edge disjoint Subgraph
- Vertex disjoint Subgraph

<u>Spanning Subgraph</u>: Contain all the vertices of G



<u>Induced subgraph</u>: It obtained by deleting a subset of vertices from G.



Isomorphism :- Two graphs are said to be isomorphic if they are perhaps the same graphs just drawn differently with different names. They have identical behaviour for any graph-theoretic property.

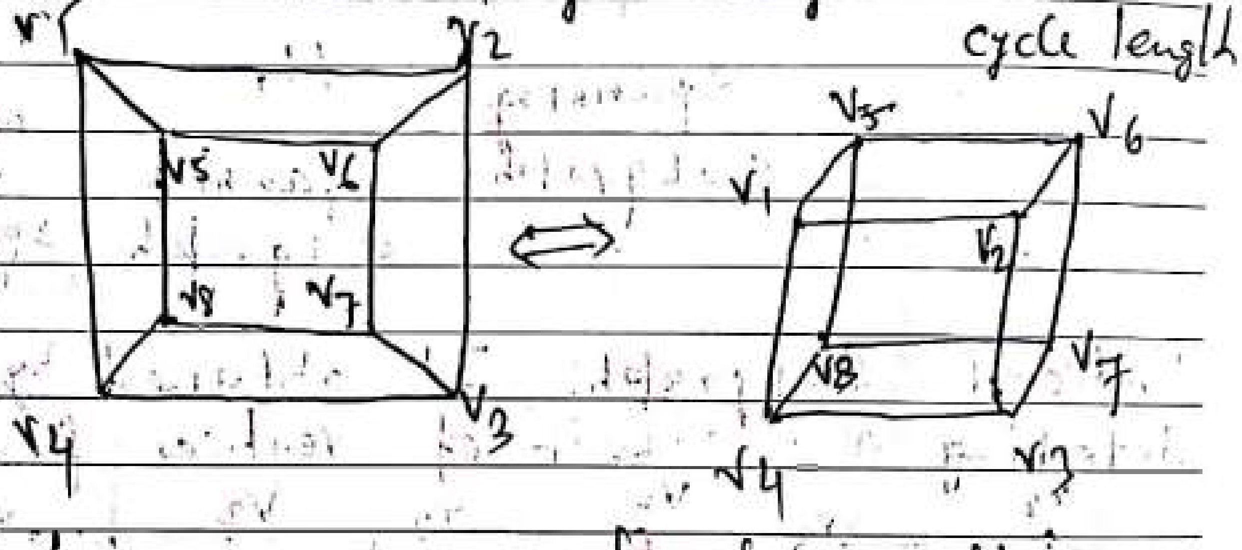
Properties:

1) No. of vertices are equal

2) No. of edges are equal

3) Degree of each node is similar in G_1 and G_2

4) Individual G_1 cycle length = C_2 cycle length



having same no. of edges, vertices and order

Isomorphic Graphs: G_1 and G_2

1) There is one-to-one correspondence b/w their vertices and edge such that incidence relationship is preserve.

II) $G_1 = G_2$ / $G_1 \simeq G_2$

III) Conditions

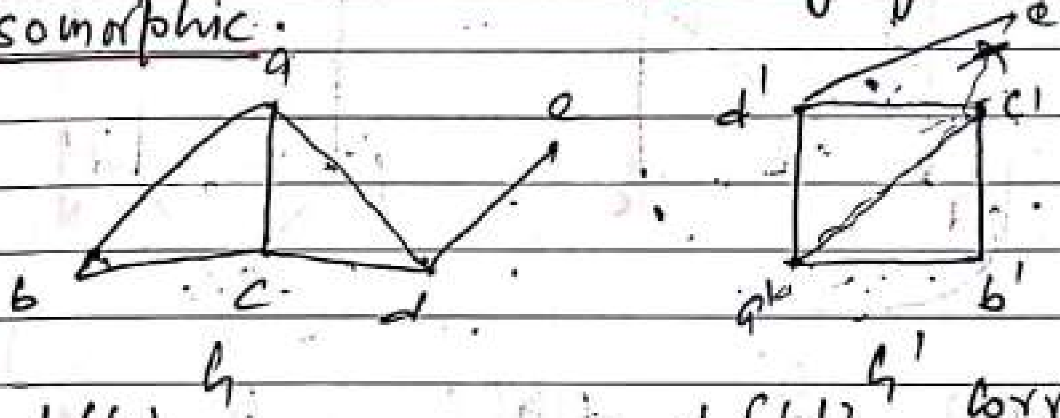
: Same no. of vertices

: Same no. of edges

: equal no. of vertices with same degree

: Same degree sequence and same cycle vector.

Q. Show that the following graphs are isomorphic.



Soln. d(G) d(G') correspondence

d(a) = 3	d(a') = 3	a - a'
d(b) = 2	d(b') = 2	b - b'
d(c) = 3	d(c') = 3	c - c'
d(d) = 3	d(d') = 3	d - d'
d(e) = 1	d(e') = 1	e - e'

d(G) = {3, 2, 3, 3, }

(1) Both graph G and G' have 5 vertices

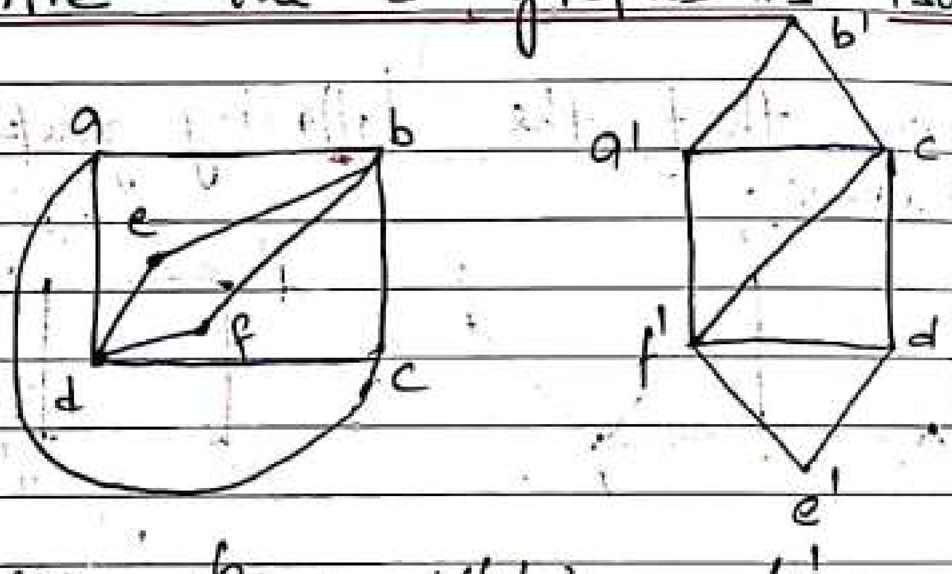
(2) " " " " " 6 edges

(3) both have degree

(4) having same degree sequence and same cycle vector.

∴ G and G' are isomorphic to each other.

2. Are the 2 - graphs is isomorphic?



d(G) d(G')

$d(a) = 3$ $d(a') = 3$

$d(b) = 4$ $d(b') = 2$

$d(c) = 3$ $d(c') = 4$

$d(d) = 4$ $d(d') = 3$

$d(e) = 2$ $d(e') = 2$

$d(f) = 2$ $d(f') = 4$

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vertices of degree 4 :- b - c' | f'

d - f' | c'

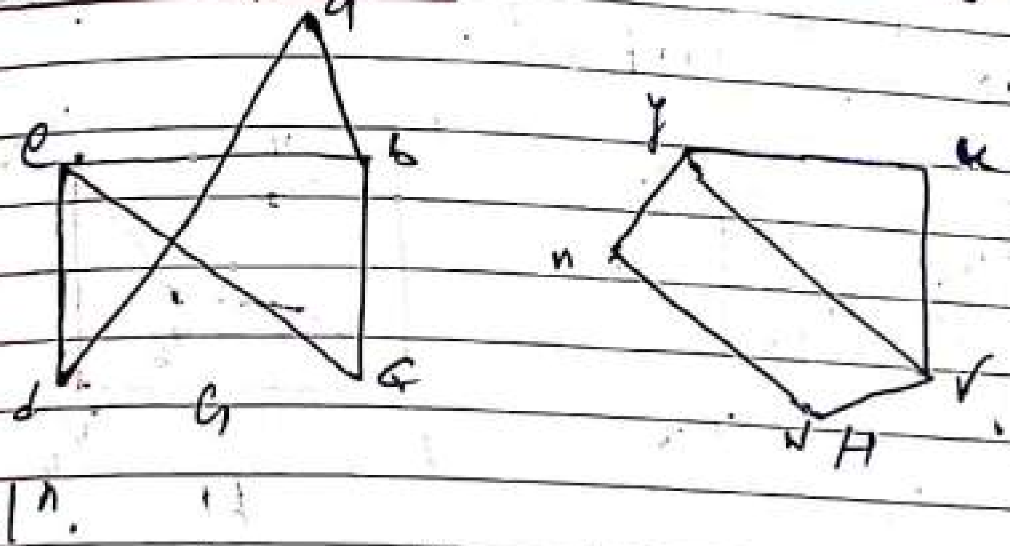
vertices of degree 3 : a - a'

c - d'

The adjacency relationship is violated in vertices having degree 3.

∴ G and G' are not isomorphic.

Q. Verify whether the following graphs are isomorphic



Sol^n.

i) No. of vertices in G = 5 = No of vertices in H.

ii) No. of edges in G = 6 = No. edges in H

iii) degree sequence of G : 2, 3, 2, 2, 3

degree sequence of H : 2, 3, 2, 2, 3.

iv) Circuits of length 3:
In G: e-b-c-e In H: u-y-v-u

Circuits of length 4, In H: y-x-w-v-y
In G: e-d-a-b-e

Circuits of length 5:
In G: a-b-c-e-d-a In H: u-v-w-x-y-u

$\therefore$ G and H are Isomorphic

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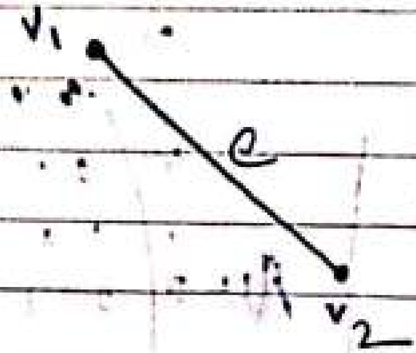
Theorem : HAND-SHAKING THEOREM

G be a graph

$$\sum_{v \in V} deg(v) = 2|E|,$$

where V is the vertex-set and E is the edge set of G.

Soln:



We Know that summation is counting. Each edge is incident on two vertices. So, on LHS summation in the contribution to degree each edge is counted twice. So, RHS, we get $2|E|$

$$\therefore \sum_{v \in V} deg(v) = 2|E|$$

Theorem :-

Number of odd degree vertices in a graph G is always even.

Soln. Let G be a graph. By

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Hand shaking theorem,

$$\sum_{v \in V} \deg(v) = 2|E|$$

partition the vertex set V into two sets V_1, V_2 such that V_1 has odd degree vertices and V_2 has even degree vertices.

$$\sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg v = 2|E|$$

LHS: Each term in 2nd summation is even and hence

$$\sum_{v \in V_2} \deg(v) \text{ is even}$$

RHS = multiplied by 2. so it is even

$$\sum_{v \in V_1} \deg(v) \text{ is even}$$

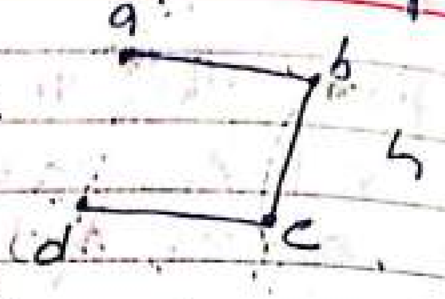
Each term in the above summation is odd. so to be even number of terms (odd degree vertices) in the (above summation) must be even.

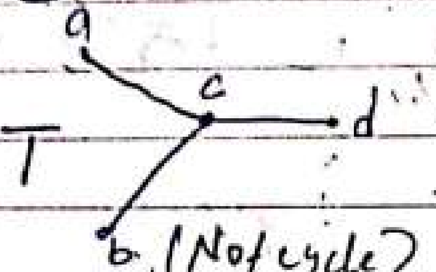
∴ Number of odd degree vertices in a graph is even

Tree :- A tree is an undirected graph that satisfies. In 1847 Krichhoff

1) T is connected and Acyclic (Not cyclic)

Their should not be cyclic





(Not cycle)

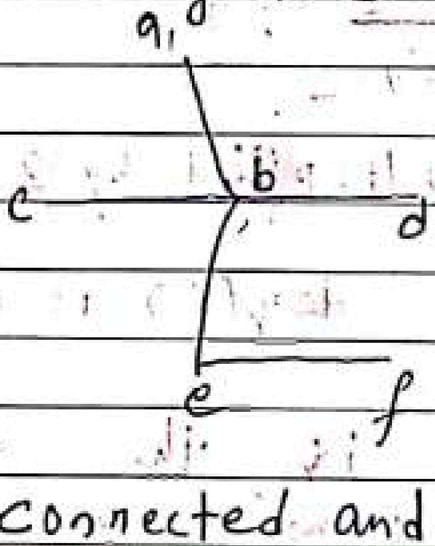
2) T is acyclic and a simple cycle c is formed if any edge is added to it.

3) Any two vertices in T can be

connected by a simple unique path.

a - b - e

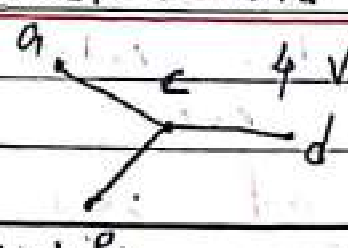
a - b - e - f.



4). An T is connected and has n-1 edges.

n = 3   4 Vertices

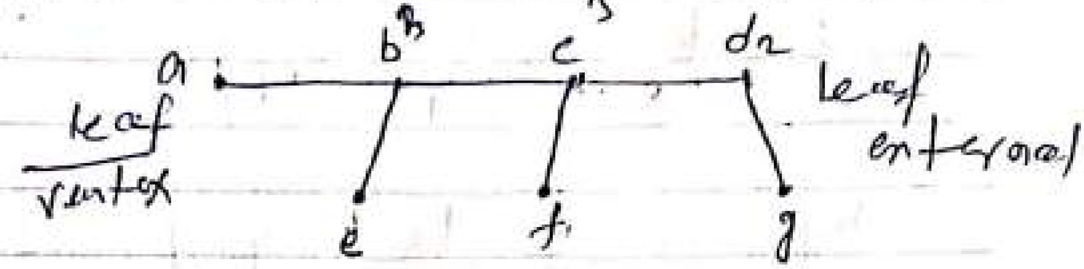
e = 2



$|e| < n-1$ disconnected, $|e| > n-1$ cycle

Internal vertex: There are minimum two edges incident on that vertex

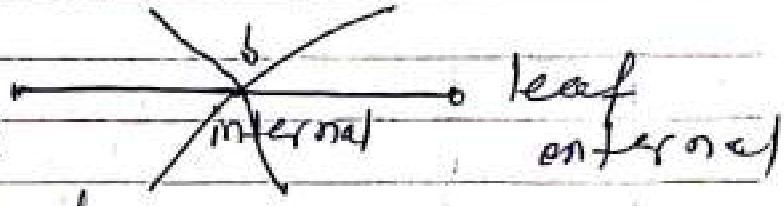
q.N



$n \geq 2$

a, b, c, d are internal vertices

a, e, f, g.
Ex vertices leaf external



External vertex: There is only one edge incident on that vertex.

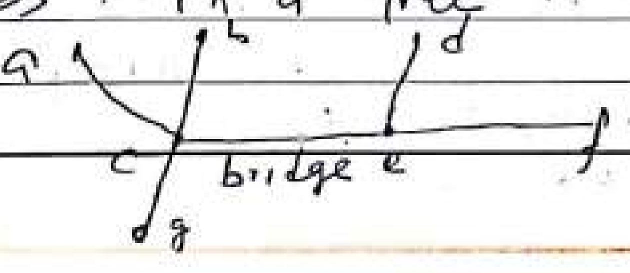
$n \geq 1$

or. Trees :- is a connected graph without any connected

or Tree is a simple graph without any circuit.

Properties

1) There is only one path b/w every pair of vertices in a tree.



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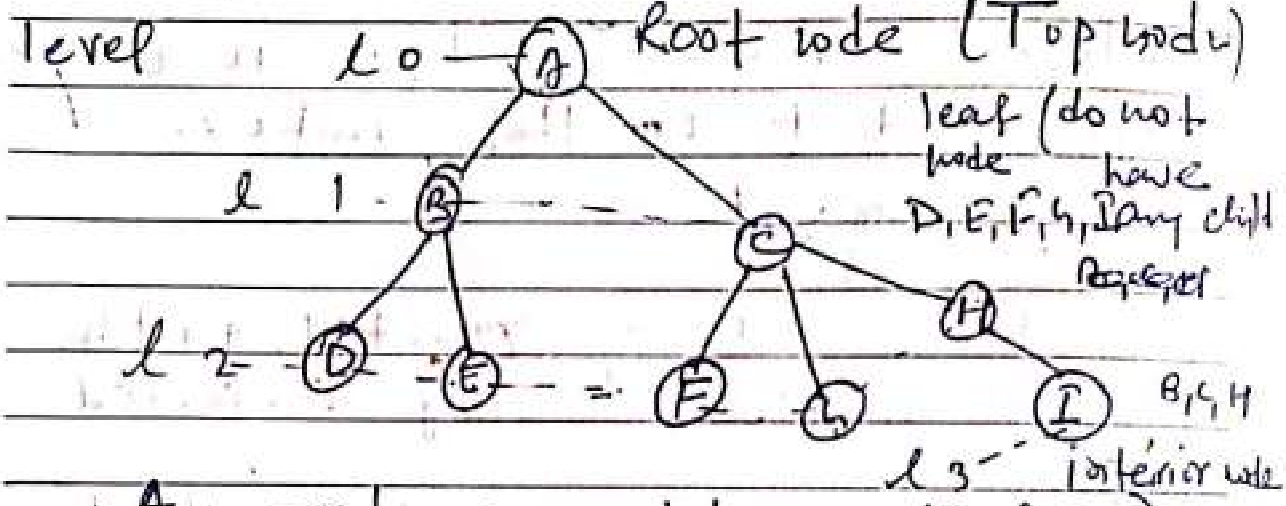
(2) Every edge in a tree is a bridge

(3) A tree with n vertices then it have n-1 edges.

(4) Any connected graph with n vertices and n-1 edge is also a tree

Rooted Tree :- (or Directed Tree) is a tree in which some node is distinguish as a root



A - root , Descendant = D, E (D of A)

B - parent of D and E node

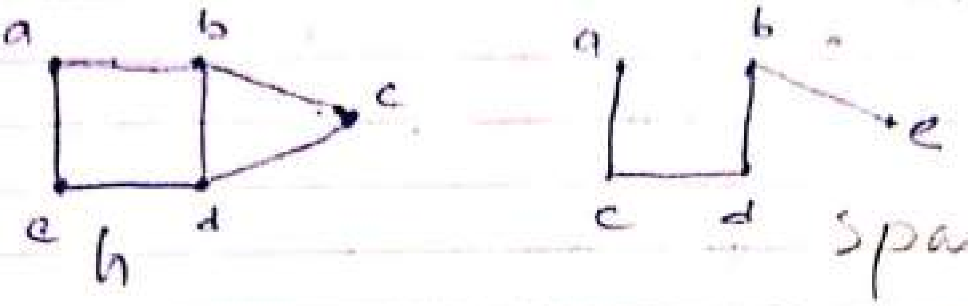
Ancestor - A node higher than the parent)

Height - length of longest path b/w root to any leaf node.

A to I (Total no. of edges across the path.

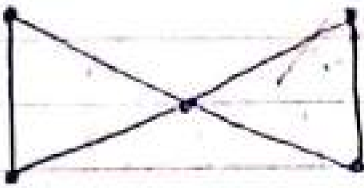
Sibling - having same (B, C) A parent = B

Spanning Tree :- A spanning tree for a graph G is a subgraph of G which contains all the vertices of G and is a tree.



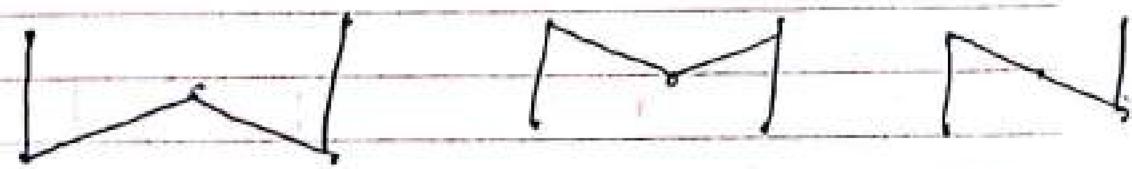
spanning tree

ex



to develop a spanning tree

soln.



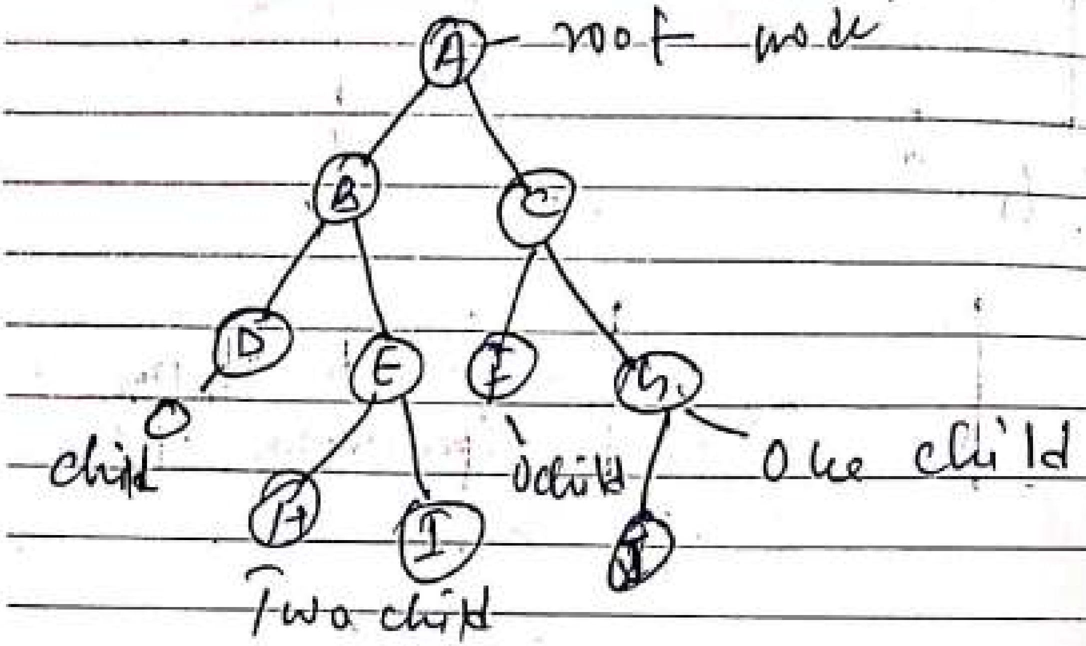
Directed Trees :- A rooted tree is a tree in which a special node is singled out. This node is called directed or rooted trees.

Height :- it is length of longest path b/w root to any leaf node. A to Z = 3

Binary Tree :- (Two)

1) Where each node have upto 2 leaves (children) or atmost 2 children

2) have 0, 1, 2, children.



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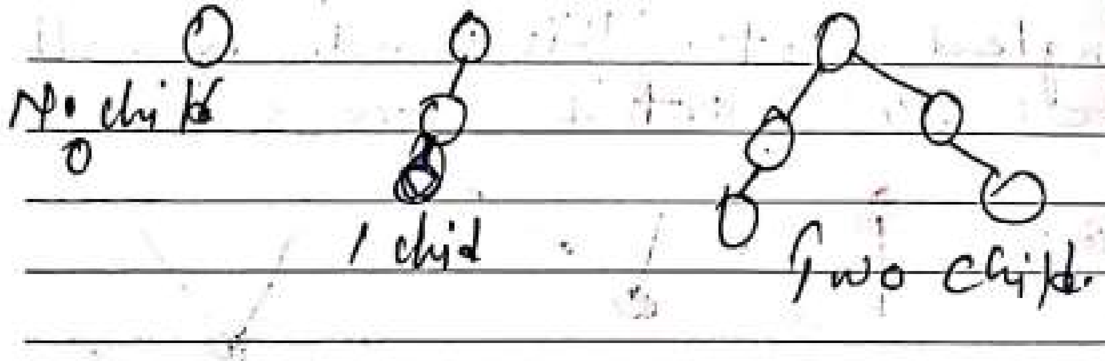
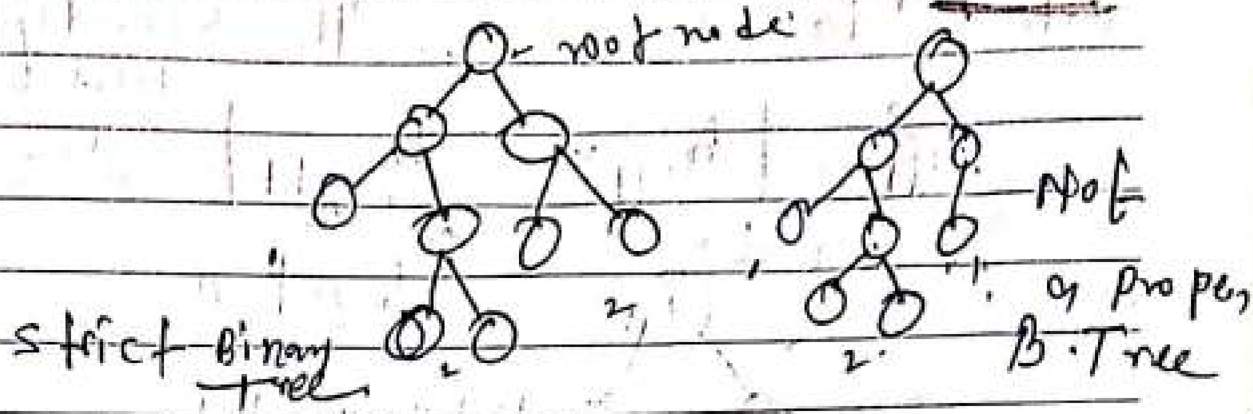
• left child node and right node contain value which can either be greater, less or equal to parent value



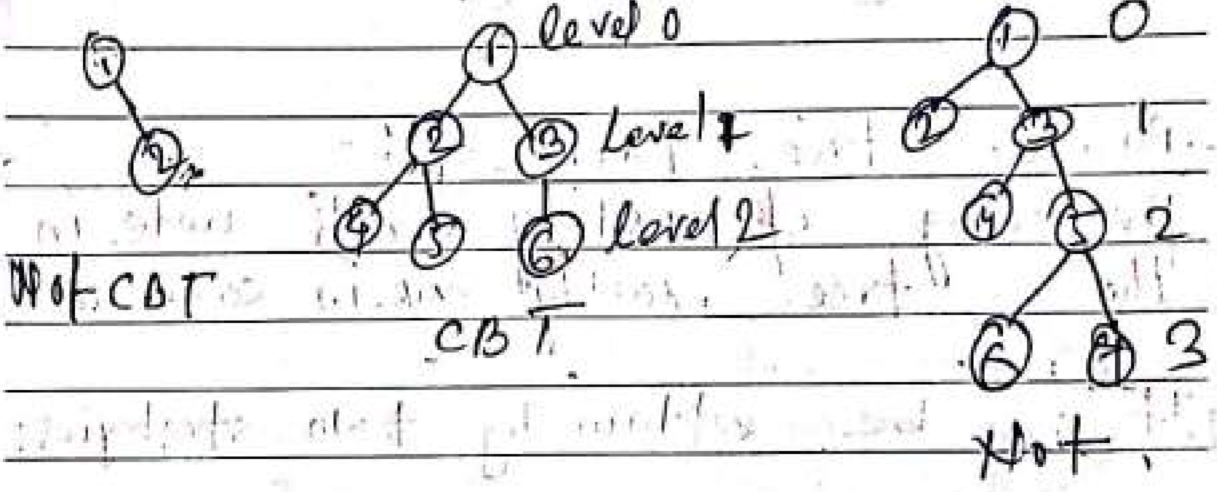
Strict / Proper / full Binary tree ~~complete~~ ~~Binary tree~~ :-

Each node can have either 0 or 2 childs



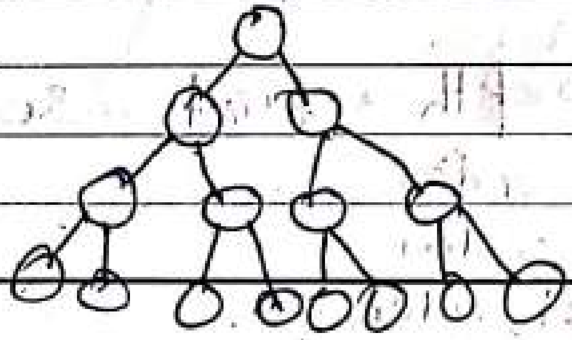
Complete Binary tree :-

All level except possible the last one level completely filled and all nodes are as left as possible.



Perfect Binary Tree :-

All level are completely filled



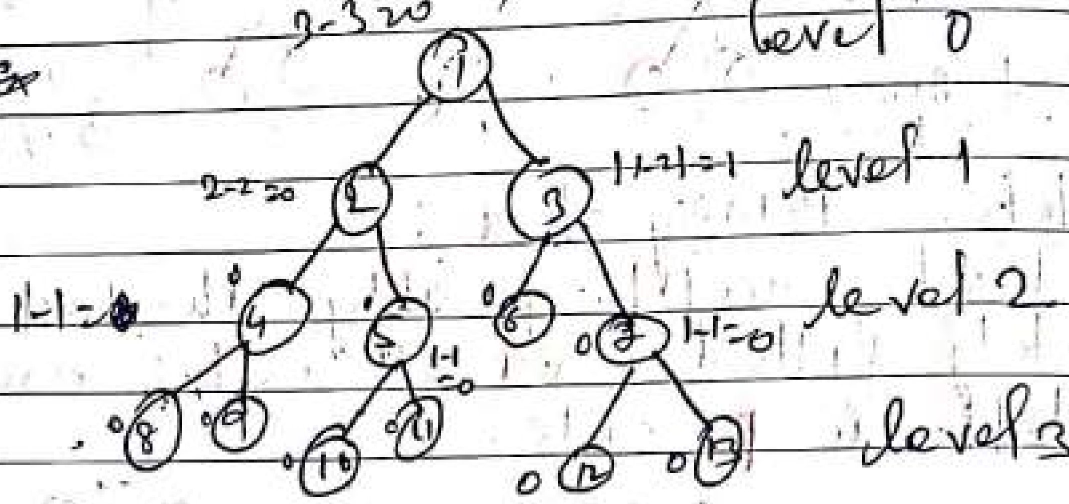
All level having two child

Balanced Binary tree :-

Absolute difference height of left and right subtree for every node is not more than K (mostly 1) (Not more than 1)

$$diff = |h_{left} - h_{right}|$$

Ex



- Binary tree traversal :-

Processing of visiting each node in the tree exactly one in some order.

- It can be explain by two strategies.

- BFS (Breadth - First - Search)
  level ordering search

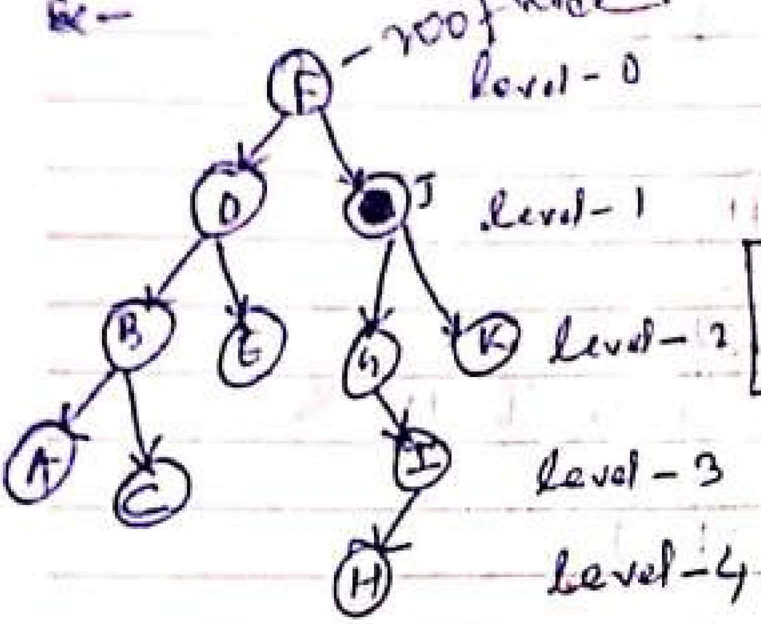
- DFS (Depth - First - Search)
  - Pre order
  - In order
  - Post order,

BFS (level ordering):-

- Visit all the nodes of same level or depth before visiting of next level.

- We will start from L-0, L-1, L-2, ... visit node from left to right.

Ex-



F - D - J - B - E - G - K - A - C - I - H

DFS (Depth First Search):- We go to child and complete the whole subtree of the child before going to next child

- Pre order (NLR
- In order (LNR)
- Post order (LRN)

- relative order of visit left - right - root

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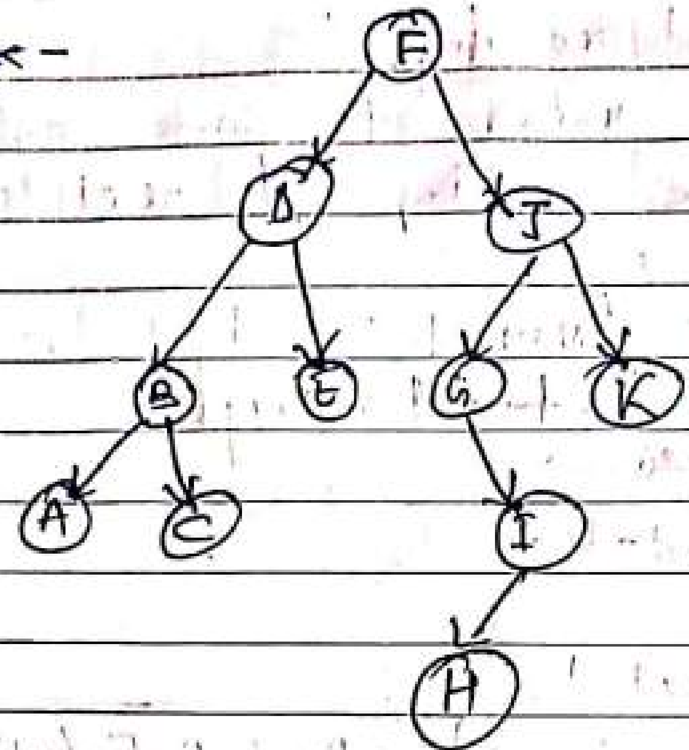
Ex -



Preorder (NLR)

F - D - B - A - C - E - J - G - I - H - K

Inorder (LNR)

A - B - C - D - E - F - G - H - I - J - K

Postorder (LRN)

A - C - B - E - D - H - I - G - K - J - F

Construction of Binary Tree

Preorder and inorder Traversal

Preorder (NLR):

A B D E F C G H J L K

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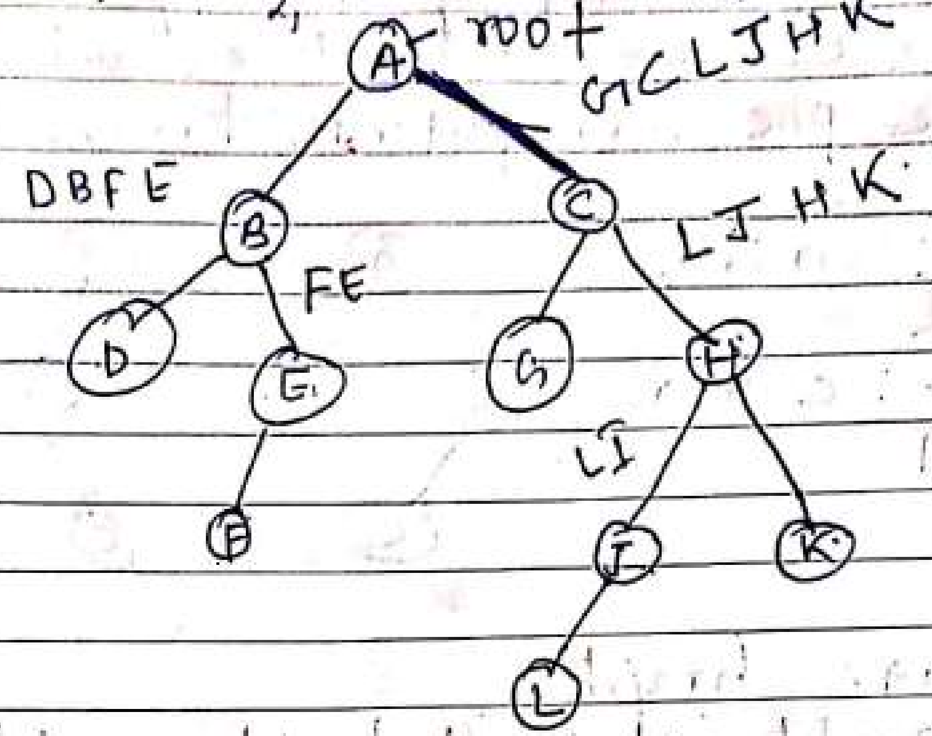
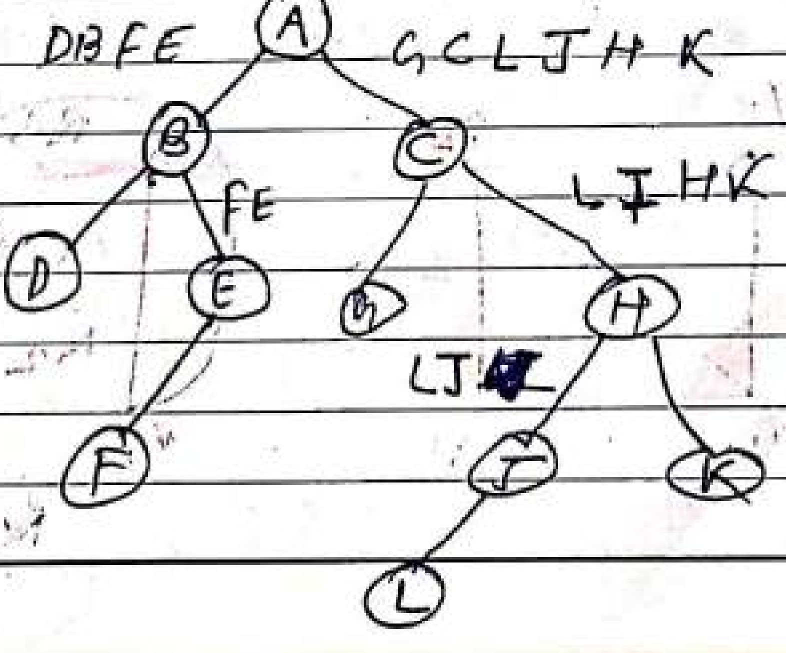
Inorder : DBFE (A) GCLJHK
(LNR)   L   R



Post order and Inorder Traversal

Postorder : DBFE BGLJKHC (A) — root
(LRN)
Inorder : D(B)FE A G (C) LJ H K
(LNR)   L   R



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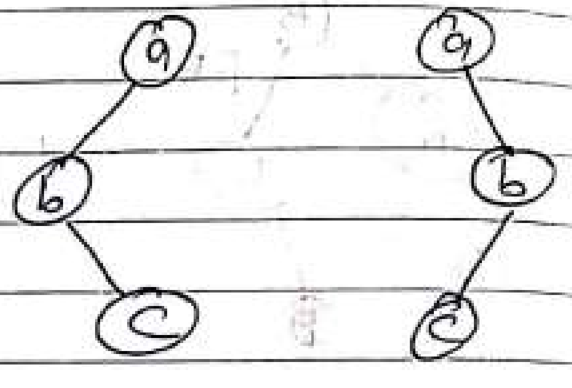
Preorder and Postorder:

- We cannot uniquely identify the tree
- There can be two trees with the same pre and postorder traversal.

Pre: a b c
NLR

Post: c b a
LRN



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Planar graph:

A graph is called planar if it can be drawn in the plane without any edges crossing and a

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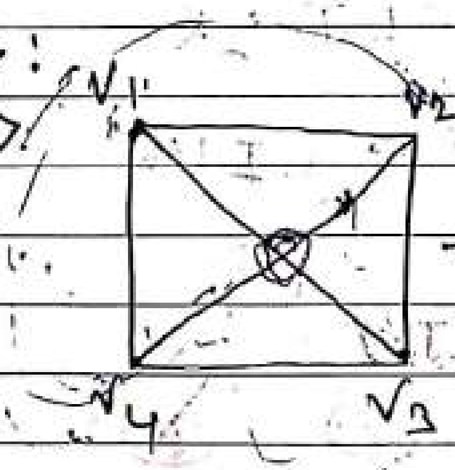
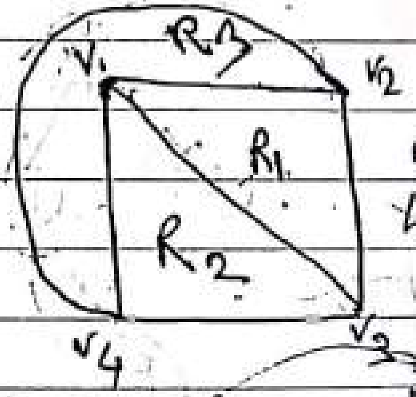
graph that cannot be drawn on a plane without a cross over b/w its edges is called Non-planar graph.

ex: 1)



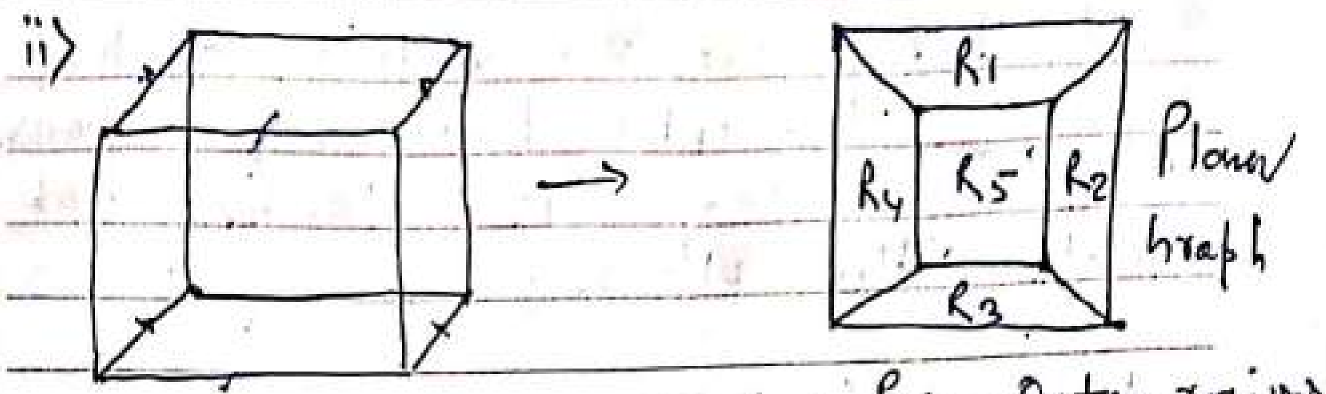
→



Planar graph

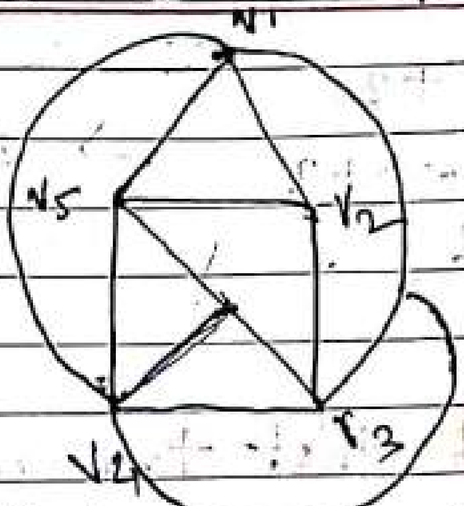
R_4 → outer regions

ii)



Theorem :- A complete graph of five vertices is non-planar.

Soln.

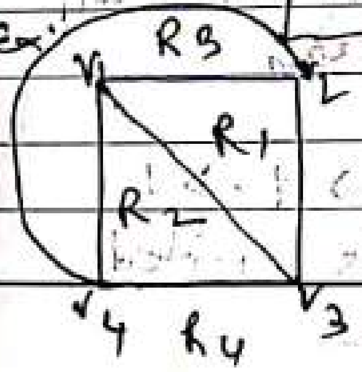


Euler's Formula :-

Let G be a connected planar simple graph with 'e' edges and vertices 'v'. Let 'r' be the regions in planar of G. Then

$$\boxed{r = e - v + 2}$$

Ex:



$e = 6,\ v = 4$

$r = e - v + 2 = 6 - 4 + 2 = 4$

Q. If there are 20 vertices, each of degree 3, then into how many regions does a rep^n of this planar graph splits the plane.

Soln. $V = 20$

We know that

$$\sum \deg v = 2e = 20 \times 3 = 60$$

$\therefore 60 = 2e$

$\Rightarrow e = 30$

$\therefore r = e - v + 2$

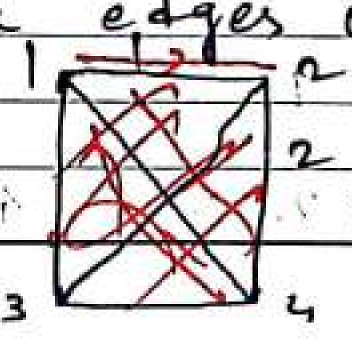
$\therefore r = 30 - 20 + 2$

$\therefore r = 12$

Euler Path :-

It is a path that traverse each edge exactly once and only once. A graph that contains an Euler Path is called as Euler Graph.

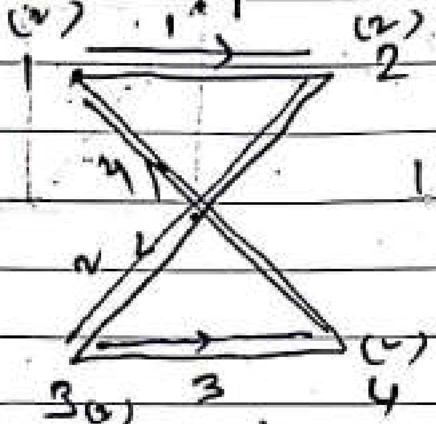
Euler Graph is always connected, because Euler path contains all the edges of the graph



$1 \to 2 \to 3 \to 4 \to 1$

Each edge is visited only once

Euler Circuit :- first and last vertex are same. It is a circuit that traverse each edge exactly once and only once.

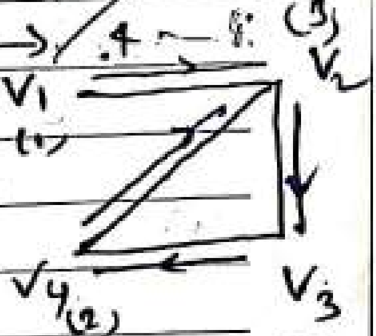


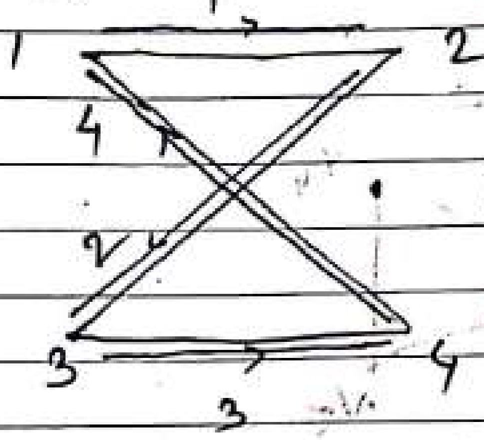
Each edge is visited only once.

Odd deg

1 → 2 → 3 → 4

Euler path





1 → 2 → 3 → 4 → 1

Euler Circuit.

Remark

A connected Graph is Euler Graph iff it has at most 2 odd degree vertices.

Euler Circuit :- Each vertex is of even degree.

Euler Path :- Max. two vertices odd degree.

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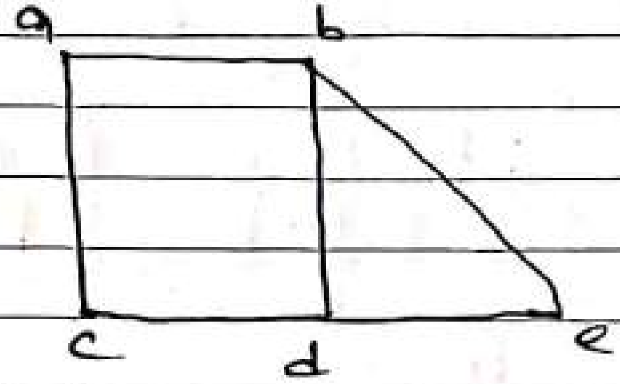
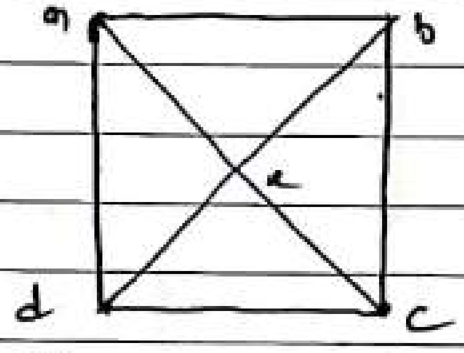
Solved Numericals on Euler Graph

Q. Find Euler Path for Graph below

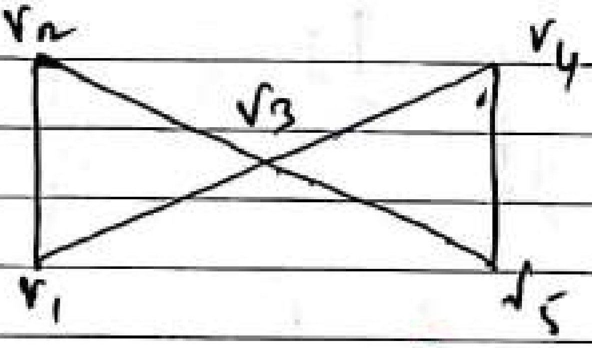
1)



ii)

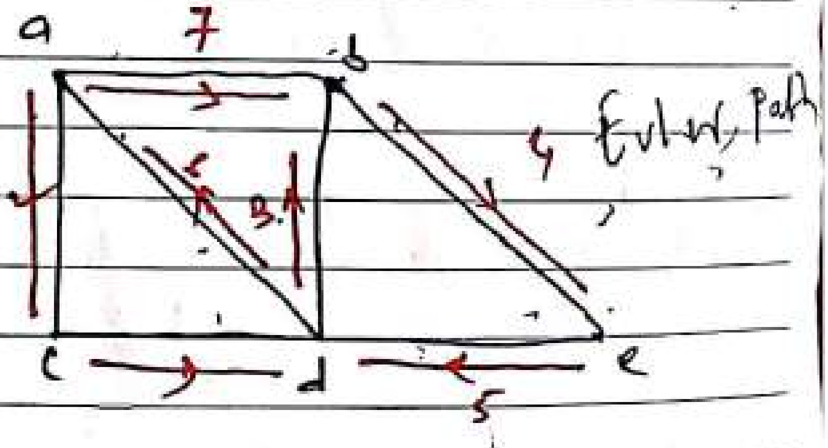


iii)

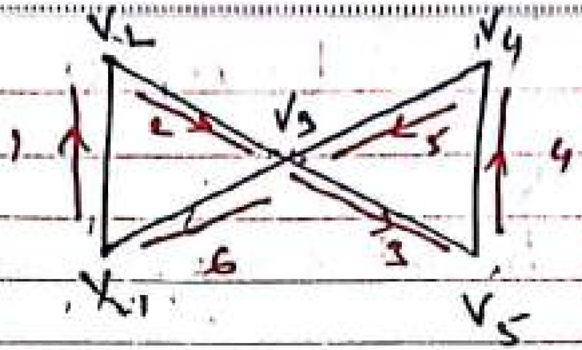


Soln. i)

a → 3
b → 3] 2 vertex with odd degree
c → 2
d → 4
e → 2



a → c → d → b → e → d → a → b

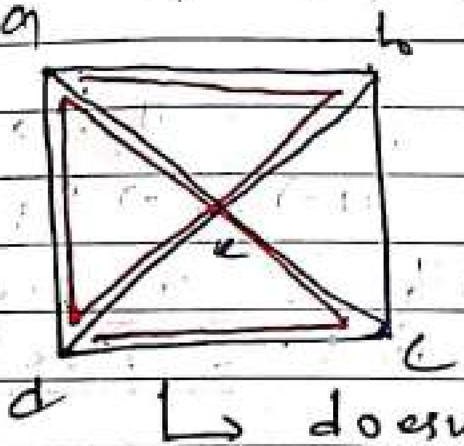
iii) 

Initial → v_1
final → v_1
Euler Circuit

$v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_5 \rightarrow v_4 \rightarrow v_3 \rightarrow v_1$

$v_1 \rightarrow 2$
$v_2 \rightarrow 2$
$v_3 \rightarrow 4$
$v_4 \rightarrow 2$
$v_5 \rightarrow 2$

Eular Circuit
even degree

iii) 

$a \rightarrow 3$
$b \rightarrow 3$
$c \rightarrow 3$
$d \rightarrow 3$
$e \rightarrow 4$

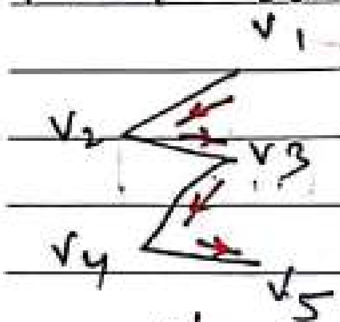
4 vertices with odd degree

↳ doesn't have any Euler path.

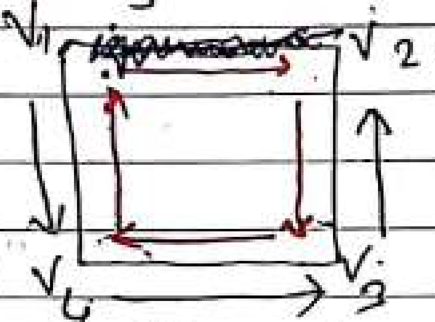
Hamiltonian Path and circuits

Hamiltonian Path :-
Path contains each vertex exactly once.

Hamiltonian Circuit:
First and last vertex are same.



H. Path → $v_1 \to v_2 \to v_3 \to v_4 \to v_5$



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H. Circuit →

$v_1 \to v_2 \to v_3 \to v_4 \to v_1$

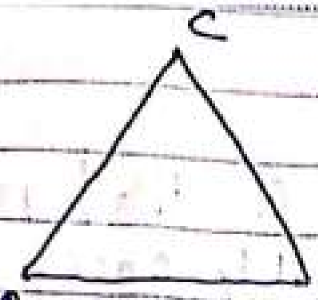
H. Path →

$v_1 \to v_4 \to v_3 \to v_2$

Length of H. Path is a connected Graph of 'n' vertices is $(n-1)$ edges

Theorem :- Let G be a graph of 'n' vertices then 'G' has a hamiltonian path if for only two vertices U and V of G.

$$\deg(U) + \deg(V) \geq n$$



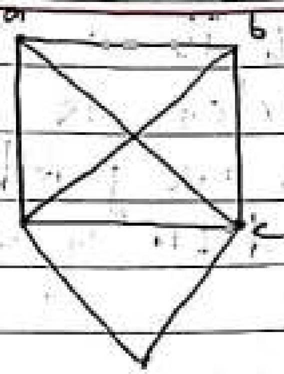
no. of vertices (n) = 3
Consider two vertex (A, B)
deg (A) = 2, deg(B) = 2

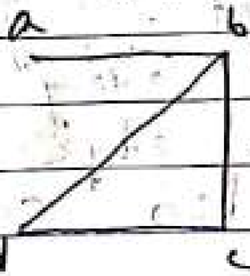
[A: 2] [B: 2] deg(A) + deg(B) = 4 ≥ n => 4 ≥ 3

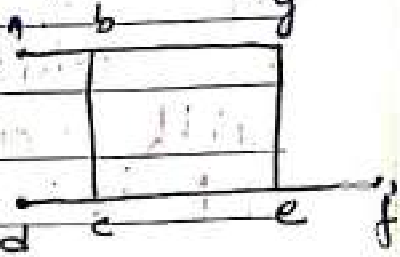
2 + 2 ≥ 3

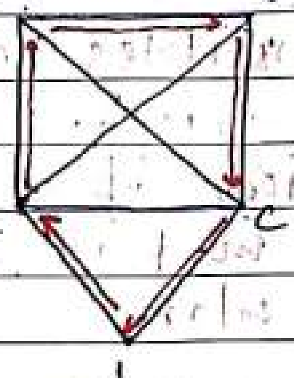
Remark On removing any one edge from a H. circuit, then we are left with H. Path

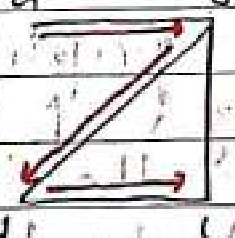
Q. Which of the following Graphs have a H. circuit, if not a H. Path.

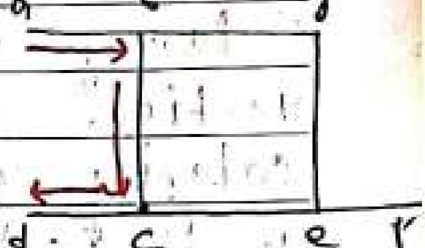
i) 

ii) 

iii) 

Soln (i) 

(ii) 

(iii) 

H. Path [a, b, d, c]

Not contain either H. Path or H. Circuit.

H. circuit [a, b, c, d, e, a]

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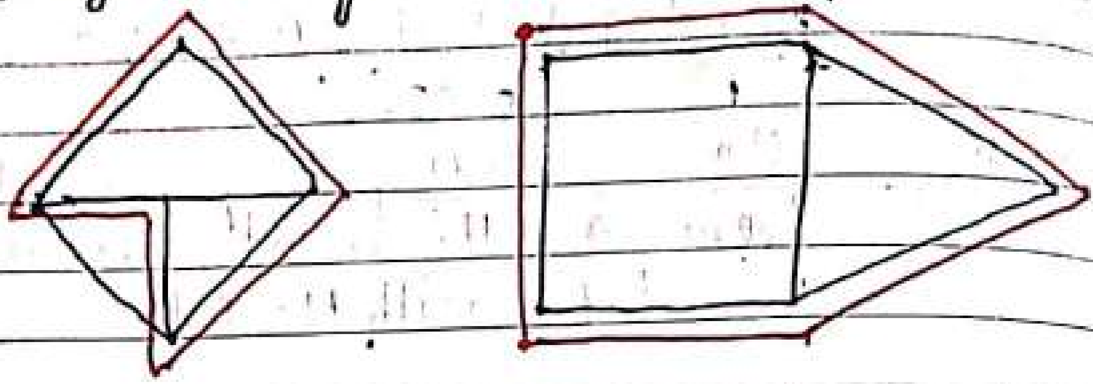
Hamiltonian Circuit :-

A simple circuit in G that passes through every vertex exactly once. S:e

a 

Hamiltonian Graph :-

A complete graph is a graph in which every vertex is connected with an edge to every other vertex. It is also called H. Graph.

Graph Coloring :-

Coloring of graph constitutes coloring vertices edges of the Graph.
Coloring all the vertices of a graph is the property that no two adjacent vertices have same color.

[Always try to color with minimum colors]

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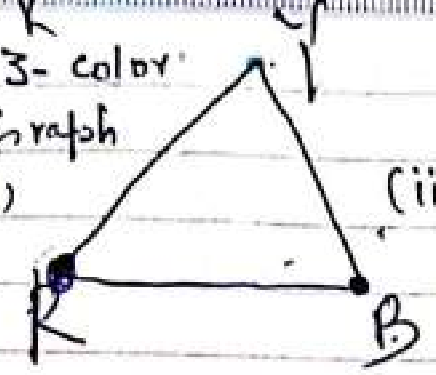
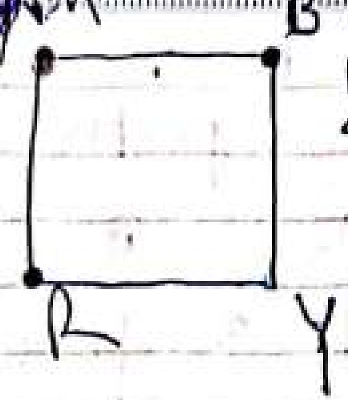
3- color Graph

(i)



(ii)



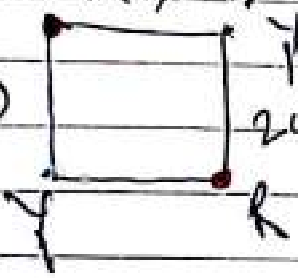
4 - color graph

chromatic Number:-

It is defined as the least no. of colors needed for coloring the graph. denoted by $\chi(G)$ or K-chromatic Graph

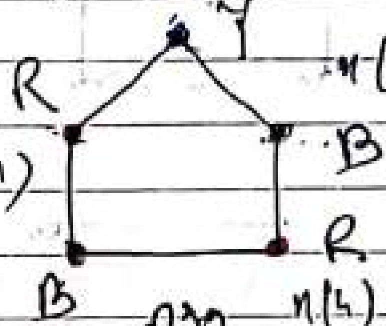
& $\chi(G) = 2$

1)



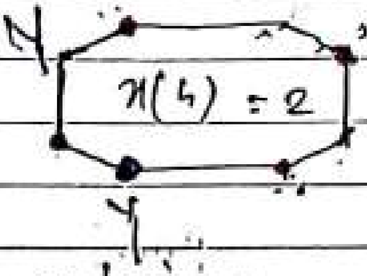
2 color Graph

11)



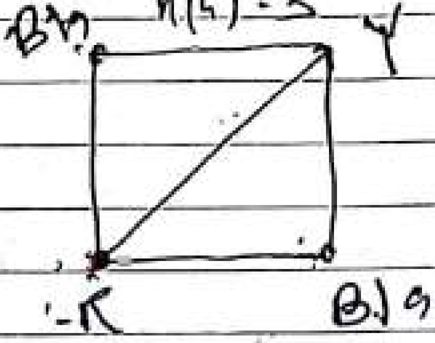
$\chi(G) = 3$

iii)



iv)

$\chi(G) = 3$

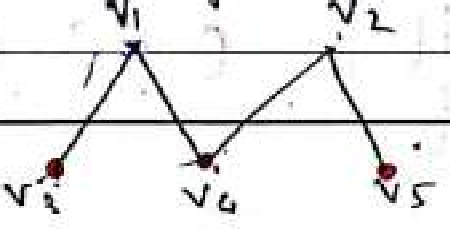


Graph coloring.

Every Bipartite Graph is 2-colorable

$\chi(G) = 2$

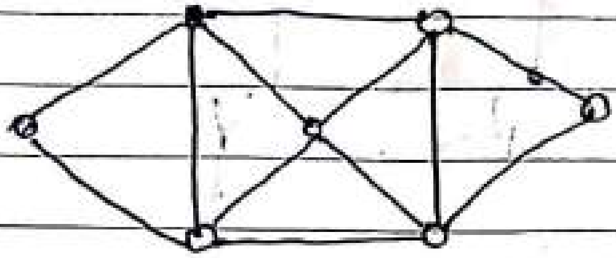
$V = \{v_1, v_2, v_3, v_4, v_5\}$. We can color the vertices of same set with one color.



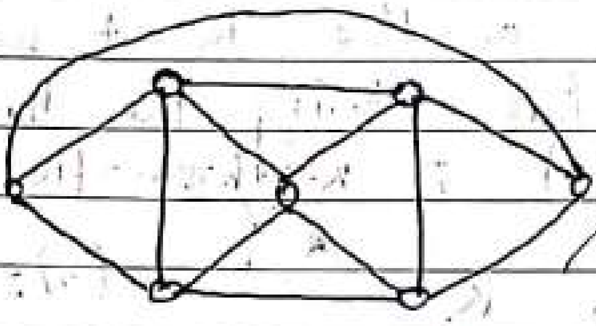
$\chi(G) = 2$

Q. What is the chromatic no. of following graph.

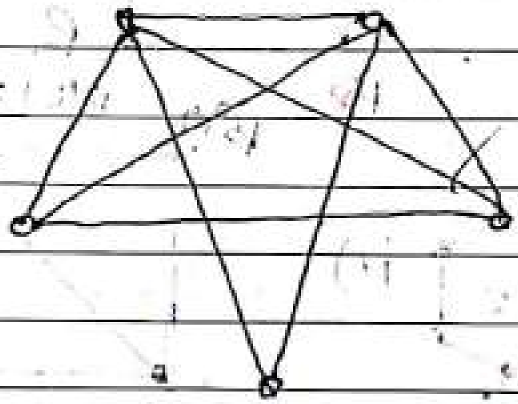
i)



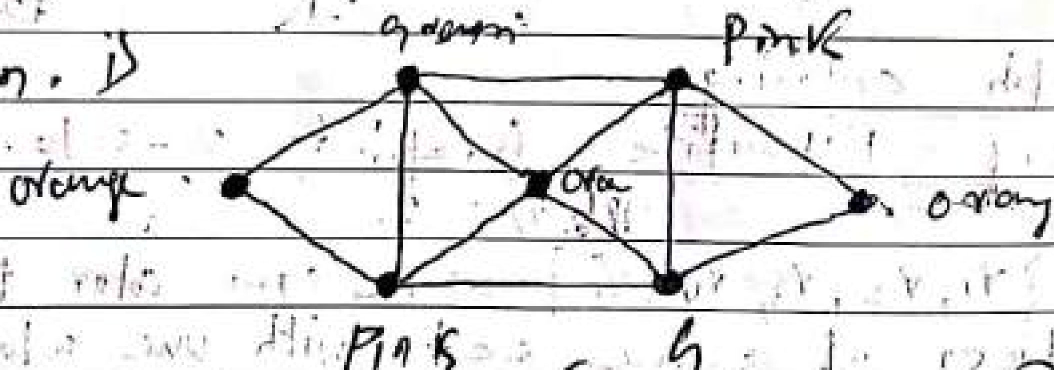
ii)



iii)

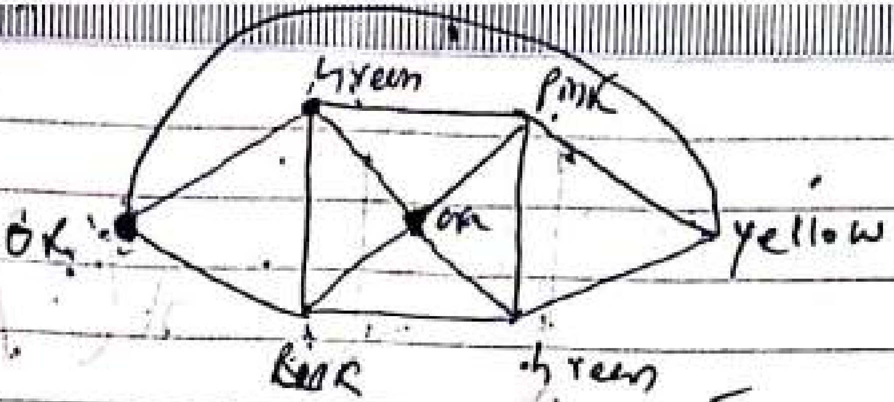


Soln. i)



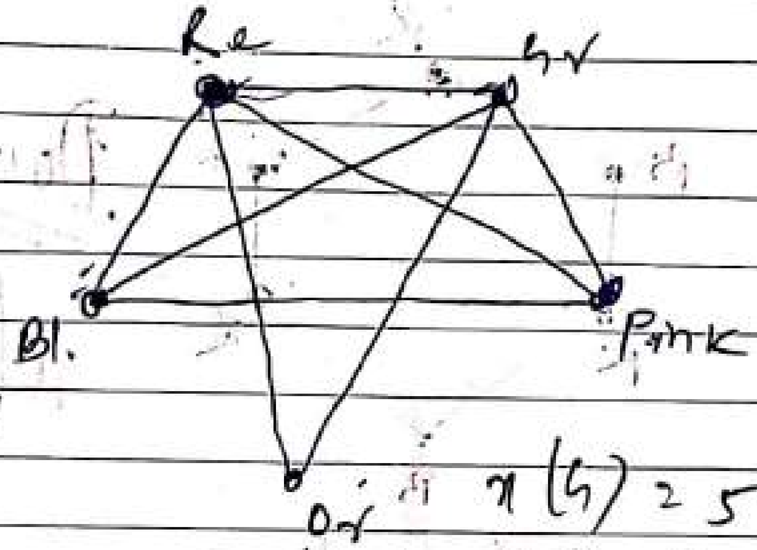
$\therefore \chi(G) = 3$ {G, O, P}

ii)



$\chi(G) = 4$ [O, G, P, Y]

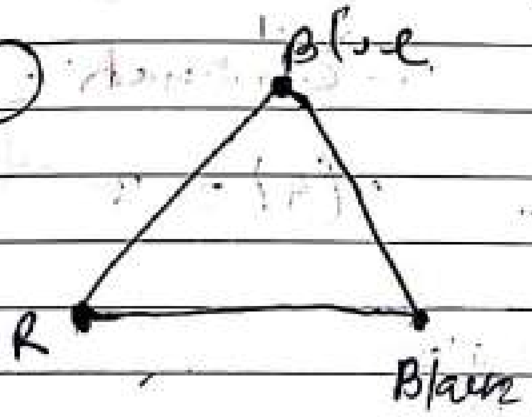
iii)



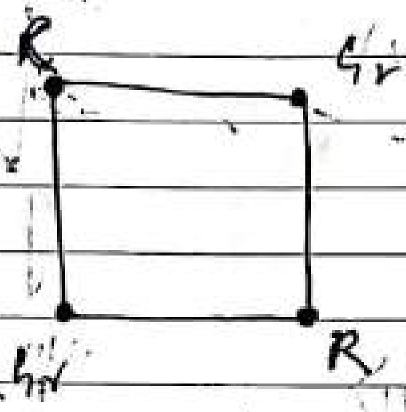
$\chi(G) = 5$

Other color Graph

(i)



(ii)



3 - color
3 - chromatic
$\chi(G) = 3$

2 - color
2 - chro
$\chi(G) = 2$

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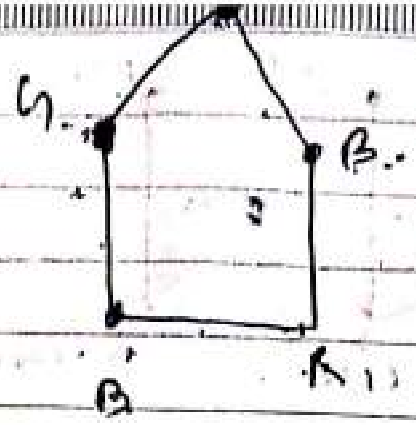
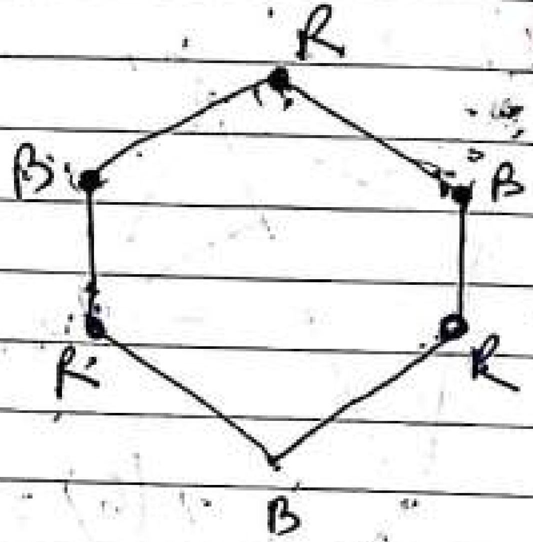
iii)



$\chi(G) = 3$

3- chromatic

iv)



~~Doubt~~

$\chi(G) = 2$

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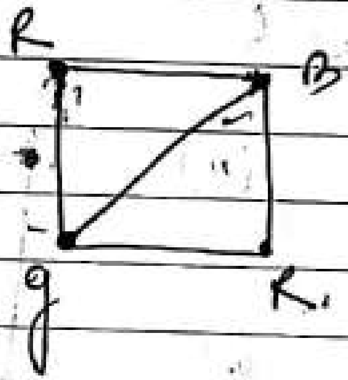
2- chromatic

v)



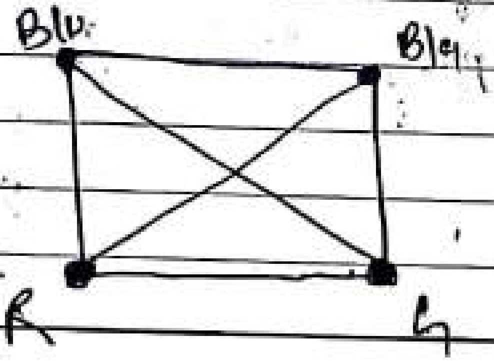
3- chromatic

$\chi(G) = 3$

vi)

K_n

$\chi(K_n) = n$

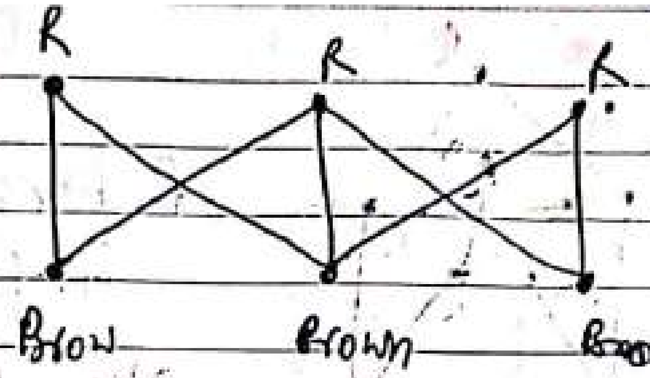


K_4

$\chi(G) = 4$

$\chi(K_4) = 4$

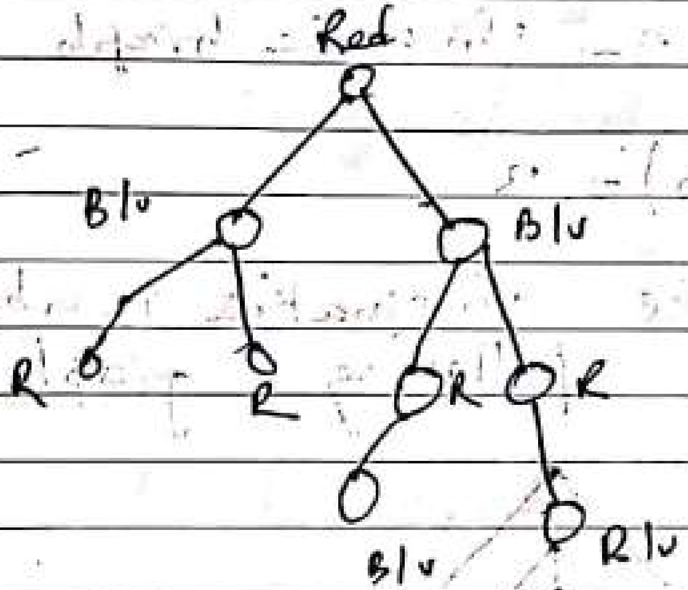
vii)



∴ Bipartite Graph

2 ∴ chromatic

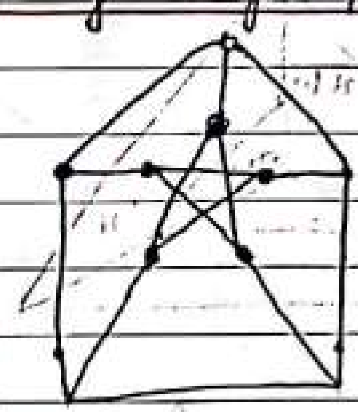
viii)



2 - chromatic Graph

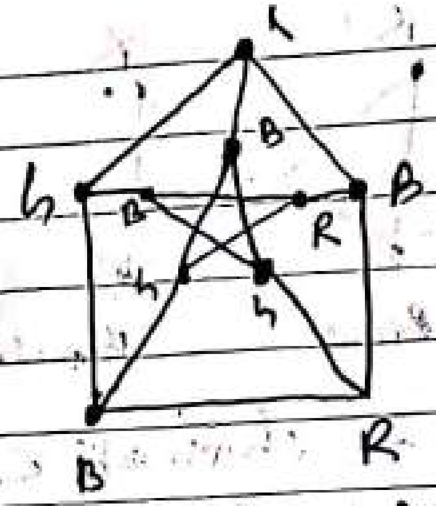
10.05

Q. Find the chromatic number for the following graph



Soln

R = Red
B = Blue
G = Green



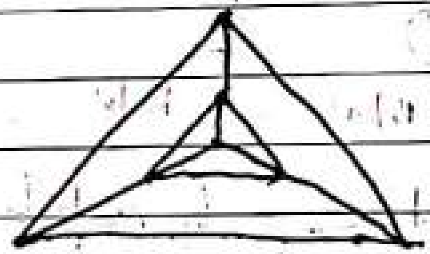
Petersen Graph

Non - Planar

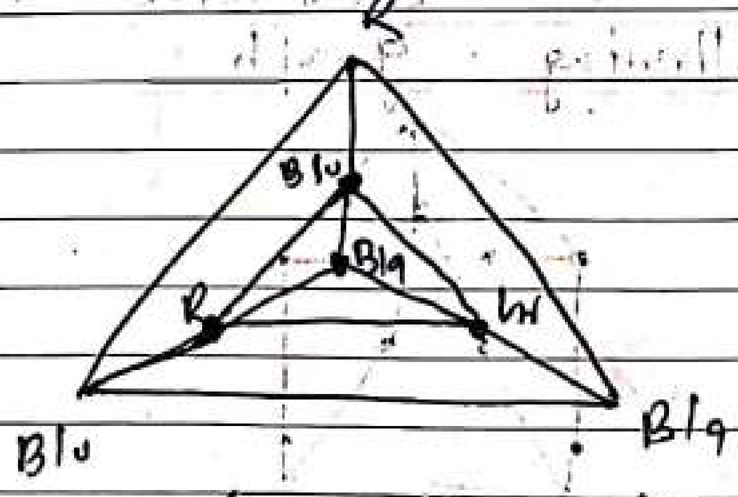
3 - chromatic graph

$\chi(G) = 3$

Q. Find the chromatic number for the following graph



Soln.



4 - chromatic

$\chi(G) = 4$

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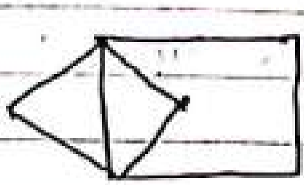
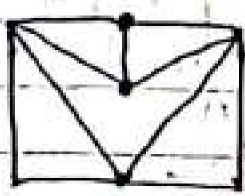
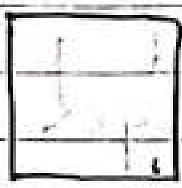
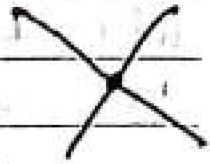
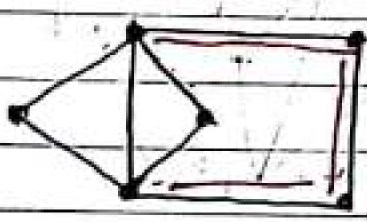
Q. Is a Hamiltonian graph Eulerian?
Is a Eulerian graph Hamiltonian?

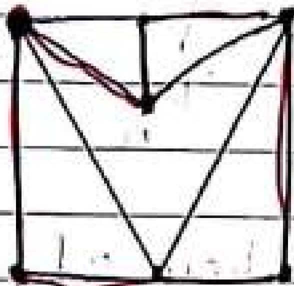
   

Soln.



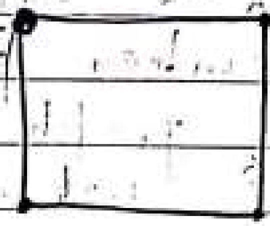
E.G = Yes

H.G = NO



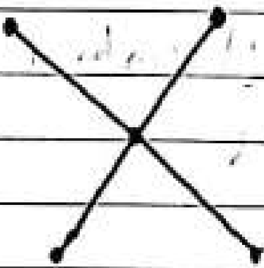
E.G = NO

H.G = Yes



E.G = Yes

H.G = Yes

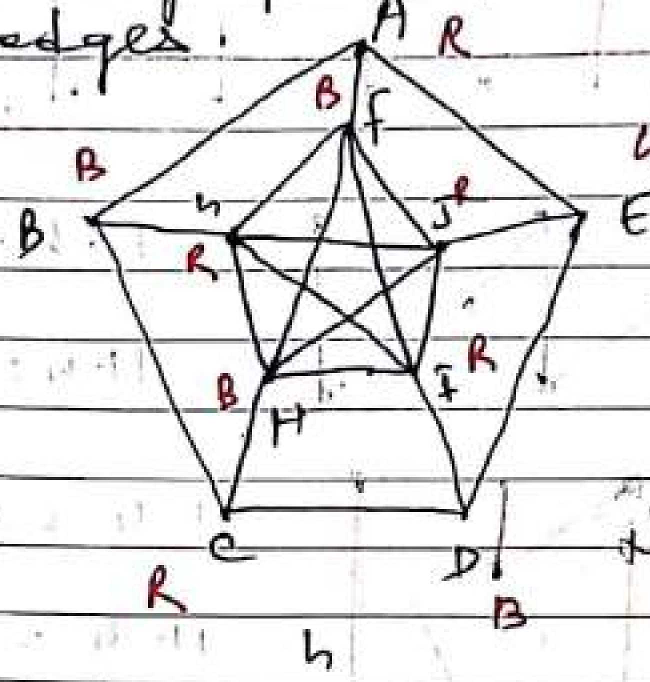


E.G = NO

H.G = NO

Q. What is peterson graph?

Soln. Peterson graph is an undirected graph with 10 vertices and 15" edges.



Non-planar

Chromatic Number of Peterson graph:
The graph G is not a complete graph with number of edges. It is not bipartite graph. In this graph has even no. of vertices. So $\chi(G) \neq 2$.

$\therefore$ chromatic Number of P.G is $\chi(G) = 3$

Remark: A complete graph of n number of vertices has $\frac{n(n-1)}{2}$ number of edges.

Q. How many edges do K_{10} have?

Soln. No of edges in K_{10} $\checkmark$

$$= \frac{10(10-1)}{2}$$

$$= 45$$

Q. How to find a graph is planar or not?

Soln. We know that

$$e \leq 3v - 6$$

$$e = \frac{4(4-1)}{2}$$

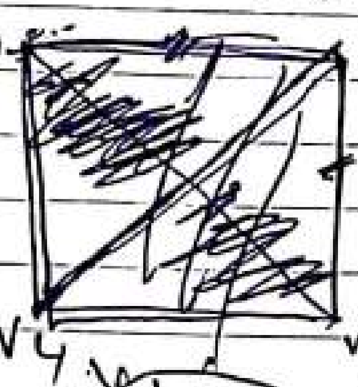
$$= 6$$

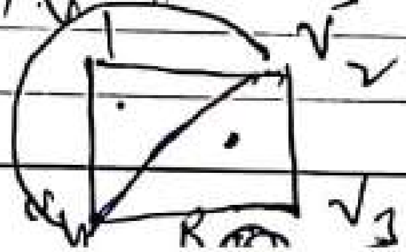
$$v = 4$$

$$e \leq 3v - 6$$

$$6 \leq 3 \times 4 - 6$$

$$6 \leq$$





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Four color theorem:-
In a graph does not required more than 4 colors to color its vertices such that no two vertices have same color.

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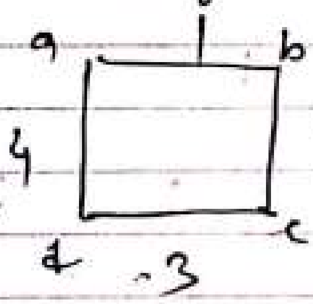
Q. What is the difference between an Eulerian graph and Eulerian circuit?

Ans. By the defn of a graph is a set of vertices and edges but circuit is a sequence of vertices and edges. so Eulerian graph is set of vertices and edges and Euler circuit is a sequence of vertices and edges



E.G

$G = \{V, E\}$

$V = \{a, b, c, d\}$

$E = \{1, 2, 3, 4\}$

$E.C = \{a\,1\,b\,2\,c\,3\,d\,4\,a\}$

Pascal's Triangle :-

Index	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1

Binomial theorem (for non-negative)

$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + \cdots + {}^nC_{n-1}\, ab^{n-1} + {}^nC_n\, b^n$

1) when $n = 4$
coeff. are ${}^4C_0, {}^4C_1, {}^4C_2, {}^4C_3, {}^4C_4$

2) when $n = 5$
coeff are ${}^5C_0, {}^5C_1, {}^5C_2, {}^5C_3, {}^5C_4, {}^5C_5$

3) $n = 6$
coeff are ${}^6C_0, {}^6C_1, {}^6C_2, {}^6C_3, {}^6C_4, {}^6C_5, {}^6C_6$

4) $n = 7$
coeff are, ${}^7C_0, {}^7C_1, {}^7C_2, {}^7C_3, {}^7C_4, {}^7C_5, {}^7C_6, {}^7C_7$

Remark

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_0 = {}^nC_n$
3. ${}^nC_1 = {}^nC_{n-1}$
4. $(1+x)^n$
$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$
$= \sum_{r=0}^{n} {}^nC_r\, x^r$
5. $(1-x)^n$
$= {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 + \cdots + (-1)^n\, {}^nC_n x^n$
$= \sum_{r=0}^{n} (-1)^r\, {}^nC_r\, x^r$

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General term

1) $(a+b)^n$ then

$t_{r+1} = {}^nC_r \cdot a^{n-r} \cdot b^r$

2) $(a-b)^n$

$t_{r+1} = (-1)^r \cdot {}^nC_r \cdot a^{n-r} \cdot b^r$

Q. find the coefficients of x^{32} and $\frac{1}{x^{17}}$ in the expansion $\left[x^4 - \frac{1}{x^3}\right]^{15}$

Soln. $t_{r+1} = (-1)^r \cdot {}^{15}C_r \cdot (x^4)^{15-r} \cdot \left(\frac{1}{x^3}\right)^r$

$= (-1)^r \cdot {}^{15}C_r \cdot x^{60-4r} \cdot x^{-3r}$

$= (-1)^r \cdot {}^{15}C_r \cdot x^{60-7r}$

for the coeff of x^{32} is must be equal to x^{60-7r}.

$x^{60-7r} = x^{32}$

$60-7r = 32$

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$\therefore \quad -7r = -28$

$r = 4$

$\therefore$ coeff of $x^{32} = {}^{15}C_4 =$

for the coeff of x^{-17} must be equal to x^{60-7r}.

$$x^{60-7r} = x^{-17}$$

$$60 - 7r = -17$$

$$-7r = -77$$

$$r = 11$$

$\therefore$ coeff of $x^{-17} = (-1) \cdot {}^{15}C_{11}$

Q. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, prove that its coefficient is

$$\frac{2n!}{\left(\frac{4n-p}{3}!\right)\left(\frac{2n+p}{3}!\right)}$$

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Sol^n. $t_{r+1} = {}^{2n}C_r \cdot (x^2)^{2n-r} \times \left(\frac{1}{x}\right)^r$

$= {}^{2n}C_r \cdot x^{4n-2r} \cdot x^{-r}$

$= {}^{2n}C_r \cdot x^{4n-2r-r}$

$= {}^{2n}C_r \cdot x^{4n-3r}$ ✓

$x^{4n-3r} = x^p$

$4n-3r = p$

$r = \frac{4n-p}{3}$

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$\therefore$ coeff of $x^p = {}^{2n}C_r$.

$= \frac{(2n)!}{(r!) \times (2n-r)!} = \frac{(2n)!}{\left(\frac{4n-p}{3}\right)! \times \left(2n - \frac{4n-p}{3}\right)!}$

$= \frac{2n!}{\frac{4n-p}{3}! \times \frac{2n+p}{3}!}$

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Q. In the binomial expansion of $(1+x)^{m+n}$, prove that the coefficients of x^m and x^n are equal.

Soln. Given expansion $(1+x)^{m+n}$

$$t_{r+1} = {}^{m+n}C_r \cdot x^r$$

$$\text{coeff of } x^m = {}^{m+n}C_m$$

$$\text{coeff of } x^n = {}^{m+n}C_n$$

$\therefore$ coeff of x^m and x^n are equal.

12.15

Q. Find the value of r when it is being given that the coeff of $(2r+4)$th and $(r-2)$th terms in the expansion of $(1+x)^{18}$ are equal.

Soln. Given expansion $(1+x)^{18}$

$$t_{m+1} = {}^{18}C_m\, x^m \qquad t_{r+1} = {}^{n}C_r\, x^{n-r} \cdot y^r$$

$$t_{2r+4} = t_{(2r+3)+1} = {}^{18}C_{2r+3}\, x^{2r+3}$$

$t_{r-2} = T_{(r-3)+1} = {}^{18}C_{r-3}\, x^{r-3}$

Remark

${}^{n}C_{r} = {}^{n}C_{p}$

$r+p = n$ ✓

$p = r$ ✓

$\therefore {}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\Rightarrow 2r+3+r-3 = 18$

$r = 6$

and $2r+3 = r-3$

$r = -6$

Q. Find the coeff of x^6y^3 in the expansion of $(x+2y)^9$.

soln. Given expansion

$(x+2y)^9$

$t_{r+1} = {}^{9}C_{r}\, x^{9-r} \cdot (2y)^r$

$= {}^{9}C_{r} \cdot 2^r \cdot x^{9-r} \cdot y^r$

for find the coeff of $x^6 y^3$

we put $r = 3$, then

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$t_{r+1} = {}^9C_3 \times 2^r \times x^6 y^3$

$\therefore$ Coeff of $x^6y^3 = {}^9C_3 \times 2^3$

$= 672$

Q. If $(1+x)^n = {}^nC_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$

1) Prove that

$C_1 + 2C_2 + 3C_3 + \cdots + nC_n = n \cdot 2^{n-1}$

Proof. LHS $= C_1 + 2C_2 + 3C_3 + \cdots + nC_n$

$= \frac{n!}{(n-1)!\,1!} + \frac{2 \cdot n!}{(n-2)!\,2!} + \frac{3 \cdot n!}{(n-3)!\,3!}$

$+ \cdots + n \cdot \frac{n!}{(n-n)!\,n!}$

$= \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} + 2 \cdot \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!} \times 2}$

$+ \frac{3 \cdot n(n-1)\cancel{(n-3)!}}{\cancel{(n-3)!} \times 3 \times 2} + \cdots + n \cdot \frac{1}{0!}$

$= n + \frac{n(n-1)}{1!} + \frac{n(n-1)(n-2)}{2!} + \cdots + n$

$$= n\left[1 + \frac{n-1}{1!} + \frac{(n-1)(n-2)}{2!} + \cdots + 1\right]$$

$$= n\,(1+1)^{n-1}$$

$$= n \cdot 2^{n-1} = RHS$$

Q. Prove that

$$C_1 - 2C_2 + 3C_3 - \cdots + n(-1)^{n-1} C_n = 0$$

Proof $LHS = C_1 - 2C_2 + 3C_3 - \cdots + n(-1)^{n-1} C_n$

$$= \frac{n!}{(n-1)!\,1!} - 2 \cdot \frac{n!}{(n-2)!\,2!} + 3 \cdot \frac{n!}{(n-3)!\,3!}$$

$$- \cdots + n(-1)^{n-1} \cdot \frac{n!}{(n-n)!\,n!}$$

$$= \frac{n(n-1)!}{(n-1)!} - 2 \cdot \frac{n(n-1)(n-2)!}{(n-2)!\,1 \cdot 2 \times 1}$$

$$+ \frac{3 \cdot n(n-1)(n-2)(n-3)!}{(n-3)!\,1 \cdot 3 \times 2!} - \cdots + n(-1)^{n-1} \cdot 1$$

$$= n - n(n-1) + \frac{n(n-1)(n-2)}{2!} + \cdots + n(-1)^{n-1}$$

$$= n\left[1 - (n-1) + \frac{(n-1)(n-2)}{2!} - \cdots + (-1)^{n-1}\right.$$

$= n(1-1)^{n-1}$

$= n \times 0^{n-1} = n \times 0 = 0$

Q. Prove that

$C_0 + 2C_1 + 3C_2 + \cdots + (n+1)C_n = (n+2) \cdot 2^{n-1}$

Proof. $LHS = C_0 + 2C_1 + 3C_2 + \cdots + (n+1) \cdot C_n$

$= C_0 + (1+1)C_1 + (2+1)C_2 + \cdots + (n+1)C_n$

$= C_0 + (C_1 + C_1) + (2C_2 + C_2) + \cdots + (nC_n + C_n)$

$= (C_0 + C_1 + C_2 + \cdots + C_n) + (C_1 + 2C_2 + \cdots + nC_n)$

$= 2^n + n \cdot 2^{n-1} = 2^n + n \cdot \frac{2^n}{2}$

$= 2^n\left(1 + \frac{n}{2}\right) = 2^n\left(\frac{n+2}{2}\right)$

$= (n+2) \cdot 2^{n-1}$

Q. Prove that $C_0 + 3C_1 + 5C_2 + \cdots + (2n+1)C_n = (n+1) \cdot 2^n$

Proof.

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Q. show that $2^{4n+4} - 15n - 16$, where $n \in N$ is divisible by 225.

soln. We have

$$2^{4n+4} - 15n - 16$$

$$= 2^{4(n+1)} - 15n - 16$$

$$= 16^{n+1} - 15n - 16$$

Now,

$$16^{n+1} - 15n - 16$$

$$= (1+15)^{n+1} - 15n - 16$$

$$= {}^{n+1}C_0 + {}^{n+1}C_1 \times 15 + {}^{n+1}C_2 \times (15)^2$$

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$$+ {}^{n+1}C_3 \times (15)^3 + \cdots + {}^{n+1}C_{n+1} \times (15)^{n+1}$$

$$-15n-16$$

$$= 1 + (n+1) \times 15 + {}^{n+1}C_2 \times (15)^2$$

$$+ {}^{n+1}C_3 \times 15^3 + (15)^{n+1} - 15n - 16$$

$$= {}^{n+1}C_2 \times (15)^2 + {}^{n+1}C_3 \times (15)^3$$

$$+ \cdots + (15)^{n+1}$$

$= (15)^2 \left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 15 + \cdots + (15)^{n-1} \right]$

$= 225 \times$ an integer.

$\therefore 2^{4n+4} - 15n - 16$ is divisible by 225

Q. Prove that

$C_0 + 3C_1 + 5C_2 + \cdots + (2n+1)C_n = (n+1) \cdot 2^n$

Proof. LHS $= C_0 + 3C_1 + 5C_2 + \cdots + (2n+1)C_n$

$= C_0 + (2+1)C_1 + (4+1)C_2 + \cdots + (2n+1)C_n$

$= C_0 + (2C_1 + C_1) + (4C_2 + C_2) + \cdots + (2nC_n + C_n)$

$= (C_0 + C_1 + C_2 + \cdots + C_n) + (2C_1 + 4C_2 + \cdots + 2nC_n)$

$= 2^n + 2(C_1 + 2C_2 + \cdots + nC_n)$

$= 2^n + 2 \times n \cdot 2^{n-1}$

$= 2^n + 2 \times n \times \frac{2^n}{2}$

$= 2^n (n+1)$

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Q. Prove that

$$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_1} + \cdots + \frac{n.C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

Proof. LHS $= \frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \cdots + \frac{nC_n}{C_{n-1}}$

$$= \frac{\frac{\cancel{n!}}{(n-1)!\cdot 1!}}{\frac{\cancel{n!}}{(n-0)!\,0!}} + \frac{\frac{\cancel{2}\,\cancel{n!}}{(n-2)!\,\cancel{2!}}}{\frac{\cancel{n!}}{(n-1)!\,1!}} + \frac{\frac{3\,\cancel{n!}}{(n-3)!\,\cancel{3!}}}{\frac{\cancel{n!}}{(n-2)!\,\cancel{2!}}} + \cdots + \frac{\frac{n\,\cancel{n!}}{(n-n)!\,n!}}{\frac{\cancel{n!}}{(\cancel{n}-\cancel{n}+1)!\,(n-1)!}}$$

$$= \frac{n\,\cancel{(n-1)!}}{\cancel{(n-1)!}} + \frac{(n-1)\,\cancel{(n-2)!}}{\cancel{(n-2)!}} + \frac{(n-2)\,\cancel{(n-3)!}}{\cancel{(n-3)!}} + \cdots + \frac{n\,\cancel{(n-1)!}}{\cancel{n!}}$$

$$= n + (n-1) + (n-2) + \cdots + 1$$

$$= 1 + 2 + \cdots + (n-2) + (n-1) + n$$

$$= \frac{n(n+1)}{2}$$

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Q. Prove that

$C_0C_2 + C_1C_3 + C_2C_4 + \dots + C_{n-2}\cdot C_n = \frac{(2n)!}{(n-2)!(n+2)!}$

Proof. L.H.S $= C_0C_2 + C_1C_3 + C_2C_4 + \dots + C_{n-2}\cdot C_n$

given expansion

$(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_{n-2}x^{n-2}$
$+ C_{n-1}x^{n-1} + C_nx^n$ —— (i)

Again

$(x+1)^n = C_0x^n + C_1\cdot x^{n-1} + C_2x^{n-2} + \dots + C_n$ —— (2)

Multiplying eqn (1) and (ii)

$(1+x)^n(x+1)^n = (C_0 + C_1x + C_2x^2 + \dots + C_{n-2}x^{n-2}$
$+ C_{n-1}\cdot x^{n-1} + C_n\cdot x^n)$
$\times (C_0\cdot x^n + C_1\cdot x^{n-1} + C_2x^{n-2}$
$+ C_3x^{n-3} + \dots + C_n)$

$(1+x)^{2n} = (\dots) \times (\dots)$

Equating the coeff of x^{n-2}

$C(2n, n-2) = C_0C_2 + C_1C_3 + C_2C_4 + \dots + C_{n-2}C_n$

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$$\frac{(2n)!}{(2n-n+2)!\,(n-2)!} = C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n$$

$$\frac{(2n)!}{(n+2)!\,(n-2)!} = C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}\cdot C_n$$

Multinomial theorem: -

$$(x+y)^n = {}^nC_0\, x^n y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 + \cdots$$

$$\therefore (x_1 + x_2 + x_3 + \cdots + x_k)^n = \sum \frac{\lfloor n\, (x_1)^{r_1} (x_2)^{r_2} \cdots (x_k)^{r_k}}{r_1!\, r_2!\, r_3! \cdots r_k!}$$

Where $r_1 + r_2 + r_3 + r_4 + \cdots + r_k = n$

and $r_1, r_2, r_3, \cdots r_k$ are non-negative

General term

$$= \frac{\lfloor n\, (x_1)^{r_1} (x_2)^{r_2} (x_3)^{r_3} \cdots (x_k)^{r_k}}{\lfloor r_1\, \lfloor r_2\, \lfloor r_3 \cdots \lfloor r_k}$$

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Total no. of terms = ${}^{n+K-1}C_{K-1}$

where K = no. of terms

Greatest coefficient in expansion of

$$(x_1 + x_2 + x_3 + \cdots + x_K)^n$$

$$= \frac{\lfloor n}{(\lfloor q)^{K-r} (\lfloor q+1)^r}$$

$K)\, n\, (q$

$\quad \underline{\quad}$

$\quad r$

$x_1 \; x_2 \; x_3 \; x_4$

Q. Find coefficient of $a^4 b^2 c d$ in the expansion of $\left(\underset{x_1}{a} + \underset{x_2}{2b} - \underset{x_3}{c} - \underset{x_4}{\frac{d}{4}}\right)^8$

Soln.

General term $= \frac{\lfloor 8}{\lfloor 4 \lfloor 2 \lfloor 1 \lfloor 1} \times (a)^4 \times (2b)^2 \times (-c)^1 \times \left(-\frac{d}{4}\right)^1$

coeff of $a^4 b^2 c d = \frac{\lfloor 8}{\lfloor 4 \lfloor 2 \lfloor 1 \lfloor 1} \times 2 \times -1 \times -\frac{1}{4}$

$= \frac{8 \times 7 \times 6 \times 5 \times \cancel{\lfloor 4}}{\cancel{\lfloor 4} \lfloor 2} \times 2 \cdot \times \frac{1}{4} = 840$

Q. find coefficient of $a^5 b^4 c^7$ in expansion of $(bc + ca + ab)^8$

Soln G.T $= \dfrac{\lfloor 8 \, (bc)^{r_1} (ca)^{r_2} (ab)^{r_3}}{\lfloor r_1 \lfloor r_2 \lfloor r_3}$

$(bc)^{r_1} (ca)^{r_2} (ab)^{r_3}$

$a^{r_2+r_3} \cdot b^{r_1+r_3} \cdot c^{r_1+r_2}$

$a^5 \cdot b^4 \cdot c^7$

Now $r_1 + r_2 + r_3 = 8$ — (1)

$r_2 + r_3 = 5$

$r_1 + r_3 = 4$

$r_1 + r_2 = 7$

Solving these $r_1 = 3$

$r_2 = 4$

$r_3 = 1$

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coeff of $a^5 b^4 c^7$

$= \dfrac{\lfloor 8}{\lfloor 3 \lfloor 4 \lfloor 1}$

$= \dfrac{8 \cdot 7 \cdot \cancel{6} \cdot 5 \, \cancel{\lfloor 4}}{\cancel{3} \times \cancel{2} \times \cancel{\lfloor 4}}$

$= 280$

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Q. find coefficient of x^4 in expansion of $\left(1+x-\frac{2}{x^2}\right)^{10}$

Soln. G.T $= \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} (1)^{r_1} (x)^{r_2} \left(\frac{-2}{x^2}\right)^{r_3}$

$= \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} (1)^{r_1} (x)^{r_2-2r_3} (-2)^{r_3}$

As $r_1+r_2+r_3 = 10$ — (1)

for coeff of x^4

$r_2 - 2r_3 = 4$

$\therefore r_2 = 2r_3+4$ — (2)

Possible cases

$(r_1, r_2, r_3) = (6, 4, 0), (3, 6, 1), (0, 8, 2)$

$(-3, 10, 3)$ ✗

Coeff of $x^4 = \frac{\lfloor 10}{\lfloor 6 \lfloor 4 \lfloor 0} + \frac{\lfloor 10\ (-2)}{\lfloor 3 \lfloor 6 \lfloor 1} + \frac{\lfloor 10 \times 4}{\lfloor 0 \lfloor 8 \lfloor 2}$

$= -1190$

Q. find Greatest coefficient in the expansion of $(a+b+c+d)^{15}$

Soln $n=15$ $\quad k)\,n\,(q$ $\quad k=4$

$k=4$

$q=3$

$r=3$

$$4)\,15\,(3=q$$
$$\underline{12}$$
$$3 \leftarrow r$$

Greatest coeff in the expansion of

$$(a+b+cd)^{15} = \frac{\lfloor n}{(\lfloor q)^{k-r}\,(\lfloor q+1)^{r}}$$

$$= \frac{\lfloor 15}{(\lfloor 3)^{4-1}\,(\lfloor 3+1)^{3}}$$

$$= \frac{\lfloor 15}{\lfloor 3\,(\lfloor 4)^{3}}$$

Q. find total no. of terms in expansion of $(a+b+c+d)^{15}$

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Soln. Total no. of terms

$= {}^{n+K-1}C_{K-1}$, $K=4$, $n=15$

$= {}^{15+4-1}C_{4-1}$

$= {}^{18}C_3$

$= 51 \times 16$

$= 816.$

Q. Coeff of x^2y^2, yzt^2 and $xyzt$ in the expansion of $(x+y+z+t)^4$ are in the ratio

a) 4:2:1 b) 1:2:4 c) 2:4:1 d) 1:4:2

Soln. $G.T = \dfrac{\lfloor 4}{\lfloor r_1 \lfloor r_2 \lfloor r_3 \lfloor r_4} (x)^{r_1} (y)^{r_2} (z)^{r_3} (t)^{r_4}$

Coeff of $x^2y^2 = \dfrac{\lfloor 4}{\lfloor 2 \lfloor 2}$

Coeff of $yzt^2 = \dfrac{\lfloor 4}{\lfloor 1 \lfloor 1 \lfloor 2}$

Coeff of $xyzt = \dfrac{\lfloor 4}{\lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1}$

$\dfrac{\lfloor 4}{\lfloor 2 \lfloor 2} : \dfrac{\lfloor 4}{\lfloor 2} : \dfrac{\lfloor 4}{\lfloor 1}$

$\dfrac{1}{4} : \dfrac{1}{2} : \dfrac{1}{1}$

$1 : 2 : 4$

Q. Find the coefficient of $x^2 y^2 z^3$, xyz^5 and $x^3 z^4$ in the expansion of $(x+y+z)^7$.

Soln. Given expansion $(x+y+z)^7$

$$\text{Coeff of } x^2y^2z^3 = \frac{7!}{2!\,2!\,3!}(1)^2\cdot(1)^2(1)^3$$

$$= \frac{7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{2\times 1\times 2\times 1\times \cancel{3!}}$$

$$= 210$$

$$\text{Coeff of } x^1y^1z^5 = \frac{7!}{1!\,1!\,5!}$$

$$= \frac{7\cdot 6\cdot \cancel{5!}}{\cancel{5!}}$$

$$= 42$$

$$\text{Coeff of } x^3 \underset{y^0}{\vee} z^4 = \frac{7!}{3!\,0!\,4!}$$

$$= \frac{7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{4\cdot 3\cdot 2\cdot 1\times \cancel{3!}}$$

$$= 35$$

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Q. Find the coefficient of $a^2 b^3 c^2 d^5 e^4$ in the expansion of $(a+2b-3c+2d+5)^{16}$.

Soln. Let $v = a$, $w = 2b$, $x = -3c$, $y = 2d$ and $z = 5$ then $(v+w+x+y+z)^{16}$

find $v^2 w^3 x^2 y^5 z^4$.

coeff of $v^2 w^3 x^2 y^5 z^4 = \dfrac{\lfloor 16}{\lfloor 2 \lfloor 3 \lfloor 2 \lfloor 5 \lfloor 4}$

change to original

coeff of $a^2 b^3 c^2 d^2 e^4$

$= \dfrac{\lfloor 16}{\lfloor 2 \lfloor 3 \lfloor 2 \lfloor 5 \lfloor 4} (1)^2 (2)^3 (-3)^2 (2)^5 . 5^4$

Q. Find the coefficient of x^4 in the expansion $(2-x+3x^2)^6$.

Soln. Given expansion $(2-x+3x^2)^6$

$G.T = \sum \dfrac{\lfloor 6}{\lfloor r_1 \lfloor r_2 \lfloor r_3} \times (2)^{r_1} (-x)^{r_2} (3x^2)^{r_3}$

$= \sum \dfrac{\lfloor 6}{\lfloor r_1 \lfloor r_2 \lfloor r_3} \times (2)^{r_1} \times (-1)^{r_2} . x^{r_2+2r_3} . 3^{r_3}$

As $r_1 + r_2 + r_3 = 6$.

for coeff of x^4,

$$r_2 + 2r_3 = 4$$

Possible cases

$(r_1, r_2, r_3) = (2, 4, 0), (3, 2, 1), (4, 0, 2)$

$\therefore$ coeff of x^4

$$= \sum \frac{6!}{r_1!\, r_2!\, r_3!} \times 2^{r_1} \times (-1)^{r_2} \times 3^{r_3}$$

$$= \frac{6!}{2!\,4!} (2)^2 (-1)^4 (3)^0$$

$$+ \frac{6!}{3!\,2!} (2)^3 (-1)^2 (3)^1$$

$$+ \frac{6!}{4!\,0!\,2!} (2)^4 (-1)^0 \times 3^2$$

$$= \frac{6\cdot5\cancel{\lfloor 4}}{2\times\cancel{\lfloor 4}}\times4 + \frac{6\cdot5\cdot4\cancel{\lfloor 3}}{\cancel{\lfloor 3}\times\cancel{2}}\times\cancel{8}^{4}\times3$$

$$+ \frac{6\cdot5\cancel{\lfloor 4}}{\cancel{\lfloor 4}\times\cancel{2}}\times\cancel{16}^{8}\times9$$

$= 60 + 1440 + 2160$

$= 3660$

Q. find the coefficient of x^7 in the expansion $(1-2x+x^3)^5$.

soln. Given expansion $(1-2x+x^3)^5$

$$G.T. = \sum \frac{\lfloor 5}{\lfloor r_1 \lfloor r_2 \lfloor r_3}(1)^{r_1}(-2x)^{r_2}(x^3)^{r_3}$$

$$= \sum \frac{\lfloor 5}{\lfloor r_1 \lfloor r_2 \lfloor r_3}(1)^{r_1}(-2)^{r_2}\cdot x^{r_2+3r_3}$$

As, $r_1+r_2+r_3 = 5$

for coeff of x^7. $r_2+3r_3 = 7$

$\Rightarrow \quad r_2+3r_3 = 7$

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Possible cases :

$(r_1, r_2, r_3) = (-2, 7, 0), (0, 4, 1), (2, 1, 2)$

$$\text{Coeff of } x^7 = \frac{\lfloor 5}{\lfloor 4}(-2)^4 + \frac{\lfloor 5}{\lfloor 2 \lfloor 2}(-2)$$

$$= \frac{5 \cancel{\lfloor 4}}{\cancel{\lfloor 4}} \times 16 + \frac{5 \times 4 \times 3 \cancel{\lfloor 2} \times (-2)}{\cancel{\lfloor 2} \times \cancel{\lfloor 2}}$$

$= 80 - 60$

$= 20$

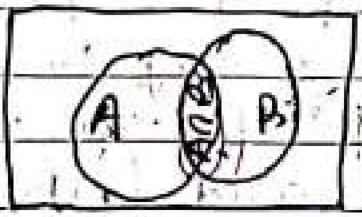
Inclusion and exclusion Principle :-
It is a counting technique which generalizes the familiar method of obtaining the number of element in the union of two finite sets. It is denoted by

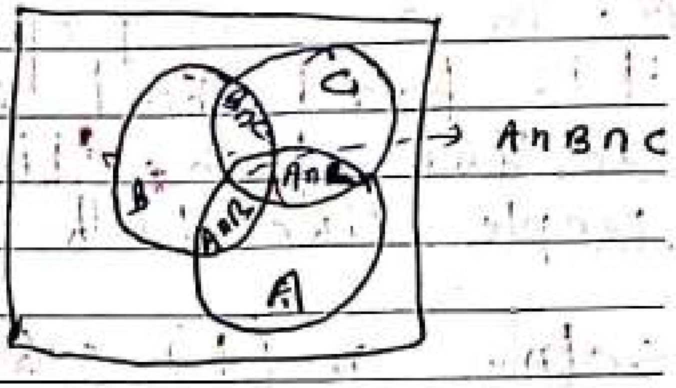
$$|A \cup B| = |A| + |B| - |A \cap B|$$

where A and B are two finite sets. The formula expresses that the sum of the sizes of the two sets may be too large since some elements may be counted twice.

for 3 sets A, B and C

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$





Properties:-

1) If A and B are not disjoint, then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

2) If A and B are disjoint, then

$$|A \cup B| = |A| + |B| \quad \because |A \cap B| = \phi$$

3) $|A \cup B| = |A| + |B - A|$

$$|A \cap B| + |B - A| = |B|$$

$$|A \cup B| = |B| + |A - B|$$

$|A \cap B| + |A - B| = |A|$

4) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B|$
$- |B \cap C| - |A \cap C|$
$+ |A \cap B \cap C|$

Q. In a survey of group of 80 people, it is found that 60 like egg and 30 like fish. Find percentage of people like both egg and fish.

Soln. $n(E) = 60$, $n(F) = 30$

$n(E \cup F) = 80$, $n(E \cap F) = ?$

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

$80 = 60 + 30 - n(E \cap F)$

$\Rightarrow n(E \cap F) = 90 - 80 = 10$

$\therefore n(E \cap F) = 10$

Q. Out of 200 students, 50 take maths. 140 of them take economics and 24

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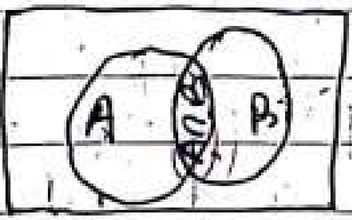
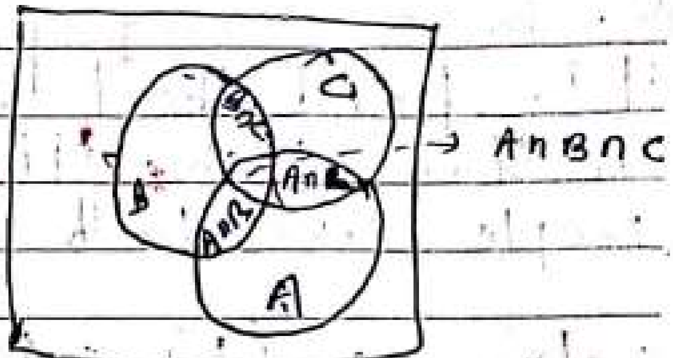
may be too large since some elements may be counted twice.
for 3 sets A, B and C

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$





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$$|A \cap B| + |B - A| = |B|$$

$$|A \cup B| = |B| + |A - B|$$

$|A \cap B| + |A - B| = |A|$

4) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B|$
$- |B \cap C| - |A \cap C|$
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$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

$80 = 60 + 30 - n(E \cap F)$

$\Rightarrow n(E \cap F) = 90 - 80 = 10$

$\therefore n(E \cap F) = 10$

Q. Out of 200 students, 50 take maths. 140 of them take economics and 24

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take both. How many of those not not take any of the course.

soln. $n(M) = 50$, $n(E) = 140$.

$n(M \cap E) = 24$.

$n(M \cup E) = 50 + 140 - 24$

$= 190 - 24$

$= 166$.

students doesn't take any course

$= 200 - 166$

$= 34$.

Q. In a survey of the wage of three toothpaste A, B, C. It is found that 60 people like A, 55 like B, 40 like C, 20 like A and B, 35 like B and C, 15 like A and C and 10 like all. Find no. of persons included in the survey.

soln. $n(A) = 60$, $n(B) = 55$, $n(C) = 40$,

$n(A \cap B) = 20$, $n(B \cap C) = 35$, $n(A \cap C) = 15$

$n(A \cap B \cap C) = 10$

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$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= 60 + 55 + 40 - 20 - 15 - 35 + 10$$

$$= 95$$

Inclusion - Exclusion Principle:

Definition:-

$A_1, A_2, \cdots, A_n \rightarrow$ finite sets. then

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$$|A_1 \cup A_2 \cdots \cup A_n| = \sum A_i - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - + \cdots + (-1)^{n+1} |A_1 \cap A_2 \cdots \cap A_n|$$

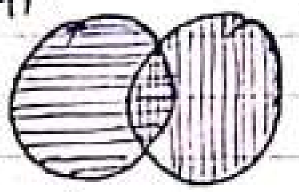
Proof: By using mathematical induction

Base step

$n = 2$

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

$$|A_2| \equiv |A_1| \# |A_1 \cap A_2|$$



<u>Inductive step</u>

Let $|A_1 \cup A_2 \cup \cdots \cup A_K| = \sum_{i=1}^{K} |A_i| - \sum |A_i \cap A_j|$

$+ - \cdots (-1)^{K+1} |A_1 \cap A_2 \cdots \cap A_K|$

$|A_1 \cup A_2 \cdots \cup A_{K+1}| = ?$

Let $B = A_1 \cup A_2 \cdots \cup A_K$. then $|B \cup A_{K+1}|$

$= |B| + |A_{K+1}| - |B \cap A_{K+1}|$

$|B \cap A_{K+1}| = |(A_1 \cap A_{K+1}) \cup (A_2 \cap A_{K+1}) \cup \cdots$

$\cup (A_K \cap A_{K+1})|$

$= \sum |A_i \cap A_{K+1}| - \sum |A_i \cap A_j \cap A_{K+1}|$

$+ - \cdots (-1)^{K+1} |A_1 \cap A_2 \cap \cdots \cap A_{K+1}|$

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$$|B \cup A_{k+1}| = |A_1 \cup A_2 \cup \cdots A_k| + |A_{k+1}| \quad -(1)$$

$$= \sum_{i=1}^{k} |A_i| - \sum |A_i \cap A_j| + \cdots + (-1)^{r+1} (A_1 \cap \cdots A_k)$$

$$+ |A_{k+1}| - \sum |A_i \cap A_{k+1}|$$

$$+ \sum |A_i \cap A_j \cap A_{k+1}|$$

$$+ \cdots (-1)^{k+1} |A_1 \cap A_2 \cdots A_{k+1}|$$

$$= \sum_{i=1}^{k+1} |A_i| - \sum_{\substack{i=1 \\ j=1 \\ i=j}}^{k+1} |A_i \cap A_j| + \cdots (-1)^{k+1} |A_1 \cap \cdots \cap A_{k+1}|$$

$$= |A_1 \cup A_2 \cdots \cup A_{k+1}|$$

Q. 270 college students, 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 28 like both

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brussels sprouts and broccoli, 18 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

soln. A_1 = like brussels sprouts
A_2 = " broccoli
A_3 = cauliflower

$|A_1| = 64$, $|A_2| = 94$, $|A_3| = 58$

$|A_1 \cap A_2| = 26$, $|A_1 \cap A_3| = 28$, $|A_2 \cap A_3| = 22$

$|A_1 \cap A_2 \cap A_3| = 14$

$$|A_1 \cup A_2 \cup A_3| = 64 + 94 + 58 - 26 - 28 - 22 + 14$$
$$= 154$$

students not like vegetables

$$= 270 - 154$$
$$= 116$$

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Q. In a survey of 60 persons, it was found that 25 read newsweek, 26 read time and 26 read fortune. Further 9 read both newsweek and fortune, 11 read both newsweek and time, 8 read both time and fortune and 8 read none of these.

i) find the number of people who read all the three magazines

ii) find the number of people who read exactly one magazine.

soln. $n(S) = 60$, $n(N) = 25$, $n(T) = 26$, $n(F) = 26$, $n(N \cap F) = 9$, $n(N \cap T) = 11$

$n(T \cap F) = 8$, $n(\bar{N} \cap \bar{T} \cap \bar{F}) = 8$

$n(\bar{N} \cap \bar{T} \cap \bar{F}) = 8$

$$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(T \cap F) - n(N \cap F) + n(N \cap T \cap F)$$

$$n(N \cup T \cup F) = n(S) - n(\bar{N} \cap \bar{T} \cap \bar{F})$$

$$= 60 - 8 = 52$$

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$52 = 25 + 26 + 26 - 11 - 8 - 9 + n(N \cap T \cap F)$

ii) $n(N \cap T \cap F) = 3$,

N and F only = $n(N \cap F) - 3) = 9 - 3 = 6$

N and T only = $n(N \cap T) - 3 = 11 - 3 = 8$

T and F only = $8 - 3 = 5$.

T only = $26 - 8 - 5 - 3 = 10$

N only = $25 - 8 - 6 - 3 = 8$

F only = $26 - 6 - 5 - 3 = 12$

Q. Out of 250 candidates who failed in either mathematics or physics or in Aggregate, it was revealed that 128 failed in mathematics and 87 in Physics and 134 in Aggregate. 31 failed in mathematics and Physics, 54 failed in Aggregate and mathematics, 30 failed in Aggregate and physics.
Find how many candidates failed.

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1) in all the three subjects.

2) in mathematics but not in physics.

3) in Aggregate but not in mathematics

Soln. $n(M \cup P \cup A) = n(M) + n(P) + n(A) - n(M \cap P) - n(A \cap M) - n(P \cap A) + n(M \cap P \cap A)$

$250 = 128 + 87 + 134 - 31 - 54 - 30 + n(M \cap P \cap A)$

16 Sunday

$\Rightarrow n(M \cap P \cap A) = 16$

16 candidates failed in all 3 subjects.

M and P both $= n(M \cap P) - 16 = 31 - 16 = 15$

M and A both $= n(M \cap A) - 16$

$= 54 - 16 = 38.$

A and P both $= n(A \cap P) - 16$

$= 30 - 16$

$= 14.$

ii) failed in M but not in P

= 128 − (15 − 16

= 97

iii) 134 − 38 − 16

= 880 failed in A but not in M

Q. Let $U = \{1, 2, 3, \ldots, 1000\}$ when $1 \le n \le 1000$ that are not divising 5, 6, and 8.

soln. $n(A) = \frac{1000}{5} = 200$

$n(B) = \frac{1000}{6} = 166$

$n(C) = \frac{1000}{8} = 125$

$n(A \cap B) = \frac{1000}{5 \times 6} = \frac{1000}{30} = 33$

$n(A \cap C) = \frac{1000}{5 \times 8} = \frac{1000}{40} = 25$

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$n(B \cap C) = \frac{1000}{48} = 20$

$n(A \cap B \cap C) = \frac{1000}{5 \times 6 \times 8} = \frac{1000}{240} = 4$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$- n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C)$

$= 200 + 166 + 125 - 33 - 25 - 20 + 4$

$= 417$

Factorial :- It is the number obtained by multiplying all the +ve integers less than or equal to a given positive integer.

$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$

$0! = $ Unity

$4! = 4 \times 3 \times 2 \times 1 = 24$

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<u>Permutation :-</u> $^nP_r = \frac{n!}{(n-r)!}, r \leq n$

$P(n,r)$

r objects chosen from n and there r objects have to be arranged in same fashion.

Ordered arrangements of objects.

[a, b, c, d] → four letters. find permutation taken two letters.

ab, ba, ac, ca, ad, da, bc, cb, bd, db, cd, dc

12 permutations.

$n=4, r=2$

$$^4P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!}$$

$$= \frac{4 \times 3 \times 2!}{2!} = 12$$

<u>Permutations with repetitions :-</u>

n objects of which n_1 are alike, n_2 are alike and so : then

$$= \frac{n!}{n_1!\, n_2! \ldots n_r!}$$

INSTITUTION :

S = 1, O = 1, n = 1, I = 3, T = 3, N = 2

$$= \frac{11!}{3! \cdot 3! \cdot 2!}$$

Q. How many words can be made by using the letters of the word "SIMPLETON" taken all at a time.

Soln. n = 9, r = 9

$$^9P_9 = \frac{9!}{(9-9)!} = \frac{9!}{0!} = 9!$$

Q. How many diff signals can be made by 5 flags from 8 flags of diff colors?

Soln. n = 8, r = 5,

$$^8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

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Q. How many permutations of the letters ABCDEFGH contains the string ABC?

Sol^n. $\underbrace{ABC}_{1}$ DEFGH $\rightarrow 6! = 720$

Single letter.

Circular Permutations

i) If clockwise and anticlockwise orders are different $= (n-1)!$

ii) If orders are not different

$$= \frac{(n-1)!}{2}$$

iii) (n, r), if orders are different

$$= \frac{^nP_r}{r}$$

iv) If orders are not different

$$= \frac{^nP_r}{2r}$$

Q. How many ways ten people can sit around a table?

Soln. $(n-1)! = (10-1)! = 9!$

Q. How may necklace of 12 beads each can be made from 18 beds of diff colours.

Soln. clockwise and anticlockwise arrangement are same

$$\frac{^nP_r}{2r} = \frac{^{18}P_{12}}{24} = \frac{18!}{6! \times 24}$$

Q. 23 Sunday In how many ways can 4 letters be posted in 3 letter boxes?

Soln. Each letter can be posted in 3 ways

∴ 4 letters can be posted

$= 3 \times 3 \times 3 \times 3 = 3^4 = 81$ ways

Q. In how many ways can 6 different rings be worn in 4 fingers of the hand?

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Soln. Each ring may be worn in 4 ways.

∴ 6 rings may be worn $= 4\times4\times4\times4\times4\times4$
$= 4^6 = 4096$ ways

Q. In how many ways can 5 apples be distributed among 6 boys, there being no restriction to the number of apples each boy may get?

Soln. Each apple may be given to any of the 6 boys. So each people may be given in 6 ways

∴ 5 apples may be given $= 6\times6\times6\times6\times6 = 6^5$

Permutations with Repetitions :-

The no. of P of n diff objects, taken r at a time when each may be repeated any number of times in each arrangement is

$= n^r$

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Permutations formulas :-

i) No of permutations of 'n' things taken 'r' at a time when a particular thing is to be always included in each arrangement

$= r^{n-1} \times P_{r-1}$

ii) $P(n,r)$ when a particular thing is fixed :-

$= {}^{n-1}P_{r-1}$

iii) $P(n,r)$ when particular thing is never taken

$= {}^{n-1}P_{r}$

iv) $P(n,r)$, when 'm' specified things always come together

$= m! \times (n-m+1)!$

v) No. of permutations of 'n' things taken all at a time, when m specified things always come together.

$= n! - [m! \times (n-m+1)!]$

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combinations:- Number of combinations of 'n' different things, taken 'r' at a time is given by.

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

Problems of circular Permutations:-

Q. In how many ways can 8 students be arranged in i) a line ii) a circle

soln. i) a line = 8!

ii) a circle = (8-1)! = 7!

Q. If 20 persons were invited for a party in how many ways can they and the host be seated at a circular table?

soln. Number of ways in which these 21 persons can be seated in round a circular table = (21-1)! = 20!

Q. In how many ways can a party 4 men and 4 women be seated at a circular table so that no two women are adjacent?

Soln. Number of ways in which these 4 men can be seated at the circular table $= 3! = 6$

4 vacant seats may be occupied by 4 women in $= {}^4P_4 = 4! = 24$ ways

$\therefore$ Required No. of ways $= 6 \times 24 = 144$.

Q. In how many ways can 7 persons sit around a table so that all shall not have the same neighbours in any two arrangements?

Soln. 7 persons sit at a round table

$= (7-1)! = 6!$ ways

In clockwise and anticlockwise arrangements, each persons will have the same neighbours.

Required number of ways $= \frac{(n-1)!}{2}$

$= \frac{(7-1)!}{2} = \frac{6!}{2} = 360$

Pigeon hole Principle:-

If n pigeonholes are occupied by $n+1$ or more pigeons, then atleast one pigeonhole occupied by more than one pigeon.

1	7, 2	3
4	5	6

Pigeonholes $= 6 = n$

pigeon $= 7 = n+1$

Generalised Pigeonhole Principle:-

No. of hole = given

Find : No. of pigeon.

If n pigeonholes are occupied by $Kn+1$ or more pigeons, where K is a positive integer, then atleast one pigeonhole is occupied by $K+1$ or more pigeons.

Q. Find the minimum number of student in a class is to that three of them are born in the same month.

Sol. $n = 12$, $m = ?$, $Kn+1 = ?$

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$K+1 = 3 \Rightarrow K = 2$

$m = Kn+1 = 2 \times 12+1 = 25$

Q. Find the minimum number of sock that one needs to choose in order to get two paires (4 socks) of same color.

Soln. 3 diff colors of socks.

$n = 3$

$K+1 = 4 \Rightarrow K = 3$

30 Sunday $m = Kn+1 = 3 \times 3+1 = 10$

Pigeon hole principle :-

If A is average no. of pigeon per pigeonhole then. If 11 pigeon and 4 holes $11/4 = 2$ (3)

principle 1 :- Some pigeon hole contains at least [A] pigeon

principle 2 :- Some other pigeon hole at most [A] pigeon.

Q. find min. no. of students in a class so that 2 students were born in same month

soln. $\underbrace{12 \text{ month} + 1}_{K+1} \Rightarrow$ 13 students

1 year $\rightarrow$ 365 days + 1

366

Q. find min no. of students in a class to be sure that 4 of them were born in same month

$n = 12$

soln. $K+1 = 4 \Rightarrow K = 3$, $n = 12$

$Kn + 1 = 3 \times 12 + 1 = 37$

Q A box contains 10 blue balls, 20 red balls, 8 green balls, 15 yellow balls and 25 white balls. How many balls must we choose to ensure that we have 12 balls of the same colour.

soln. $n = 5$, $K+1 = 12 \Rightarrow K = 11$

$Kn+1 = 11 \times 5 + 1 = 56$

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Q. find the minimum number of students in a class to be sure that six of them are born in same month.

Soln. $n = 12$, $K+1 = 6 \Rightarrow K = 5$

$$Kn+1 = 5 \times 12 + 1 = 61$$

Q. Suppose a loundary bag contains number of socks that one needs to choosen in order to get two socks of of the same colour.

Soln. $n = 3$, $K+1 = 3 \Rightarrow K = 2$

$$Kn+1 = 2 \times 3 + 1$$

$$= 7$$

Generalized Pigeonhole Principle:-

If n objects (Pigeons) are placed into K $(n > K)$ boxes (pigeonhole then there is atleast one box containing atleast $\left\lceil \frac{n}{K} \right\rceil$ objects

or

or $\left[\frac{n-1}{K}\right]+1$

Q. How many students must be in a class to guarantee that atleast 2 students receive same score.

[0 to 100]

Soln. $0 \longrightarrow 100$] 101 scores.

102 students so that 2 student score same marks

$[1 \longrightarrow 4] \longrightarrow 5$

1, 1	2, 2, 5
3, 3	4, 4

Q. How many people atleast in a group of 110 people have the same last initials

Soln. $n = 110$, $m = 26$ [English word have 26 letters]

$\therefore \left[\frac{n-1}{m}+1\right] = \left[\frac{110-1}{26}+1\right]$

$= \frac{109}{26}+1 = 4+1 = 5$

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Q. Show that at least 2 people out of 13 must have their birthday in the same month when they are assembled in a room.

Soln: $n = 13$, $m = 12$

$\therefore \left\lfloor \frac{n-1}{m} \right\rfloor + 1 = \frac{13-1}{12} + 1 = 1+1 = 2$

Q. Show that if 10 colours are used to paint 201 houses, at least 21 houses will be same colour.

Soln. $n = 201$, $m = 10$

$\frac{n-1}{m} + 1 = \frac{201-1}{10} = \frac{200}{10} + 1$

$= 20 + 1 = 21$

Q. How many people must have to guarantee that at least 9 of them will have birthday in the same day of week?

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Soln. Let n = No. of people

m = 7 [7 days in a week]

$\therefore \frac{n-1}{m} + 1 \Rightarrow \frac{n-1}{7} + 1 = 9$

$\Rightarrow \frac{n-1}{7} = 8 \Rightarrow n = 57$

Q. How many people among 120000 are born at the same time (h, m, s)?

Soln. Min^m no. of people born at the same $= \frac{n-1}{24} + 1$

$= \frac{120000-1}{24} + 1$

Q. Suppose we have 27 distinct odd positive integers all less than 100. Show that there is a pair of numbers whose sum is 102. What if there were only 26 odd positive integers?

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