Chapter	Name of the Topic	Hours	Marks
01	**1. Function, Limit and Continuity** 1.1 Function • Definition of variable, constant, intervals and their type • Definition of Function, value of a function and types of functions, Simple Examples • Definition of sinh x, cosh x and tanh x and some hyperbolic identities 1.2 **Use the concepts of Limit for solving the problems** • Explain the concept of limit and intuitive meaning of $\lim_{x\to a} f(x)=l$ and its properties. • Derive the Standard limits $\lim_{x\to a}\frac{x^n-a^n}{x-a}$, $\lim_{x\to 0}\frac{\sin x}{x}$, $\lim_{x\to 0}\mathrm{Cos}x$, $\lim_{x\to 0}\frac{\tan x}{x}$, $\lim_{x\to 0}\frac{a^x-1}{x}$, $\lim_{x\to 0}\frac{e^x-1}{x}$, $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$, $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ with simple example. • Evaluate the limits of the type $\lim_{x\to a}\frac{f(x)}{g(x)}$. • Explain the Concept of continuity of a function at a point and in interval with some examples whether a given function is continuous or not.	06	12

KK COLLEGE OF ENGINEERING AND MANAGEMENT
DIPLOMA 2ND SEMESTER
SUB- ENGINEERING MATHEMATICS

	2. **Differentiation and its meaning in engineering situations** • Concept of derivative of a function y = f(x) from the first principle as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and Standard notations to denote the derivative of a function. • Derivatives of elementary functions like x^n, a^x, e^x, log x, sin x, cos x, tanx, secx, cosecx, cot x and Invise Trigonometrical function using the first principles. • Rules for differentiation of sum, difference, scalar multiplication, product and quotient of functions with illustrative and simple examples. • Differentiation of a function of a function (Chain rule) with illustrative examples such as (i) $\sqrt{t^2+\frac{2}{t}}$ (ii) $x^2\sin 2x$ (iii) $\frac{x}{\sqrt{x^2+1}}$ (iv) $\log(\sin(\cos x))$.etc • Differentiation of a function with respect to another function and also differentiation of parametric functions with examples. • Derivatives of some simple hyperbolic functions (without Proof). • Differentiation of implicit function with examples. • Logarithmic differentiation of some functions with examples like $[f(x)]^{g(x)}$. • Concept of higher order derivatives (second and third order) with examples. • Concept of functions of several variables, partial derivatives and difference between the ordinary and partial derivatives with simple examples.	12	24

	3. Applications of the Differentiation 3.1 Geometrical Applications of Derivatives ▪ State the Geometrical meaning of the derivative as the slope of the tangent to the curve y=f(x) at any point on the curve. ▪ Equation of tangent and normal to the curve y=f(x) at any point on it. ▪ The concept of angle between two curves and procedure for finding the Angle between two given curves with illustrative examples. 3.2 Use of Derivatives to find extreme values of functions ▪ The concept and condition of increasing and decreasing functions with illustrative examples. ▪ Find the extreme values (maxima or minima) of a function of single variable - simple problems yielding maxima and minima. 3.3 Concept of Derivatives as Rate Measure with illustraribe examples. 3.4 Concept of Derivatives to find Radius of Curvature with illustraribe examples.	**14**	**24**
	4. Statistics ▪ Measures of Central tendency (mean, median, mode) for ungrouped and grouped frequency distribution. ▪ Graphical representation (Histogram and Ogive Curves) to find mode and median ▪ Measures of Dispersion such as range, mean deviation, Standard Deviation, Variance and coefficient of variation. Comparison of two sets of observations.	**04**	**08**
	5. Complex Number. ▪ Represent the complex number in various forms like modulus-amplitude, polar form, Exponential (Euler) form – illustrate with examples ▪ Modulus, Conjugate and Argument of Complex Number and their properties. ▪ Operations on complex numbers (Equality, Addition, Subtraction, Multiplication and Division) with examples. ▪ Square root of complex number ▪ Cube roots of units and their properties, simple problems based on them. ▪ De-Moivre's theorem (without proof) and simple problems.	**6**	**12**
	Total	**42**	**80**

Variable:- The quantities which change their values are called variable. It is denoted by x, y, z, ...

Example

i) The temperature of a city

ii) The velocity of a vehicle moving on a road.

iii) The marks of students in Mathematics of class XII.

All change with time

Constant:- The quantities which do not change their values are called constant. It is denoted by a, b, c, ...

Example

i) Volume of cube

ii) Weight of object.

Do not change with time

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Type of constant

1) Arbitrary constant :- The variable which has fixed values but different in another is called an arbitrary constant.

Example - $y = mx + c$

where m and c are fixed for a particular line but different in another position of same line.

$$y = ae^{x} + be^{-x},$$

$a, b =$ Arbitrary constants.

ii) Pure constants :- The variable which has fixed values anywhere, any time is called pure.

Example.

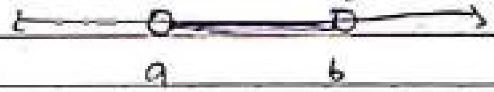
1) The ratio of the circumference to the diameter of a circle is a pure constant.

2) The counting numbers are pure constants

Intervals :- It is a collection of all real numbers which lie between two given real numbers.

Open interval :- If the variable takes all the values between a and b but not a and b. Then the interval is called an open interval. It is denoted by (a, b)

$\therefore (a, b) = \{x / x \in \mathbb{R},\ a < x < b\}$

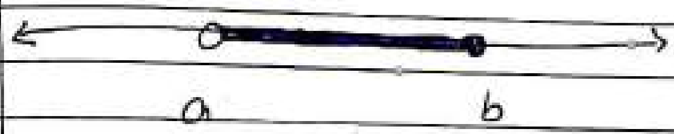


Closed interval :- If the variable takes all the values between a and b including a and b. Then the interval is called a closed interval

$[a,b] = \{x / x \in R,\ a \leq x \leq b\}$

Semi - Open interval :- If the variable (x) takes values between a and b such that $a < x \leq b$ or $a \leq x < b$ are called semi-open or semi-closed intervals and are denoted by $(a,b]$ and $[a,b)$

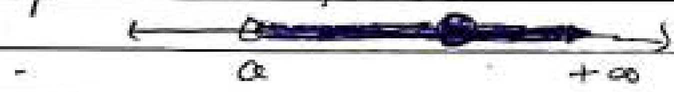
$(a,b] = \{x / x \in R,\ a < x \leq b\}$, $[a,b) = \{x / x \in R,\ a \leq x < b\}$



Other intervals :-

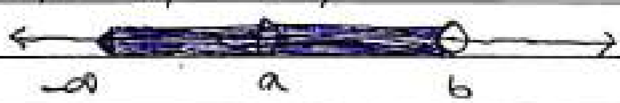
a) The set of all real numbers greater than 'a'. It is denoted by (a, ∞)

$(a, \infty) = \{x / x \in R,\ x > a\}$



b) The set of all real number less than b. It is denoted by $(-\infty, b)$

$(-\infty, b) = \{x / x \in R,\ x < b\}$



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Even function

for $y = f(x)$, if $f(-x) = f(x)$, then $f(x)$ is called an even function.

If $y = f(x) = x^2$, then $f(-x) = (-x)^2 = x^2 = f(x)$

$\therefore$ $f(x) = x^2$ is an even function.

Ex - i) $f(x) = \cos x$

$f(-x) = \cos(-x) = \cos x$

$f(x) = f(-x) = \cos x$

$\therefore$ f is even

ii) $f(x) = x^4$

$f(-x) = (-x)^4 = x^4$

odd function

$y = f(x)$,

if $f(-x) = -f(x)$,

then $f(x)$ is called an odd function.

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Q. (1) $f(x) = \dfrac{x}{1+x^2}$

$$f(-x) = \frac{-x}{1+(-x)^2} = \frac{-x}{1+x^2} = -\frac{x}{1+x^2}$$

$= -f(x)$

$\therefore f(-x) = -f(x)$

$\therefore f(x)$ is an odd function

(2) $f(x) = \sin x$

$f(-x) = \sin(-x) = -\sin x = -f(x)$

$\therefore f(-x) = -f(x)$

$\therefore f$ is an odd function.

Q. If $f(x) = 4x^4 + 3\cos x + x\sin x + 1$ show that the function is even

Soln. $f(x) = 4x^4 + 3\cos x + x\sin x + 1$

$f(-x) = 4(-x)^4 + 3\cos(-x) + (-x)\sin(-x) + 1$

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$= 4x^4 + 3\cos x + x\sin x + 1 = f(x)$

$\therefore f$ is even fun

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Q. If $f(x) = \cos x$, show that

$f(3x) = 4.f^3(x) - 3f(x)$

Soln. $f(x) = \cos x$

$$f(3x) = \cos 3x$$
$$= 4\cos^3 x - 3\cos x$$
$$= 4(\cos x)^3 - 3\cos x$$
$$= 4\{f(x)\}^3 - 3f(x)$$

$$\therefore f(3x) = 4.f^3(x) - 3f(x)$$

Q. If $f(x) = \frac{1}{x}$,

Prove that $f(x) - f(x+1) = f(x^2+x)$

Soln. $f(x) = \frac{1}{x}$

$$f(x+1) = \frac{1}{x+1}$$

$$f(x^2+x) = \frac{1}{x^2+x}$$

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$$f(x) - f(x+1) = \frac{1}{x} - \frac{1}{x+1}$$

$$= \frac{(x+1) - x}{x(x+1)}$$

$$= \frac{1}{x^2+x} = f(x^2+x)$$

$$\therefore \quad f(x) - f(x+1) = f(x^2+x)$$

Q. If $f(x) = \frac{1}{1-x}$,

Show that $f[f\{f(x)\}] = x$

Soln. $f(x) = \frac{1}{1-x}$

Change x by $f(x)$

$$\therefore f\{f(x)\} = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}}$$

$$= \frac{1-x}{1-x-1} = \frac{1-x}{-x}$$

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$$\therefore f\{f(x)\} = \frac{x-1}{x}$$

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Again replacing x by f(x)

$$\therefore f[f\{f(x)\}] = \frac{f(x)-1}{f(x)}$$

$$= \frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}}$$

$$= \frac{1-1+x}{x} = x$$

$$\therefore f[f\{f(x)\}] = x$$

Solve these

1. show that $f(x) = \frac{a^x + a^{-x}}{2}$ is even f^n

2. show that $f(x) = x^3 + 3\sin x + x$ is odd f^n

3. $f(x) = \frac{\cos x}{1+\sin^2 x}$

determine whether it is an odd or even f^n.

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function

Q. If $f(x) = 16^x + \log_2 x$. find the value of $f(\frac{1}{4}), f(\frac{1}{2})$

Soln. $f(1/4) = 16^{1/4} + \log_2 1/4 = (2^4)^{1/4} + \log_2 2^{-2}$

$= 2^{4\times 1/4} + \log_2 2^{-2}$ ⇒ $2^1 + (-2)\log_2 2 = 2 + (-2)\times 1$
$= 2 - 2 = 0$

$f(1/2) = 16^{1/2} + \log_2 1/2 = (2^4)^{1/2} + \log_2 2^{-1} = 2^2 + (-1).1$
$= 4 - 1 = 3$

Q. $f(x) = x^2 + x + 1$. find $f(x+1)$

Sln. $f(x+1) = (x+1)^2 + x+1+1 = x^2 + 2x + 1 + x + 1 + 1$
$= x^2 + 3x + 3$

Q. If $f(x) = x^3 - 3x + \sin x + x\cos x$, Prove that $f(x) + f(-x) = 0$

Soln. $f(x) = x^3 - 3x + \sin x + x\cos x$
$f(-x) = (-x)^3 - 3(-x) + \sin(-x) + \{-x\cos(-x)\}$
$f(-x) = -x^3 + 3x - \sin x - x\cos x$
$\therefore f(x) + f(-x) = x^3 - 3x + \sin x + x\cos x - x^3 + 3x - \sin x - x\cos x$
$= 0$

Q. If $f(t) = 10\sin(100\pi t)$, find $f(\frac{1}{200} - t)$

Soln. $f(\frac{1}{200} - t) = 10\sin[100\pi(\frac{1}{200} - t)]$
$= 10\sin[\frac{\pi}{2} - 100t\pi]$
$= 10\cos(100t\pi)$ $\because \sin(\frac{\pi}{2} - \theta) = \cos\theta$

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Q. If $\phi(t) = I_m \sin(20t - 0.2)$, find $\phi(\pi/40 + t)$

Soln. $\phi\left(\frac{\pi}{40} + t\right) = I_m \sin\left[20\left(\frac{\pi}{40} + t\right) - 0.2\right]$

$= I_m \sin\left(\frac{\pi}{2} + 20t - 0.2\right)$

$= I_m \sin\left(\pi/2 + 20t - 0.2\right)$

$= I_m \cos(20t - 0.2)$

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Q. If $f(t) = 4.5\left[\sin pt + \frac{1}{2}\sin 2pt\right]$, show that $f\left(\frac{2\pi}{p}+t\right) = f(t)$

Soln. $f\left(\frac{2\pi}{p}+t\right) = 4.5\left[\sin p\left(\frac{2\pi}{p}+t\right) + \frac{1}{2}\sin 2p\left(\frac{2\pi}{p}+t\right)\right]$

$= 4.5\left[\sin(2\pi + pt) + \frac{1}{2}\sin(4\pi + 2pt)\right]$

$= 4.5\left[\sin pt + \frac{1}{2}\sin 2pt\right]$

$\because \sin(2\pi+\theta) = \sin\theta$

$\sin(4\pi+\theta) = \sin\theta$

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2(a) If $f(x) = x^3 - 5x^2 - 4x + 20$, show that $f(0) = -2f(3)$

Soln. $f(0) = 0^3 - 5.0^2 - 4\times 0 + 20 = 20$

$f(3) = 3^3 - 5.3^2 - 4.3 + 20 = 27 - 45 - 12 + 20 = -10$

$-2f(3) = -2 \times -10 = 20$

$\therefore \quad f(0) = -2f(3)$

(b) If $f(x) = 1+x$ for $-1 \le x < 0$

$= 1-x$ for $0 \le x \le 1$

$= x-1$ for $1 < x \le 2$, find $f(\frac{8}{7})$ and $f(-\frac{2}{7})$

Soln. $f(8/7) = x-1 = \frac{8}{7} - 1 = \frac{8-7}{7} = 1/7$

$f(-2/7) = 1+x = 1 - \frac{2}{7} = \frac{7-2}{7} = \frac{5}{7}$

(c) If $f(x) = x^2 + 6x - 8$, $t = z+2$, find $f(t)$

Soln. $f(t) = f(z+2) = (z+2)^2 + 6(z+2) - 8$

$= z^2 + 2z\cdot 2 + 2^2 + 6z + 12 - 8$

$= z^2 + 4z + 6z + 4 + 4 = z^2 + 10z + 4 + 4$

$= z^2 + 10z + 8$

(d) If $\phi(x) = \frac{2x-3}{x-2}$, find $\phi[\phi(x)]$

Soln. $\phi[\phi(x)] = \phi\left[\frac{2x-3}{x-2}\right] = \frac{2\left(\frac{2x-3}{x-2}\right) - 3}{\frac{2x-3}{x-2} - 2}$

$= \frac{\frac{4x-6-3(x-2)}{\cancel{x-2}}}{\frac{2x-3-2(x-2)}{\cancel{x-2}}} = \frac{4x-\cancel{6}-3x+\cancel{6}}{\cancel{2x}-3-\cancel{2x}+4} = \frac{x}{1} = x$

(e) find the range of $f(x) = x^2+1$ for $[-5, 2]$

soln. $-5 \le x \le 2$

$(-5)^2 \le x^2 \le 2^2$

$25 \le x^2 \le 4$

Adding 1 each term

$25+1 \le x^2+1 \le 4+1$

$26 \le x^2+1 \le 5$

$\therefore$ Range $= [5, 26]$

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Q. If $f(x) = \log\left(\frac{x-1}{x}\right)$, show that

$$f(y^2) = f(y) + f(-y)$$

Soln. Given $f(x) = \log\left(\frac{x-1}{x}\right)$

$$\therefore \quad f(y) = \log\left(\frac{y-1}{y}\right) \quad \text{--- (i)}$$

$$f(-y) = \log\left(\frac{-y-1}{-y}\right) \quad \text{--- (ii)}$$

$$= \log\left(\frac{-(y+1)}{-y}\right)$$

$$= \log\left(\frac{y+1}{y}\right)$$

$$f(y^2) = \log\left(\frac{y^2-1}{y^2}\right) \text{ ---}$$

Adding eqn (i) and (ii)

$$\therefore \quad f(y) + f(-y) = \log\left(\frac{y-1}{y}\right) + \log\left(\frac{y+1}{y}\right)$$

$$= \log\left(\frac{y-1}{y} \times \frac{y+1}{y}\right)$$

$$= \log\frac{y^2-1}{y^2}$$

$$= f(y^2)$$

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Q. If $f(x) = \dfrac{x+2}{4x-3}$ and $t = \dfrac{2+3x}{4x-1}$

show thet $f(t) = x$

Soln. Given $f(x) = \dfrac{x+2}{4x-3}$ and $t = \dfrac{2+3x}{4x-1}$

$$\therefore f(t) = \frac{t+2}{4t-3}$$

$$= \frac{\dfrac{2+3x}{4x-1} + 2}{4\left(\dfrac{2+3x}{4x-1}\right) - 3}$$

$$= \frac{2+3x+8x-2}{8+12x-12x+3}$$

$$= \frac{11x}{11}$$

$$= x$$

Q. If $y = f(x) = \dfrac{2x-3}{3x-2}$

show thet $f(y) = x$

Soln. Given $y = f(x) = \dfrac{2x-3}{3x-2}$

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that means

$$y = \frac{2x-3}{3x-2} \quad \text{and} \quad f(x) = \frac{2x-3}{3x-2}$$

$$\therefore f(y) = \frac{2y-3}{3y-2}$$

$$= \frac{2\left(\frac{2x-3}{3x-2}\right) - 3}{3\left(\frac{2x-3}{3x-2}\right) - 2}$$

$$= \frac{4x-6-9x+6}{6x-9-6x+4}$$

$$= \frac{-5x}{5}$$

$\therefore f(y) = x$

Q. If $f(x) = y = \frac{9x+1}{5x-9}$, show that

(i) $f(y) = x$

(ii) $f[f(x)] = f(y)$

soln. Given $f(x) = y = \frac{9x+1}{5x-9}$

$y = f(x)$

$\therefore y = \frac{9x+1}{5x-9}, \quad f(x) = \frac{9x+1}{5x-9}$

(i). $f(y) = \dfrac{ay+1}{5y-a}$

$= \dfrac{a\left(\dfrac{ax+1}{5x-a}\right)+1}{5\left(\dfrac{ax+1}{5x-a}\right)-a}$

$= \dfrac{a^2x + a + 5x - a}{5ax + 5 - 5ax + a^2}$

$= \dfrac{a^2x + 5x}{a^2+5}$

$= \dfrac{x\,\cancel{(a^2+5)}}{\cancel{(a^2+5)}}$

$= x$

(ii) $f(x) = \dfrac{ax+1}{5x-a}$ $\therefore f[f(x)] = \dfrac{af(x)+1}{5f(x)-a}$

$= \dfrac{ay+1}{5y-a}$

$= f(y)$

$\therefore f[f(x)] = f(y)$

Solve it

1. If $f(x) = x^2 + \frac{1}{x^2}$

show that $f(x) + f(-x) = 2 \cdot f(x)$

2. If $f(x) = \frac{ax+b}{cx-a}$, show that $f[f(x)] = x$

3. If $f(x) = x - \frac{1}{x}$

show that $f^3(x) = f(x^3) - 3f\left(\frac{1}{x}\right)$

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Q. If $f(x) = \frac{1}{1-x}$, show that $f[f\{f(x)\}] = x$

Soln. Given $f(x) = \frac{1}{1-x}$

$$\therefore \quad f[f(x)] = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}}$$

$$= \frac{1}{\frac{1-x-1}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x}$$

$$\therefore \quad f[f\{f(x)\}] = \frac{f(x)-1}{f(x)} = \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}$$

$$= \frac{\frac{1-(1-x)}{\cancel{1-x}}}{\frac{1}{\cancel{1-x}}} = \frac{1-1+x}{1}$$

$$= x$$

Q. If $y = f(x) = \frac{2x-3}{3x-2}$, show that $f(y) = x$

Sol. Given $y = \frac{2x-3}{3x-2}$, and $f(x) = \frac{2x-3}{3x-2}$

$$f(y) = \frac{2y-3}{3y-2} = \frac{2\left(\frac{2x-3}{3x-2}\right)-3}{3\left(\frac{2x-3}{3x-2}\right)-2}$$

$$= \frac{\frac{4x-\cancel{6}-9x+\cancel{6}}{\cancel{3x-2}}}{\frac{\cancel{6x}-9-\cancel{6x}+4}{\cancel{3x-2}}} = \frac{-\cancel{5}x}{-\cancel{5}} = x$$

Q. Show that $f(x) = \frac{a^x + a^{-x}}{2}$ is an even function.

Sol: $f(x) = \frac{a^x + a^{-x}}{2}$

$\therefore f(-x) = \frac{a^{-x} + a^{-(-x)}}{2} = \frac{a^{-x} + a^x}{2}$

$\therefore f(x) = f(-x)$ is an even function

Q. If $f(x) = \frac{\cos x}{1+\sin^2 x}$, determine if is an odd or even.

Sol. $f(x) = \frac{\cos x}{1+\sin^2 x}$

$f(-x) = \frac{\cos(-x)}{1+\sin^2(-x)} = \frac{\cos x}{1+\sin^2 x}$

$\therefore f(x) = f(-x)$ is an even function.

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Q. If $f(x) = \frac{2}{1+x} - 1$, show that $f[f\{f(x)\}] = f(x)$

Sol. Given $f(x) = \frac{2}{1+x} - 1$

$\therefore f[f(x)] = \frac{2}{1+f(x)} - 1 = \frac{2}{1+\frac{2}{1+x} - \cancel{1}} - 1$

$= \frac{2}{\frac{2}{1+x}} - 1 = \frac{\cancel{2}(1+x)}{\cancel{2}} - 1$

$= 1+x-\cancel{1} = x$

$\therefore f[f\{f(x)\}] = f(x)$

Q. If $f(x) = \frac{x-4}{4x-1}$, show that $(f\circ f)(x) = x$

Sol. Given $f(x) = \frac{x-4}{4x-1}$

Now, $(f\circ f)(x) = f[f(x)] = \frac{f(x)-4}{4f(x)-1}$

$= \frac{\frac{x-4}{4x-1} - 4}{4\left(\frac{x-4}{4x-1}\right) - 1} = \frac{\frac{x-4-4(4x-1)}{\cancel{4x-1}}}{\frac{4(x-4)-1(4x-1)}{\cancel{4x-1}}}$

$= \frac{x-\cancel{4}-16x+\cancel{4}}{\cancel{4x}-16-\cancel{4x}+1} = \frac{-15x}{\cancel{-15}} = x$

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Limits

Algebraic Limits:-

In this method we directly put $x = a$

Q. Evaluate $\lim_{x \to 1} \dfrac{2x^2+3x+1}{4x-1}$

Ans. $L = \lim_{x \to 1} \dfrac{2x^2+3x+1}{4x-1}$

$= \dfrac{2(1)^2+3\cdot 1+1}{4\cdot 1-1}$

$= \dfrac{2+3+1}{3} = \dfrac{6}{3} = 2$

Q. Evaluate $\lim_{x \to -3} \dfrac{x^3+27}{x^2+2x+3}$

Ans. $L = \cancel{\lim_{x \to -3}} \dfrac{(-3)^3+27}{(-3)^2+2\times(-3)+3}$

$= \dfrac{\cancel{-27}+\cancel{27}}{9-6+3}$

$= \dfrac{0}{6}$

$= 0$

Method of factorization :-

Given $\lim_{x \to \pm a} \frac{f(x)}{g(x)}$: $\frac{f(\pm a)}{g(\pm a)} = \frac{0}{0}$ then

$(x \pm a)$ is a factor of both $f(x)$ and $g(x)$

Q. Evaluate $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^2 - 3x - 4}$

Soln. $L = \lim_{x \to 4} \frac{\cancel{(x-4)}(x-3)}{\cancel{(x-4)}(x+1)}$

$= \lim_{x \to 4} \frac{x-3}{x+1}$

$= \frac{4-3}{4+1} = \frac{1}{5}$

Q. $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x^2 - 9}$

Ans. $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x^2 - 9}$

$= \lim_{x \to 3} \frac{x^2 + 5x - 3x - 15}{x^2 - 3^2}$

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$$= \lim_{x \to 3} \frac{x(x+5) - 3(x+5)}{(x+3)(x-3)}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}(x+3)}$$

$$= \frac{3+5}{3+3} = \frac{\cancel{8}^{4}}{\cancel{6}_{3}} = \frac{4}{3}$$

Q. Evaluate $\lim_{x \to -2} \frac{x^3+8}{x^2+3x+2}$

Ans. $L = \lim_{x \to -2} \frac{x^3+2^3}{x^2+2x+x+2}$

$$= \lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{x(x+2)+1(x+2)}$$

$$= \lim_{x \to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{(x+1)\cancel{(x+2)}}$$

$$= \frac{(-2)^2 - 2\times(-2)+4}{-2+1}$$

$$= \frac{4+4+4}{-1} = -12$$

Q. Evaluate $\lim\limits_{x \to 1} \dfrac{x^3+4x^2+x-6}{x^3-1}$

Soln.

$$\cancel{x^3-1 \,\big)\, x^3+4x^2+x-6 \,\big(\, 1}$$
$$\cancel{x^3 \quad -1}$$
$$\cancel{4x^2+x}$$

Ans. $L = \lim\limits_{x \to 1} \dfrac{x^3-x^2+5x^2-5x+6x-6}{x^3-1^3}$

$= \lim\limits_{x \to 1} \dfrac{x^2(x-1)+5x(x-1)+6(x-1)}{(x-1)(x^2+x+1)}$

$= \lim\limits_{x \to 1} \dfrac{(x^2+5x+6)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)}$

$= \dfrac{1^2+5.1+6}{1^2+1+1} = \dfrac{12}{3} = 4$

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Q. Evaluate $\lim_{x \to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

Soln. $L = \lim_{x \to 1} \dfrac{x^{1/2}-1}{x^{1/3}-1}$

L.C.M of 2 and 3 = 6

put $x^{1/6} = t$

$(x^{1/6})^2 = t^2 \Rightarrow x^{1/3} = t^2$

$(x^{1/6})^3 = t^3 \Rightarrow x^{1/2} = t^3$

when $x \to 1$, then $t \to 1$

$L = \lim_{t \to 1} \dfrac{t^3-1}{t^2-1} = \lim_{t \to 1} \dfrac{\cancel{(t-1)}(t^2+t+1)}{\cancel{(t-1)}(t+1)}$

$= \dfrac{1^2+1+1}{1+1} = 3/2$

Solve these

(1) $\lim_{x \to 2} \dfrac{x-2}{x^2+x-6}$ (2) $\lim_{x \to 1} \dfrac{x^2+3x-4}{x^2-1}$

(3) $\lim_{x \to 3} \dfrac{x^2-5x+6}{x^3+3x^2-18x}$ (4) $\lim_{x \to -1} \dfrac{x^3+1}{x^4-1}$

(5) $\lim_{x \to 2} \dfrac{2x^2-7x+6}{5x^2-11x+2}$

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18.06.2020

Q. Evaluate $\lim_{x\to 2} \frac{x^{10} - 1024}{x^5 - 32}$

soln. $L = \lim_{x\to 2} \frac{x^{10} - 1024}{x^5 - 32}$

$= \lim_{x\to 2} \frac{x^{10} - 2^{10}}{x^5 - 2^5}$

$= \lim_{x\to 2} \frac{x^{10} - 2^{10}}{x^5 - 2^5}$

$= \frac{10 \cdot 2^{10-1}}{5 \cdot 2^{5-1}} = \frac{\overset{2}{\cancel{10}} \cdot 2^9}{\cancel{5} \cdot 2^4} = \frac{2^{10}}{2^4}$

$= 2^6 = 64$

Method of simplification:-

In this we first simplify the function.

Q. Evaluate $\lim_{x\to 2} \frac{1}{x-2} - \frac{2}{x^2 - 2x}$

soln. $L = \lim_{x\to 2} \left[\frac{1}{x-2} - \frac{2}{x^2 - 2x} \right]$

$= \lim_{x\to 2} \left[\frac{1}{x-2} - \frac{2}{x(x-2)} \right]$

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$$= \lim_{x \to 2} \frac{1}{x-2}\left[1 - \frac{2}{x}\right]$$

$$= \lim_{x \to 2} \frac{1}{\cancel{x-2}}\left[\frac{\cancel{x-2}}{x}\right]$$

$$= \lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$$

Q. Evaluate $\lim_{x \to 2}\left[\frac{1}{x-2} + \frac{6x}{8-x^3}\right]$

soln. $L = \lim_{x \to 2}\left[\frac{1}{x-2} + \frac{6x}{-(x^3-8)}\right]$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{6x}{x^3-2^3}\right]$$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{6x}{x^3-2^3}\right]$$

$$= \lim_{x \to 2}\left[\frac{1}{x-2} - \frac{6x}{(x-2)(x^2+2x+4)}\right]$$

$$= \lim_{x \to 2} \frac{1}{x-2}\left[1 - \frac{6x}{x^2+2x+4}\right]$$

$$= \lim_{x \to 2} \frac{1}{x-2}\left[\frac{x^2+2x+4-6x}{x^2+2x+4}\right]$$

$$= \lim_{x \to 2} \frac{1}{x-2}\left[\frac{x^2-4x+4}{x^2+2x+4}\right]$$

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$$= \lim_{x\to 2} \frac{1}{\cancel{x-2}}\left[\frac{(x-2)\cancel{x}}{(x+2)^2}\right]$$

$$= \lim_{x\to 2} \frac{x-2}{(x+2)^2}$$

$$= \frac{2-2}{(2+2)^2} = \frac{0}{4^2} = 0$$

Q. $\lim_{x\to 3}\left[\frac{1}{x-3} - \frac{3}{x^3-5x^2+6x}\right]$

$$= \lim_{x\to 3}\left[\frac{1}{x-3} - \frac{3}{x(x^2-5x+6)}\right]$$

$$= \lim_{x\to 3}\left[\frac{1}{x-3} - \frac{3}{x(x-3)(x-2)}\right]$$

$$= \lim_{x\to 3} \frac{x(x-3)(x-2) - 3(x-3)}{x(x-3)(x-2)}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}\left[x(x-2)-3\right]}{x\cancel{(x-3)}(x-2)}$$

$$= \lim_{x\to 3} \frac{x^2-2x-3}{x(x-2)} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{x\cancel{(x-3)}(x-2)}$$

$$= \frac{3+1}{3(3-2)} = \frac{4}{3}$$

Q. Evaluate $\lim_{x \to 2} \frac{1}{x-2} - \frac{4}{x^3 - 2x^2}$

soln. $L = \lim_{x \to 2} \frac{1}{x-2} - \frac{4}{x^2(x-2)}$

$= \lim_{x \to 2} \frac{1}{x-2} \left[1 - \frac{4}{x^2} \right]$

$= \lim_{x \to 2} \frac{1}{x-2} \left[\frac{x^2 - 4}{x^2} \right]$

$= \lim_{x \to 2} \frac{1}{\cancel{x-2}} \frac{(x+2)\cancel{(x-2)}}{x^2}$

$= \frac{2+2}{2^2} = 1$

Solve these

1. $\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{2}{x(x-1)(x-2)} \right]$

2. $\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{1}{x^2 - 3x + 2} \right]$

3. $\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3 - 3x^2 + 2x} \right]$

4. $\lim_{x \to 2} \left[\frac{4}{x^2 - 4} + \frac{1}{2-x} \right]$

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Limit

Q. Evaluate $\lim_{n\to 0} \dfrac{(1+n)^5 - (1+5n+10n^2)}{5n^3 - 4n^5}$

soln. Let $L = \lim_{n\to 0} \dfrac{(1+n)^5 - (1+5n+10n^2)}{5n^3 - 4n^5}$

$$= \lim_{n\to 0} \frac{1+5n+\frac{5(5-1)}{2!}.n^2+\frac{5(5-1)(5-2)}{3!}.n^3 + \frac{5(5-1)(5-2)(5-3)}{4!}.n^4 + \frac{5(5-1)(5-2)(5-3)(5-4)}{5!}.n^5 - (1+5n+10n^2)}{5n^3-4n^5}$$

$$= \lim_{n\to 0} \frac{1+5n+\frac{5.\cancel{4}^2}{2.1}.n^2 + \frac{5.4.\cancel{3}}{\cancel{3}.2.1}.n^3 + \frac{5.\cancel{4}.\cancel{3}.\cancel{2}}{\cancel{4}.\cancel{3}.\cancel{2}.1}.n^4 + \frac{\cancel{5.4.3.2.1}}{\cancel{5.4.3.2.1}}.n^5 - (1+5n+10n^2)}{5n^3-4n^5}$$

$$= \lim_{n\to 0} \frac{1+5n+10n^2+10n^3+5n^4+n^5-(1+5n+10n^2)}{5n^3-4n^5}$$

$$= \lim_{n\to 0} \frac{\cancel{1}+\cancel{5n}+\cancel{10n^2}+10n^3+5n^4+n^5-\cancel{1}-\cancel{5n}-\cancel{10n^2}}{5n^3-4n^5}$$

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$$= \lim_{x \to 0} \frac{\cancel{x^3}(10 + 5.x + x^2)}{\cancel{x^3}(5 - 4x^2)}$$

$$= \frac{10 + 5.0 + 0}{5 - 4.0} = \frac{10}{5} = 2 \text{ Ans}$$

Trigonometric limits by substitution:-

In this case, evaluate limit by the change of variables

i) If $\lim_{x \to a}$, then by putting $x = a+h$, $x \to a$, h

ii) If $\lim_{x \to \infty}$, then by putting $x = \frac{1}{t}$, $t \to 0$, x

Q. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$

soln. Let $L = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$

putting $x = \frac{\pi}{2} + h$, when $x \to \frac{\pi}{2}$, $h \to 0$

$$L = \lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \to 0} \frac{-\sin h}{\cancel{\pi} - \cancel{\pi} - 2h}$$

$$= \lim_{h \to 0} \frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$= \lim_{h \to 0} \frac{1}{2} \cdot \frac{\sin h}{h} = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

2. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\left(\frac{\pi}{2} - x\right)^2}$

Soln. Let $L = \lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\left(\frac{\pi}{2} - x\right)^2}$

Put $x = \frac{\pi}{2} + h$, when $x \to \frac{\pi}{2}$, $h \to 0$

$$= \lim_{h \to 0} \frac{1 - \sin\left(\frac{\pi}{2} + h\right)}{\left\{\frac{\pi}{2} - \left(\frac{\pi}{2} + h\right)\right\}^2} = \lim_{h \to 0} \frac{1 - \cos h}{\left(\frac{\pi}{2} - \cancel{\frac{\pi}{2}} - h\right)^2}$$

$$= \lim_{h \to 0} 2 \cdot \frac{\sin^2 \frac{h}{2}}{h^2} = \cancel{2} \lim_{h \to 0} \frac{\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4}}{\cancel{h^2}}$$

$$= 2 \cdot 1 \left(\frac{1}{2}\right)^2 = 2 \cdot \frac{1}{4} = \frac{1}{2}$$

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Infinity type

In this we replace x by $\frac{1}{t}$.

when $x \to \infty$, then $t \to 0$

Q. Evaluate $\lim_{x \to \infty} \frac{3x^2+4}{5x^2+7}$

soln. Let $L = \lim_{x \to \infty} \frac{3x^2+4}{5x^2+7}$

put $x = \frac{1}{t}$, when $x \to \infty$, then $t \to \infty$

$$L = \lim_{t \to 0} \frac{3\left(\frac{1}{t^2}\right)+4}{5\left(\frac{1}{t^2}\right)+7}$$

$$= \lim_{t \to 0} \frac{3+4t^2}{5+7t^2} = \frac{3+0}{5+0} = 3/5$$

Q. Evaluate $\lim_{x \to \infty} \frac{4x+1}{\sqrt{5x^2+2}}$

soln. put $x = \frac{1}{t}$, when $x \to \infty$, $t \to 0$

$$= \lim_{t \to 0} \frac{4\left(\frac{1}{t}\right)+1}{\sqrt{5\left(\frac{1}{t^2}\right)+2}}$$

$$= \lim_{t \to 0} \frac{\frac{4+t}{t}}{\sqrt{\frac{5+2t^2}{t}}} = \lim_{t \to 0} \frac{4+t}{\sqrt{5+2t^2}}$$

$$= \frac{4+0}{\sqrt{5+0}} = \frac{4}{\sqrt{5}}$$

7. Evaluate $\lim_{x \to \infty} \left(\sqrt{x^2+x} - x\right)$

Soln. Put $x = \frac{1}{t}$; when $x \to \infty$, $t \to 0$

$$= \lim_{t \to 0} \sqrt{\frac{1}{t^2} + \frac{1}{t}} - \frac{1}{t}$$

$$= \lim_{t \to 0} \frac{\sqrt{1+t}}{t} - \frac{1}{t}$$

$$= \lim_{t \to 0} \frac{\sqrt{1+t} - 1}{t} \times \frac{\sqrt{1+t}+1}{\sqrt{1+t}+1}$$

$$= \lim_{t \to 0} \frac{1+t-1}{t\left[\sqrt{1+t}+1\right]}$$

$$= \lim_{t \to 0} \frac{\cancel{t}}{\cancel{t}\left[\sqrt{1+t}+1\right]}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{1+1} = 2$$

Q. Evaluate $\lim_{x \to \infty} x\left[\sqrt{x^2+1} - \sqrt{x^2-1}\right]$

soln. put $x = \frac{1}{t}$, when $x \to \infty$, $t \to 0$

$$L = \lim_{t \to 0} \frac{1}{t}\left[\sqrt{\frac{1}{t^2}+1} - \sqrt{\frac{1}{t^2}+1}\right.$$

$$= \lim_{t \to 0} \frac{1}{t}\left[\sqrt{\frac{1+t^2}{t}} - \sqrt{\frac{1+t^2}{t}}\right]$$

$$= \lim_{t \to 0} \frac{1}{t}\left[\frac{\sqrt{1+t^2} - \sqrt{1-t^2}}{t} \times \frac{\sqrt{1+t^2} + \sqrt{1-t^2}}{\sqrt{1+t^2} + \sqrt{1-t^2}}\right.$$

$$= \lim_{t \to 0} \frac{1}{t}\left[\frac{\cancel{1}+t^2 - \cancel{1}+t^2}{t\left[\sqrt{1+t^2}+\sqrt{1-t^2}\right]}\right]$$

$$= \lim_{t \to 0} \frac{2\cancel{t^2}}{\cancel{t^2}\left[\sqrt{1+t^2}+\sqrt{1-t^2}\right]}$$

$$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1$$

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Q. Evaluate $\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{n-1}{n^2} \right]$

Soln. Let $L = \lim_{n \to \infty} \frac{1+2+3+\cdots+n-1}{n^2}$

$$= \lim_{n \to \infty} \frac{\frac{(n-1)\,n}{2}}{n^2}$$

$$= \lim_{n \to \infty} \frac{n(n-1)}{2n^2}$$

Put $n = \frac{1}{t}$, when $n \to \infty$ then, $t \to 0$

$$L = \lim_{t \to 0} \frac{\frac{1}{t}\left(\frac{1}{t} - 1\right)}{2 \cdot \frac{1}{t^2}} = \lim_{t \to 0} \frac{\frac{1}{t}\left(\frac{1-t}{t}\right)}{2 \cdot \frac{1}{t^2}}$$

$$= \lim_{t \to 0} \frac{1-t}{2} = \frac{1-0}{2} = \frac{1}{2}$$

Do the these (1) $\lim_{n \to \infty} \frac{n^2 - 3}{\sqrt{5n + 9n^4}}$

(2) $\lim_{n \to \infty} \frac{\sqrt{n^2 - 3}}{\sqrt[3]{n^3 + 1}}$ (3) $\lim_{n \to \infty} n \cdot \sin\left(\frac{1}{n}\right)$

(4) $\lim_{n \to \infty} \sqrt{n^2 + 5n} - n$ (5) $\lim_{n \to \infty} \frac{1+2+3+\cdots+n}{n^2}$

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Exponential Limits :-

1) $$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

2) $$\lim_{x \to 0} \frac{a^{Kx} - 1}{Kx} = \log_e a$$

Q. Evaluate $\lim_{x \to 0} \frac{a^{4x} - 1}{3x}$

soln. $$L = \lim_{x \to 0} \frac{a^{4x} - 1}{3x}$$

$$= \lim_{x \to 0} \frac{a^{4x} - 1}{4x} \times \frac{4}{3}$$

$$= \log_e a \cdot \frac{4}{3}$$

Q. Evaluate $\lim_{x \to 0} \frac{a^{\tan x} - 1}{2x}$

soln. $$L = \lim_{x \to 0} \frac{a^{\tan x} - 1}{2x}$$

$$= \lim_{x \to 0} \frac{a^{\tan x} - 1}{\tan x} \cdot \frac{\tan x}{2x}$$

$$= \lim_{x \to 0} \frac{a^{\tan x} - 1}{\tan x} \times \lim_{x \to 0} \frac{\tan x}{x} \cdot \frac{1}{2} = \frac{1}{2} \log_e a$$

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Q. Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$

Soln. $L = \lim_{x \to 0} \frac{a^x - 1 - b^x + 1}{x}$

$= \lim_{x \to 0} \frac{a^x - 1 - (b^x - 1)}{x}$

$= \lim_{x \to 0} \left[\frac{a^x - 1}{x} - \frac{b^x - 1}{x} \right]$

$= \lim_{x \to 0} \frac{a^x - 1}{x} - \lim_{x \to 0} \frac{b^x - 1}{x}$

$= \log_e a - \log_e b = \log_e \left(\frac{a}{b} \right)$

Q. Evaluate $\lim_{x \to 0} \frac{3^{2x} - 1}{2^{3x} - 1}$

Soln. $L = \lim_{x \to 0} \frac{3^{2x} - 1}{2^{3x} - 1}$

$= \lim_{x \to 0} \frac{(3^2)^x - 1}{(2^3)^x - 1}$

$= \lim_{x \to 0} \frac{9^x - 1}{8^x - 1} = \lim_{x \to 0} \frac{\frac{9^x - 1}{x}}{\frac{8^x - 1}{x}}$

$= \frac{\log_e 9}{\log_e 8}$

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Q. Evaluate $\lim_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

soln. $L = \lim_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

$= \lim_{x\to 0} \frac{(3\times 2)^x - 3^x - 2^x + 1}{x^2}$

$= \lim_{x\to 0} \frac{3^x \cdot 2^x - 3^x - 2^x + 1}{x^2}$

$= \lim_{x\to 0} \frac{3^x (2^x - 1) - 1(2^x - 1)}{x^2}$

$= \lim_{x\to 0} \frac{(3^x - 1)(2^x - 1)}{x^2}$

$= \lim_{x\to 0} \frac{3^x - 1}{x} \cdot \lim_{x\to 0} \frac{2^x - 1}{x}$

$= \log_e 2 \times \log_e 3$

Q. $\lim_{x\to 0} \frac{a^x + a^{-x} - 2}{x^2}$

soln. $L = \lim_{x\to 0} \frac{a^x + a^{-x} - 2}{x^2} = \lim_{x\to 0} \frac{a^x + \frac{1}{a^x} - 2}{x^2}$

$= \lim_{x\to 0} \frac{(a^x)^2 + 1 - 2 \cdot a^x}{x^2 \cdot a^x} = \lim_{x\to 0} \frac{(a^x - 1)^2}{x^2 \cdot a^x} = \left\{\lim_{x\to 0} \frac{(a^x - 1)}{x}\right\}^2 \cdot \lim_{x\to 0} \frac{1}{a^x}$

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$= (\log_e a)^2 \cdot \frac{1}{a^0} = (\log_e a)^2$

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Method of Rationalization

Q. Evaluate $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

Soln. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x} \times \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$= \lim\limits_{x\to 0} \dfrac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim\limits_{x\to 0} \dfrac{1+x-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim\limits_{x\to 0} \dfrac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})} = \lim\limits_{x\to 0} \dfrac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})} = \dfrac{2}{\sqrt{1+0}+\sqrt{1-0}} = \dfrac{2}{1+1} = 1$

Q. Evaluate $\lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{x^2-16}$

Soln. $\lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{x^2-16} \times \boxed{\cancel{\dfrac{x^2+16}{x^2+16}}} \times \dfrac{2+\sqrt{x}}{2+\sqrt{x}} = \lim\limits_{x\to 4} \dfrac{2^2-(\sqrt{x})^2}{(x^2-4^2)(2+\sqrt{x})}$

$= \lim\limits_{x\to 4} \dfrac{4-x}{(x-4)(x+4)(2+\sqrt{x})} = \lim\limits_{x\to 4} \dfrac{-\cancel{(x-4)}}{\cancel{(x-4)}(x+4)(2+\sqrt{x})}$

$= -\dfrac{1}{(4+4)(2+\sqrt{4})} = \dfrac{-1}{8(2+2)} = \dfrac{-1}{32}$

Q. $\lim\limits_{x\to 2} \dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$

Soln. $\lim\limits_{x\to 2} \dfrac{x^2-2^2}{\sqrt{x+2}-\sqrt{3x-2}} \times \dfrac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{3x-2})}{(\sqrt{x+2})^2-(\sqrt{3x-2})^2}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{x-2})}{x+2-3x+2}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{x-2})}{4-2x}$

$= \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)(\sqrt{x+2}+\sqrt{x-2})}{-2\cancel{(x-2)}} = \dfrac{(2+2)(\sqrt{2+2}+\sqrt{2-2})}{-2} = \dfrac{4\times 2}{-2} = -4$

Q. Evaluate $\lim_{x \to 1} \dfrac{3-\sqrt{4+5x}}{5-\sqrt{12+13x}}$

Soln. $\lim_{x\to 1} \dfrac{3-\sqrt{4+5x}}{5-\sqrt{12+13x}} \times \dfrac{3+\sqrt{4+5x}}{3+\sqrt{4+5x}} \times \dfrac{5+\sqrt{12+13x}}{5+\sqrt{12+13x}}$

$= \lim_{x\to 1} \dfrac{[3^2-(\sqrt{4+5x})^2][5+\sqrt{12+13x}]}{[5^2-(\sqrt{12+13x})^2](3+\sqrt{4+5x})}$

$= \lim_{x\to 1} \dfrac{[9-(4+5x)][5+\sqrt{12+13x}]}{[25-(12+13x)][3+\sqrt{4+5x}]}$

$= \lim_{x\to 1} \dfrac{(9-4-5x)(5+\sqrt{12+13x})}{(25-12-13x)(3+\sqrt{4+5x})}$

$= \lim_{x\to 1} \dfrac{(5-5x)(5+\sqrt{12+13x})}{(13-13x)(3+\sqrt{4+5x})}$

$= \lim_{x\to 1} \dfrac{5\cancel{(1-x)}(5+\sqrt{12+13x})}{13\cancel{(1-x)}(3+\sqrt{4+5x})} = \lim_{x\to 1} \dfrac{5(5+\sqrt{12+13x})}{13(3+\sqrt{4+5x})}$

$= \dfrac{5(5+\sqrt{12+13\times 1})}{13(3+\sqrt{4+5\times 1})} = \dfrac{5(5+5)}{13(3+3)} = \dfrac{5\times \cancel{10}\,5}{13\times\cancel{6}\,3} = \dfrac{25}{39}$

Solve it ① $\lim_{x\to 9} \dfrac{3-\sqrt{x}}{x^2-81}$

(2) $\lim_{x\to 0} \dfrac{x}{\sqrt{9-x+x^2}-3}$

③ $\lim_{x\to 1} \dfrac{\sqrt{x^2-1}+\sqrt{x+1}}{\sqrt{x^3-1}}$

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Limits of type $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

Q. Evaluate $\lim_{x \to a} \frac{x^{10} - a^{10}}{x - a}$

Soln. Let $L = \lim_{x \to a} \frac{x^{10} - a^{10}}{x - a}$

$n = 10$

$$= 10 a^{10-1}$$

$$= 10 a^{9}$$

Q. Evaluate $\lim_{x \to 1} \frac{x^{7/2} - 1}{x^{5/2} - 1}$

Soln. Let $L = \lim_{x \to 1} \frac{x^{7/2} - 1^{7/2}}{x^{5/2} - 1^{5/2}}$

$$= \lim_{x \to 1} \frac{\frac{x^{7/2} - 1^{7/2}}{x - 1}}{\frac{x^{5/2} - 1^{5/2}}{x - 1}}$$

$$= \frac{\frac{7}{2} (1)^{7/2 - 1}}{\frac{5}{2} (1)^{5/2 - 1}}$$

$$= \frac{\frac{7}{\cancel{2}}}{\frac{5}{\cancel{2}}}$$

$$= \frac{7}{5}$$

Infinity Type of Limits

Put $x = \frac{1}{t} \Rightarrow t = \frac{1}{x}$

When $x \to \infty$ then $t \to 0$

Q. Evaluate $\lim\limits_{x \to \infty} \frac{2x^2+1}{3x^2-5}$

Soln.

Let $L = \lim\limits_{x \to \infty} \frac{2x^2+1}{3x^2-5}$

Put $x = \frac{1}{t}$ then $x \to \infty$, $t \to 0$

$$\therefore \quad L = \lim_{t \to 0} \frac{\frac{2}{t^2}+1}{\frac{3}{t^2}-5}$$

$$= \lim_{t \to 0} \frac{\frac{2+t^2}{\cancel{t^2}}}{\frac{3-5t^2}{\cancel{t^2}}}$$

$$= \lim_{t \to 0} \frac{2+t^2}{3-5t^2}$$

$$= \frac{2+0}{3-5\times 0}$$

$$= \frac{2}{3}$$

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Q. $\lim_{x \to 2} \dfrac{x^5 - 32}{x^{10} - 1024}$

soln. Let $L = \lim_{x \to 2} \dfrac{x^5 - 32}{x^{10} - 1024} = \lim_{x \to 2} \dfrac{x^5 - 2^5}{x^{10} - 2^{10}}$

$$= \lim_{x \to 2} \frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^{10} - 2^{10}}{x - 2}} = \frac{5(2)^{5-1}}{10(2)^{10-1}} = \frac{5 \cdot 2^4}{10 \cdot 2^9}$$

$$= \frac{\cancel{5} \times \cancel{16}}{\underset{2}{\cancel{10}} \times \underset{32}{\cancel{512}}} = \frac{1}{2 \times 32} = \frac{1}{64}$$

solve it (1) $\lim_{x \to 2} \dfrac{x^9 - 512}{x - 2}$

(2) $\lim_{x \to a} \dfrac{x^{2/7} - a^{2/7}}{x - a}$

(3) $\lim_{x \to 1} \dfrac{x^5 - 1}{x - 1}$

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1. $\lim_{x \to 0} \dfrac{1 - \cos 4x}{1 - \cos 6x}$

2. $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin^2 x}$

3. $\lim_{x \to 0} \dfrac{\operatorname{cosec} x - \cot x}{x}$

4. $\lim_{x \to 0} x \operatorname{cosec} x$

5. $\lim_{x \to 0} \dfrac{\sin x/4}{x}$

Important Notes

Important Phones

Important Works

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1. $$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8}$$

2. $$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

3. $$\lim_{x \to 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

4. $$\lim_{x \to 0} \frac{e^x - x - 1}{x}$$

5. $$\lim_{x \to 0} \frac{a^x - 1}{x}$$

solve it

1. $\lim_{n\to\infty} \sqrt{n^2+1} - \sqrt{n^2-1}$

2. $\lim_{n\to\infty} \sqrt{n^2+n+1} - \sqrt{n^2-n-1}$

3. $\lim_{n\to\infty} \dfrac{1+2+3+\dots+n}{n}$

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Trigonometric limits

(1) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

(2) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

(3) $\lim_{\theta \to 0} \cos\theta = 1$

(4) $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$

(5) $\lim_{\theta \to 0} \tan\theta = 0$

(6) $\lim_{\theta \to 0} \frac{\theta}{\tan\theta} = 1$

Q. Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$

Soln. $L = \lim_{x \to 0} \frac{\frac{\sin 4x}{4x} \times 4\cancel{x}}{3\cancel{x}}$

$= \frac{1 \times 4}{3} = \frac{4}{3}$

Q. Evaluate $\lim_{x \to 0} \frac{\sin x^\circ}{x}$ $\quad \because x^\circ = \frac{\pi x}{180}$

Soln. $L = \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x}$

$= \lim_{x \to 0} \frac{\frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi \cancel{x}}{180}}{\cancel{x}}$

$= 1 \times \frac{\pi}{180} = \frac{\pi}{180}$

Q Evaluate $\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$

Soln. $L = \lim_{x \to 0} \frac{\frac{\tan 7x}{7x} \times 7\cancel{x}}{\frac{\sin 3x}{3x} \times 3\cancel{x}} = \frac{1 \times 7}{1 \times 3} = \frac{7}{3}$

Q. Evaluate $\lim\limits_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]$

soln. Put $x=\frac{1}{t}$, $x\to\infty$ then $t\to 0$

$$= \lim_{t\to 0} \frac{1}{t}\left[\sqrt{\frac{1}{t^2}+1}-\sqrt{\frac{1}{t^2}-1}\right]$$

$$= \lim_{t\to 0} \frac{1}{t}\left[\sqrt{\frac{t^2+1}{t^2}}-\sqrt{\frac{1-t^2}{t^2}}\right]$$

$$= \lim_{t\to 0} \frac{1}{t}\left[\frac{\sqrt{t^2+1}}{t}-\frac{\sqrt{1-t^2}}{t}\right]$$

$$= \lim_{t\to 0} \frac{1}{t^2}\left[\sqrt{t^2+1}-\sqrt{1-t^2} \times \frac{\sqrt{t^2+1}+\sqrt{1-t^2}}{\sqrt{t^2+1}+\sqrt{1-t^2}}\right]$$

$$= \lim_{t\to 0} \frac{1}{t^2}\left[\frac{(\sqrt{t^2+1})^2-(\sqrt{1-t^2})^2}{\sqrt{t^2+1}+\sqrt{1-t^2}}\right]$$

$$= \lim_{t\to 0} \frac{1}{t^2}\left[\frac{t^2+\cancel{1}-\cancel{1}+t^2}{\sqrt{t^2+1}+\sqrt{1-t^2}}\right]$$

$$= \lim_{t\to 0} \frac{2\cancel{t^2}}{\cancel{t^2}\left[\sqrt{1+t^2}+\sqrt{1-t^2}\right]}$$

$$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}}$$

$$= \frac{2}{1+1}$$

$$= \frac{2}{2}$$

$$= 1$$

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Q. Evaluate $\lim\limits_{x \to 0} \dfrac{7x - 3\tan 2x}{5x + 2\sin 3x}$

Soln. $L = \lim\limits_{x \to 0} \dfrac{7x - 3\dfrac{\tan 2x}{2x} \times 2x}{5x + 2\dfrac{\sin 3x}{3x} \times 3x}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x}\left[7 - 3\dfrac{\tan 2x}{2x} \times 2\right]}{\cancel{x}\left[5 + 2\dfrac{\sin 3x}{3x} \times 3\right]}$

$= \dfrac{7 - 3 \times 1 \times 2}{5 + 2 \times 1 \times 3} = \dfrac{7 - 6}{5 + 6} = \dfrac{1}{11} = \mathrm{Ans}$

Q. $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x \sin x}$

Soln. $L = \lim\limits_{x \to 0} \dfrac{1 - \cos x}{x \sin x}$

$= \lim\limits_{x \to 0} \dfrac{1 - \cos x}{x \sin x} \times \dfrac{1 + \cos x}{1 + \cos x}$

$= \lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{x \sin x\,(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{\sin^2 x}{x \sin x\,(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{\left(\dfrac{\sin x}{x}\right)^2 \times x^2}{x \dfrac{\sin x}{x} \times x\,(1 + \cos x)}$

$= \lim\limits_{x \to 0} \dfrac{\left(\dfrac{\sin x}{x}\right)^2 \times \cancel{x^2}}{\dfrac{\sin x}{x} \times \cancel{x^2}\,(1 + \cos x)}$

$= \dfrac{1^2}{1(1 + \cos 0)} = \dfrac{1}{1 + 1} = \dfrac{1}{2}$

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Q. Evaluate $\lim_{x\to 0} \frac{\sin^2 3x}{27x^2}$

Sol^n. $L = \lim_{x\to 0} \frac{\sin^2 3x}{27x^2}$

$= \lim_{x\to 0} \frac{\left(\frac{\sin 3x}{3x}\right)^2 \times 9\cancel{x^2}}{_3\cancel{27}\,\cancel{x^2}}$

$= \frac{1}{3}$

Q. Evaluate $\lim_{x\to 0} \frac{1-\cos 10x}{1-\cos 4x}$

Sol^n. $L = \lim_{x\to 0} \frac{1-\cos 10x}{1-\cos 4x}$

$= \lim_{x\to 0} \frac{\cancel{2}\sin^2 \frac{10x}{2}}{\cancel{2}\sin^2 \frac{4x}{\cancel{2}}}$

$= \lim_{x\to 0} \frac{\sin^2 5x}{\sin^2 2x} = \lim_{x\to 0} \frac{\left(\frac{\sin 5x}{5x}\right)^2 \times (5x)^2}{\left(\frac{\sin 2x}{2x}\right)^2 \times (2x)^2}$

$= \lim_{x\to 0} \frac{\left(\frac{\sin 5x}{5x}\right)^2 \times 25\cancel{x^2}}{\left(\frac{\sin 2x}{2x}\right)^2 \times 4\cancel{x^2}} = \frac{1\times 25}{1\times 4} = \frac{25}{4}$

Solve these

1. $\lim_{x\to 0} \frac{\sin 5x}{\sin 2x}$

2. $\lim_{x\to 0} \frac{\sin 5x}{2x}$

3. $\lim_{x\to 0} x \operatorname{cosec} 3x$

AUGUST 2008

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

JULY '08 DAY 213-153 **31** Thursday WEEK 31

Continuity

A function, $f(x)$ is said to be continuous at $x = a$ if it satisfies the following condition

1) $LHL = \lim_{h \to 0} f(a-h)$, $x \leq a$

$RHL = \lim_{h \to 0} f(a+h)$, $x > a$

$LHL = RHL = f(a)$

Q. Let $f(x) = 2x+3$, $x \leq 0$

$3(x+1)$, $x > 0$

Soln. At $x = 0$ (a)

$LHL = \lim_{h \to 0} f(a-h)$

$= \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} f(-h)$

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$$\lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} 2(-h)+3$$

$$\lim_{h\to 0} -2h+3$$

$$= -2\times 0+3$$

$$= 3$$

$$RHL = \lim_{h\to 0} f(a+h)$$

$$= \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0} 3(h+1)$$

$$= 3(0+1)$$

$$= 3$$

$$f(a) = f(0) = 3(x+1) = 3(0+1) = 3$$

$$LHL = RHL = f(0) = 3$$

it is contiguous

Q. Let $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$

Soln. At $x = 1$

$$LHL = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(1-h)$$

$$= \lim_{h \to 0} (1-h)^2 - 1$$

$$(1-0)^2 - 1 = 0.$$

$$RHL = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} f(1+h)$$

$$= \lim_{h \to 0} -(1+h)^2 - 1$$

$$-(1+0)^2 - 1 = -2$$

SUNDAY 3

$$f(1) = -1^2 - 1 = -2$$

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$$LHL \neq RHL = f(1)$$

It is discontinuous

$f(x)$ does not exist

Q. Let $f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, & x>1 \end{cases}$

Find the values of a and b

Soln. At $x=1$

$$LHL = \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} a+b(1-h)$$

$$= a+b(1-0) = a+b$$

$$RHL = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} b-a(1+h) = b-a$$

Important Notes | Important Phones | Important Works

$f(1) = 4$

hw

1) $f(x) = \begin{cases} 4x-5, & x \le 2 \\ x-a, & x > 2 \end{cases}$

find a

2) $f(x) = \begin{cases} \cos x, & x \geq 0 \\ x+K, & x < 0 \end{cases}$

find the value of K

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It is continuous when

$LHL = RHL = f(1)$.

$a+b = b-a = 4$

$a+b = 4 \Rightarrow a+4 = 8$

$b-a = 4 \qquad a = 4$

$2b = 8$

$b = 4$.

1 (a) $3x^5$

Let $y = 3x^5$

$$\frac{dy}{dx} = 3\frac{dx^5}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 3.5x^{5-1}$$

$$\Rightarrow \frac{dy}{dx} = 15x^4.$$

(b). Let $y = \sin x - \cos x + \frac{2}{x^3}$

$$y = \sin x - \cos x + 2x^{-3}$$

$$\frac{dy}{dx} = \frac{d\sin x}{dx} - \frac{d\cos x}{dx} + 2\frac{dx^{-3}}{dx}$$

$$\frac{dy}{dx} = \cos x - (-\sin x) + 2(-3)x^{-3-1}$$

$$\frac{dy}{dx} = \cos x + \sin x - 6x^{-4}$$

$$\frac{dy}{dx} = \cos x + \sin x - \frac{6}{x^4}$$

(c) Let $y = x^2 + 2^x + \log_{10} x$

$$\frac{dy}{dx} = \frac{dx^2}{dx} + \frac{d2^x}{dx} + \frac{d\log_{10} x}{dx}$$

$$\frac{dy}{dx} = 2x^{2-1} + 2^x \cdot \log 2 + \frac{1}{x\log 10}$$

$$\frac{dy}{dx} = 2x + 2^x \cdot \log 2 + \frac{1}{x\log 10}$$

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tiation

(d) Let $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$\Rightarrow y = (\sqrt{x})^2 + 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2$

$\Rightarrow y = x^1 + 2 + \frac{1}{x}$

$y = x^1 + 2 + x^{-1}$

$\frac{dy}{dx} = \frac{dx^1}{dx} + \frac{d2}{dx} + \frac{dx^{-1}}{dx}$

$\frac{dy}{dx} = 1 + 0 - 1 \cdot x^{-1-1}$

$\frac{dy}{dx} = 1 - x^{-2} = 1 + \frac{1}{x^2}$

(e) Let $y = 5a^x + \sqrt[4]{x^3} + \log_e x^2$

$\frac{dy}{dx} = \frac{5\,da^x}{dx} + \frac{d\,x^{3/4}}{dx} + 2\frac{d\log_e x}{dx}$

$\frac{dy}{dx} = 5a^x \cdot \log a + \frac{3}{4} x^{\frac{3}{4}-1} + \frac{2}{x\log e}$

$\frac{dy}{dx} = 5a^x \cdot \log a + \frac{3}{4} x^{-1/4} + \frac{2}{x\log e}$

$\frac{dy}{dx} = 5 \cdot a^x \log a + \frac{3}{4x^{1/4}} + \frac{2}{x\log e}$

(f) Let $y = \frac{5}{x} + \frac{1}{\sqrt{x}} - \frac{1}{x^n}$

$y = 5x^{-1} + \frac{1}{x^{1/2}} - \frac{1}{x^n}$

$y = 5x^{-1} + x^{-1/2} - x^{-n}$

$\frac{dy}{dx} = 5(-1)x^{-1-1} + \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} - (-n)x^{-n-1}$

$\frac{dy}{dx} = -5x^{-2} - \frac{1}{2}x^{-3/2} + n\,x^{-(n+1)}$

$\frac{dy}{dx} = \frac{-5}{x^2} - \frac{1}{2x^{3/2}} + \frac{n}{x^{n+1}}$

$y = u \times v \Rightarrow \frac{dy}{dx} =$

2(a) Let $y = (x^2+1)(x^2-x+1)$

$$\frac{dy}{dx} = (x^2-x+1)\frac{d(x^2+1)}{dx} + (x^2+1)\frac{d(x^2-x+1)}{dx}$$

$$\frac{dy}{dx} = (x^2-x+1)2x + (x^2+1)(2x-1)$$

(b) Let $y = e^x \cdot 5^x$

$$\frac{dy}{dx} = 5^x\frac{de^x}{dx} + e^x\frac{d5^x}{dx}$$

$$\frac{dy}{dx} = 5^x \cdot e^x + e^x \cdot 5^x \log 5$$

(c) Let $y = x \cdot \tan^{-1}x$

$$\frac{dy}{dx} = \tan^{-1}x\frac{dx}{dx} + x\frac{d\tan^{-1}x}{dx}$$

$$\frac{dy}{dx} = \tan^{-1}x \cdot 1 + x \cdot \frac{1}{1+x^2}$$

(d) Let $y = e^x \cdot \sin x \cdot \cos x$

$$\frac{dy}{dx} = e^x\frac{d\sin x \cdot \cos x}{dx} + \sin x\cos x\frac{de^x}{dx}$$

$$\frac{dy}{dx} = e^x[\sin x(-\sin x) + \cos x \cdot \cos x] + \sin x \cdot \cos x \cdot e^x$$

$$\frac{dy}{dx} = e^x[\cos^2 x - \sin^2 x + \sin x \cdot \cos x]$$

$$\frac{dy}{dx} = e^x[\cos 2x + \sin x \cdot \cos x]$$

If $y = \frac{u}{v}$ then

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

3 (a) Let $y = \frac{1+\sqrt{x}}{1-\sqrt{x}}\begin{matrix} u \\ v \end{matrix}$

$$\frac{dy}{dx} = \frac{(1-\sqrt{x})\frac{1}{2\sqrt{x}} - (1+\sqrt{x})\left(\frac{-1}{2\sqrt{x}}\right)}{(1-\sqrt{x})^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}\left[1-\cancel{\sqrt{x}}+1+\cancel{\sqrt{x}}\right]}{(1-\sqrt{x})^2}$$

$$= \frac{\frac{1}{\cancel{2}\sqrt{x}} \times \cancel{2}}{(1-\sqrt{x})^2}$$

$$= \frac{1}{\sqrt{x}\,(1-\sqrt{x})^2}$$

(b) Let $y = \frac{e^x - 1}{e^x + 1}$

$$\frac{dy}{dx} = \frac{(e^x+1)\frac{d\,e^x-1}{dx} - (e^x-1)\frac{d\,e^x+1}{dx}}{(e^x+1)^2}$$

$$\frac{dy}{dx} = \frac{(e^x+1)e^x - (e^x-1)e^x}{(e^x+1)^2}$$

$$\frac{dy}{dx} = \frac{e^x(e^x+1-e^x+1)}{(e^x+1)^2}$$

$$\frac{dy}{dx} = \frac{2e^x}{(e^x+1)^2}$$

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(c) Let $y = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x) \cdot \frac{d(\cos x + \sin x)}{dx} - (\cos x + \sin x)\frac{d(\cos x - \sin x)}{dx}}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)(-\sin x + \cos x) - (\cos x + \sin x)(-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)(\cos x - \sin x) + (\sin x + \cos x)(\sin x + \cos x)^2}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)^2 + (\sin x + \cos x)^2}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x - \cancel{2\sin x \cos x} + \sin^2 x + \cos^2 x + \cancel{2\sin x \cos x}}{(\cos x - \sin x)^2}$$

$$\frac{dy}{dx} = \frac{1 + 1}{(\cos x - \sin x)^2} = \frac{2}{(\cos x - \sin x)^2}$$

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(1) Differentiate the following question

i) $e^{mx} + e^{-mx}$

let $y = e^{mx} + e^{-mx}$

$$\frac{dy}{dx} = \frac{de^{mx}}{dmx} \times \frac{dmx}{dx} + \frac{de^{-mx}}{d(-mx)} \times \frac{d-mx}{dx}$$

$$\frac{dy}{dx} = e^{mx} \times m + e^{-mx} \times -m$$

$$\frac{dy}{dx} = m\left[e^{mx} - e^{-mx}\right]$$

ii) e^{a^x}

let $y = e^{a^x}$

$$\frac{dy}{dx} = \frac{de^{a^x}}{da^x} \times \frac{da^x}{dx}$$

$$\therefore \frac{dy}{dx} = e^{a^x} \times a^x \cdot \log a$$

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iii) $7^{x+1/x}$

Let $y = 7^{x+1/x}$

$$\frac{dy}{dx} = \frac{d\,7^{x+1/x}}{d\,x+1/x} \times \frac{d\,x+1/x}{dx}$$

$$\frac{dy}{dx} = 7^{x+1/x}\left[x - \frac{1}{x^2}\right]$$

iv) $e^{\cos^{-1}x}$

Let $y = e^{\cos^{-1}x}$

$$\frac{dy}{dx} = \frac{d\,e^{\cos^{-1}x}}{d\cos^{-1}x}$$

$$\frac{dy}{dx} = e^{\cos^{-1}x} \times \frac{-1}{\sqrt{1-x^2}}$$

v) $a^{\sqrt{\sin x}}$

Let $y = a^{\sqrt{\sin x}}$

$$\frac{dy}{dx} = \frac{d\,a^{\sqrt{\sin x}}}{d\sqrt{\sin x}} \times \frac{d\sqrt{\sin x}}{dx}$$

$$\frac{dy}{dx} = a^{\sqrt{\sin x}} \cdot \log a \times \frac{1}{2\sqrt{\sin x}}$$

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vii) If $y = x \cdot \tan^{-1}x + (1+x^2)$.

$$\frac{dy}{dx} = x\frac{d\tan^{-1}x}{dx} + \tan^{-1}x \cdot \frac{dx}{dx} + \frac{d(1+x^2)}{dx}$$

$$\frac{dy}{dx} = x \cdot \frac{1}{1+x^2} + \tan^{-1}x \cdot 1 + 2x$$

viii) If $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$, show that $\frac{dy}{dx} = \frac{-4}{(e^x-e^{-x})^2}$

$$\frac{dy}{dx} = \frac{(e^x-e^{-x})\frac{d(e^x+e^{-x})}{dx} - (e^x+e^{-x})\frac{d(e^x-e^{-x})}{dx}}{(e^x-e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{(e^x-e^{-x})(e^x-e^{-x}) - (e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{(e^x-e^{-x})^2 - (e^x+e^{-x})^2}{(e^x-e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{e^{2x}+e^{-2x} - 2e^x \cdot e^{-x} - e^{2x} - e^{-2x} - 2e^x \cdot e^{-x}}{(e^x-e^{-x})^2}$$

$$\frac{dy}{dx} = \frac{-2e^{x-x} - 2e^{x-x}}{(e^x-e^{-x})^2} = \frac{-2-2}{(e^x-e^{-x})^2} = \frac{-4}{(e^x-e^{-x})^2}$$

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vi) $y = ae^{3x} + be^{-3x}$

$$\frac{dy}{dx} = a \cdot \frac{de^{3x}}{d3x} \times \frac{d3x}{dx} + b \frac{de^{-3x}}{d(-3x)} \times \frac{d(-3x)}{dx}$$

$$\frac{dy}{dx} = a \cdot e^{3x} \cdot 3 + b \cdot e^{-3x} \times -3$$

$$\therefore \frac{dy}{dx} = 3 \cdot a \cdot e^{3x} - 3 b \cdot e^{-3x}$$

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Derivative of logarithmic function

formula

1) $y = \log_e x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$

2) $y = \log_a x \Rightarrow \frac{dy}{dx} = \frac{1}{x \cdot \log_e a}$

3) $y = \log_e (ax+b) \Rightarrow \frac{dy}{dx} = \frac{1}{ax+b} \cdot a$

Ex

Differentiate the following Questions:

1) $\log(\tan^{-1} x)$

Let $y = \log(\tan^{-1} x)$

$\frac{dy}{dx} = \frac{d \log(\tan^{-1} x)}{d \tan^{-1} x} \times \frac{d \tan^{-1} x}{dx}$

$\frac{dy}{dx} = \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$

2) $\log_x 2 = \frac{\log 2}{\log x}$ [by the rule change of base]

Let $y = \frac{\log 2}{\log x} \Rightarrow \frac{dy}{dx} = \frac{\log x \cdot \frac{d \log 2}{dx} - \log 2 \cdot \frac{d \log x}{dx}}{(\log x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\log x \times 0 - \log 2 \cdot \frac{1}{x}}{(\log x)^2} = -\frac{\log 2}{x (\log x)^2}$

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(3) $y = \log_{10} x - \log_x 10 + (\log x)^{10} = \log_{10} x - \frac{\log 10}{\log x} + (\log x)^{10}$

$$\frac{dy}{dx} = \frac{d\log_{10} x}{dx} - \frac{\log x \frac{d \log 10}{dx} - \log 10 \cdot \frac{d \log x}{dx}}{(\log x)^2} + \frac{d(\log x)^{10}}{d \log x} \times \frac{d \log x}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\log 10 \cdot \frac{x}{1}} - \frac{\log x \times 0 - \log 10 \cdot \frac{1}{x}}{(\log x)^2} + 10(\log x)^{10-1} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x \log 10} + \frac{\log 10}{x(\log x)^2} + \frac{10(\log x)^9}{x}$$

(4) $y = \log(\sin x^\circ + \cos x^\circ)$; $x^\circ = \frac{\pi x}{180}$

$$\frac{dy}{dx} = d\log\left(\sin\frac{\pi x}{180} + \cos\frac{\pi x}{180}\right)$$

$$\frac{dy}{dx} = \frac{d \log\left(\sin\frac{\pi x}{180} + \cos\frac{\pi x}{180}\right)}{d\left(\sin\frac{\pi x}{180} + \cos\frac{\pi x}{180}\right)} \times \frac{d\left(\sin\frac{\pi x}{180} + \cos\frac{\pi x}{180}\right)}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sin\frac{\pi x}{180} + \cos\frac{\pi x}{180}}\left[\frac{\pi}{180}\cos\frac{\pi x}{180} - \frac{\pi}{180}\sin\frac{\pi x}{180}\right]$$

$$= \frac{\pi}{180}\left[\frac{\cos x^\circ - \sin x^\circ}{\sin x^\circ + \cos x^\circ}\right]$$

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(6) $\log\left(x+\sqrt{x^2-a^2}\right)$

Let $y = \log\left(x+\sqrt{x^2-a^2}\right)$

$$\frac{dy}{dx} = \frac{d\log\left(x+\sqrt{x^2-a^2}\right)}{d\left(x+\sqrt{x^2-a^2}\right)} \times \left\{\frac{d(x)}{dx} + \frac{d\sqrt{x^2-a^2}}{d\,x^2-a^2} \times \frac{d(x^2-a^2)}{dx}\right\}$$

Write on both side

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2-a^2}}\left[1+\frac{1}{2\sqrt{x^2-a^2}}\cdot 2x\right]$$

$$\frac{dy}{dx} = \frac{1}{\cancel{x+\sqrt{x^2-a^2}}}\left[\frac{\cancel{x+\sqrt{x^2-a^2}}}{\sqrt{x^2-a^2}}\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2-a^2}}$$

(7) $y = \log\left[\sqrt{x-a}+\sqrt{x-b}\right]$

$$\frac{dy}{dx} = \frac{d\log\left(\sqrt{x-a}+\sqrt{x-b}\right)}{d\left[\sqrt{x-a}+\sqrt{x-b}\right]} \times \left[\frac{d\sqrt{x-a}}{d\,x-a} \times \frac{d(x-a)}{dx} + \frac{d\sqrt{x-b}}{d(x-b)} \times \frac{d(x-b)}{dx}\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\left[\frac{1}{2\sqrt{x-b}}+\frac{1}{2\sqrt{x-a}}\right] = \frac{\left[\cancel{\sqrt{x-a}+\sqrt{x-b}}\right]}{2\sqrt{x-a}\sqrt{x-b}}$$

$$= \frac{1}{2\sqrt{x-a}\cdot\sqrt{x-b}}$$

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Implicit function

Let $f(x,y) = a$ be a function of x and y defined in such that y is not expressible directly in terms of x. Then $f(x,y) = a$ is called an implicit function of x and y

Ex $\frac{d(y^2)}{dx} = \frac{dy^2}{dy} \times \frac{dy}{dx} = 2y . \frac{dy}{dx}$

Q. If $x^3+y^3 = 3axy$. find $\frac{dy}{dx}$

Soln. Given, $x^3+y^3 = 3axy$

Diff —— to 'x'

$$\frac{dx^3}{dx} + \frac{dy^3}{dy} \times \frac{dy}{dx} = 3a\left[x\frac{dy}{dx} + y.\frac{dx}{dx}\right]$$

$$\Rightarrow 3x^2 + 3y^2 \cdot \frac{dy}{dx} = 3a\left[x\frac{dy}{dx} + y.1\right]$$

$$\Rightarrow 3x^2 + 3y^2 \cdot \frac{dy}{dx} = 3ax\frac{dy}{dx} + 3ay$$

$$\Rightarrow 3y^2 \cdot \frac{dy}{dx} - 3ax \cdot \frac{dy}{dx} = 3ay - 3x^2$$

$$\Rightarrow \cancel{3}(y^2 - ax) \cdot \frac{dy}{dx} = \cancel{3}(ay - x^2)$$

$$\therefore \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

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Q. If $x^3+y^3=a^3$. find $\frac{dy}{dx}$

Soln. Given $x^3+y^3=a^3$

Diff — to 'x'

$$\frac{dx^3}{dx}+\frac{dy^3}{dy}\times\frac{dy}{dx}=\frac{da^3}{dx}$$

$$\Rightarrow x^2\cdot\cancel{\frac{dx}{dx}}+3y^2\cdot\frac{dy}{dx}=0 \quad \left[\because \frac{d}{dx}(c)=0\right.$$

$$\Rightarrow \cancel{3}y^2\cdot\frac{dy}{dx}=-3x^2 \Rightarrow \therefore \frac{dy}{dx}=\frac{-x^2}{y^2}$$

Q. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
find dy/dx.

Soln. Given

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Diff —— to 'x'

$$a \cdot \frac{dx^2}{dx} + 2h\left[x \cdot \frac{dy}{dx} + y \cdot \frac{dx}{dx}\right] + b \cdot \frac{dy^2}{dy} \times \frac{dy}{dx} + 2g\frac{dx}{dx} + 2f\frac{dy}{dx} + \frac{dc}{dx} = 0$$

$$\Rightarrow a \cdot 2x + 2hx\frac{dy}{dx} + 2hy \cdot 1 + b \cdot 2y \cdot \frac{dy}{dx} + 2g \cdot 1 + 2f \cdot \frac{dy}{dx} + 0 = 0$$

$$\Rightarrow 2hx\frac{dy}{dx} + 2by\frac{dy}{dx} + 2f\frac{dy}{dx} = -2ax - 2hy - 2g$$

$$\Rightarrow \cancel{2}[hx + by + f]\frac{dy}{dx} = -\cancel{2}[ax + hy + g]$$

$$\therefore \frac{dy}{dx} = \frac{-[ax + hy + g]}{hx + by + f}$$

Q. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$

Prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Soln. Given $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides

$x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y - y^2 - xy^2 = 0$

$\Rightarrow x^2 - y^2 + x^2y - xy^2 = 0$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow$ Either $x - y = 0$

and $x+y+xy = 0$

$$\boxed{\begin{array}{l}\text{Diff w.r.t. } x \\ \frac{dx}{dx} + \frac{dy}{dx} + x\frac{dy}{dx} + y\cdot\frac{dx}{dx} = 0 \\ \Rightarrow 1 + \frac{dy}{dx} + x\frac{dy}{dx} + y\cdot 1 = 0\end{array}}$$

$y(1+x) = -x \Rightarrow y = \frac{-x}{1+x}$

Diff w.r.t. x

$$\frac{dy}{dx} = -\left[\frac{(1+x)\cdot\frac{dx}{dx} - x\cdot\frac{d(1+x)}{dx}}{(1+x)^2}\right]$$

$$= -\frac{(1+x-x)}{(1+x)^2} \Rightarrow \therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Q. If $\sin y = x \sin(a+y)$.

Prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

Sol^n. $\sin y = x \sin(a+y)$

$\therefore \quad x = \frac{\sin y}{\sin(a+y)}$

Diff — to 'y'

$$\frac{dx}{dy} = \frac{\sin(a+y) \cdot \frac{d \sin y}{dy} - \sin(y) \cdot \frac{d \sin(a+y)}{dy}}{(\sin(a+y))^2}$$

$$\frac{dx}{dy} = \frac{\sin(a+y) \cdot \cos y - \sin y \cdot \cos(a+y)}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin(a+\cancel{y}-\cancel{y})}{\sin^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Q. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$

Soln. Given $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$$y = \frac{x+1}{\sqrt{x}} \Rightarrow \sqrt{x}\cdot y = x+1$$

Diff ___ to 'x'

$$\sqrt{x}\cdot\frac{dy}{dx} + y\cdot\frac{d\sqrt{x}}{dx} = \frac{dx}{dx} + \frac{d\cdot 1}{dx}$$

$$\Rightarrow \sqrt{x}\cdot\frac{dy}{dx} + y\cdot\frac{1}{2\sqrt{x}} = 1$$

$$\frac{2x\frac{dy}{dx} + y}{2\sqrt{x}} = 1$$

$$\Rightarrow 2x\frac{dy}{dx} + y = 2\sqrt{x}$$

Q. If $\cos(x+y) = y\sin x$. find $\frac{dy}{dx}$

Soln. $\cos(x+y) = y\cdot\sin x$

Diff ___ to 'x'

$$\frac{d\cos(x+y)}{d(x+y)}\cdot\frac{d(x+y)}{dx} = y\frac{d\sin x}{dx} + \sin x\cdot\frac{dy}{dx}$$

$$\Rightarrow -\sin(x+y)\left[\frac{dx}{dx} + \frac{dy}{dx}\right] = y\cdot\cos x + \sin x\cdot\frac{dy}{dx}$$

$$-\sin(x+y)\cdot 1 - \sin(x+y)\cdot\frac{dy}{dx} = y\cos x + \sin x\cdot\frac{dy}{dx}$$

$$[-\sin(x+y) - \sin x]\cdot\frac{dy}{dx} = y\cos x + \sin(x+y)$$

$$\therefore \frac{dy}{dx} = \frac{-[y\cos x + \sin(x+y)]}{\sin(x+y) + \sin x}$$

Q. If $e^x + e^y = e^{x+y}$. Prove that $\frac{dy}{dx} = -e^{y-x}$

Soln. Given $e^x + e^y = e^{x+y}$

Dividing by e^{x+y}

$$\frac{e^x}{e^{x+y}} + \frac{e^y}{e^{x+y}} = \frac{\cancel{e^{x+y}}}{\cancel{e^{x+y}}}$$

$$e^{x-x-y} + e^{y-x-y} = 1$$

$$e^{-y} + e^{-x} = 1$$

Diff to 'x'

$$\frac{d e^{-y}}{d(-y)} \times \frac{d(-y)}{dx} + \frac{d e^{-x}}{d(-x)} \times \frac{d(-x)}{dx} = 0$$

$$-e^{-y} \cdot \frac{dy}{dx} - e^{-x} \cdot \frac{dx}{dx} = 0$$

$$-e^{-y} \cdot \frac{dy}{dx} = e^{-x}$$

$$\frac{dy}{dx} = -\frac{e^{-x}}{e^{-y}} = e^{-x+y} = -e^{y-x}$$

Solve it (1) $x^2 + y^2 = 4$ (2) $\sqrt{x} + \sqrt{y} = \sqrt{a}$

(3) $xy = c^2$ (4) $x^2 + y^2 - 3xy = 1$

(5) If $y = x \sin y$. Prove that $x \cdot \frac{dy}{dx} = \frac{y}{1 - x\cos y}$

(6) If $\cos y = x\cos(y+a)$. Prove that $\frac{dy}{dx} = \frac{\cos^2(y+a)}{\sin a}$

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Parametric Function

In this case x and y are given as function of a variable t. then, t is called parameter.

Let $x = f(t)$ and $y = g(t)$

$$\frac{dx}{dt} = f'(t) \text{ and } \frac{dy}{dt} = g'(t)$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$$

Q. find $\frac{dy}{dx}$, when $x = a(t+\sin t)$ and $y = a(1-\cos t)$

soln. $x = a(t+\sin t)$, $y = a(1-\cos t)$

$$\frac{dx}{dt} = a\left[\frac{dt}{dt} + \frac{d\sin t}{dt}\right], \quad \frac{dy}{dt} = a\left[\frac{d\,1}{dt} - \frac{d\cos t}{dt}\right]$$

$$\frac{dx}{dt} = a[1+\cos t], \quad \frac{dy}{dt} = a(+\sin t)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{+a\cdot\sin t}{a(1+\cos t)}$$

$$= \frac{+\cancel{a}\cdot\cancel{2}\sin t/_2\cdot\cancel{\cos t/_2}}{\cancel{a}\times\cancel{2}\cos^{\cancel{2}} t/_2} = +\frac{\sin t/_2}{\cos t/_2}$$

$$\therefore \frac{dy}{dx} = \tan t/_2$$

Q. If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$,

find $\frac{dy}{dx}$ at $t = \pi/3$

Soln. $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$

$\frac{dx}{dt} = 3 \cdot \frac{d\sin t}{dt} - \frac{d\sin 3t}{d3t} \times \frac{d3t}{dt}$, $\frac{dy}{dt} = 3\frac{d\cos t}{dt} - \frac{d\cos 3t}{d3t} \times \frac{d3t}{dt}$

$\Rightarrow \frac{dx}{dt} = 3\cos t - 3\cos 3t$, $\frac{dy}{dt} = 3(-\sin t) - (-\sin 3t) \cdot 3$

$\frac{dy}{dt} = -3\sin t + 3\sin 3t$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t} = \frac{\cancel{3}(\sin 3t - \sin t)}{\cancel{3}(\cos t - \cos 3t)}$

$\frac{dy}{dx} = \frac{\cancel{2}\cos\frac{3t+t}{2} \cdot \sin\frac{3t-t}{2}}{\cancel{2}\sin\frac{3t+t}{2} \cdot \sin\frac{3t-t}{2}} = \frac{\cos 2t \cdot \cancel{\sin t}}{\sin 2t \cdot \cancel{\sin t}}$

$\frac{dy}{dx} = \cot 2t$

At $t = \pi/3$

$\frac{dy}{dx} = \cot\frac{2\pi}{3} = \cot\left(\pi - \frac{\pi}{3}\right) = -\cot\pi/3$

$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{3}}$

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Q. If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$

Show that $\frac{dy}{dx} = -y/x$

Soln. $x = \sqrt{a^{\sin^{-1}t}}$

$$\frac{dx}{dt} = \frac{d\sqrt{a^{\sin^{-1}t}}}{d a^{\sin^{-1}t}} \cdot \frac{d\, a^{\sin^{-1}t}}{d \sin^{-1}t} \cdot \frac{d \sin^{-1}t}{dt}$$

$$= \frac{1}{2\sqrt{a^{\sin^{-1}t}}} \cdot a^{\sin^{-1}t} \log a \cdot \frac{1}{\sqrt{1-t^2}}$$

$$= \frac{\log a}{2\sqrt{1-t^2}} \cdot \frac{(a^{\sin^{-1}t})^1}{(a^{\sin^{-1}t})^{1/2}}$$

$$= \frac{\log a}{2\sqrt{1-t^2}} \cdot (a^{\sin^{-1}t})^{1-1/2}$$

$$= \frac{\log a}{2\sqrt{1-t^2}} \cdot (a^{\sin^{-1}t})^{1/2}$$

$$= \frac{\log a}{2\sqrt{1-t^2}} \cdot \sqrt{a^{\sin^{-1}t}}$$

$$= \frac{\log a}{2\sqrt{1-t^2}} \cdot x$$

Similarly, $y = \sqrt{a^{\cos^{-1}t}}$

$$\frac{dy}{dt} = \frac{d\sqrt{a^{\cos^{-1}t}}}{d\,a^{\cos^{-1}t}} \times \frac{d\,a^{\cos^{-1}t}}{d\cos^{-1}t} \times \frac{d\cos^{-1}t}{dt}$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \times a^{\cos^{-1}t}.\log a. -\frac{1}{\sqrt{1-t^2}}$$

$$= \frac{-\log a}{2\sqrt{1-t^2}} \cdot \sqrt{a^{\cos^{-1}t}}$$

$$= \frac{-\log a}{2\sqrt{1-t^2}} \cdot y$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-\log a \cdot y}{2\sqrt{1-t^2}}}{\frac{\log a \cdot x}{2\sqrt{1-t^2}}}$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x}$$

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Q. If $x = (\log t + \cos t)$ and $y = e^t + \sin t$ find dy/dx.

soln. $x = \log t + \cos t$, $y = e^t + \sin t$

$\frac{dx}{dt} = \frac{d\log t}{dt} + \frac{d\cos t}{dt}$, $\frac{dy}{dt} = \frac{de^t}{dt} + \frac{d\sin t}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{1}{t} - \sin t$, $\Rightarrow \frac{dy}{dt} = e^t + \cos t$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t + \sin t}{\frac{1}{t} - \sin t}$

Q. find $\frac{dy}{dx}$, $x = \sqrt{\sin 2\theta}$, $y = \sqrt{\cos 2\theta}$

soln. $x = \sqrt{\sin 2\theta}$, $y = \sqrt{\cos 2\theta}$

$\frac{dx}{d\theta} = \frac{d\sqrt{\sin 2\theta}}{d\sin 2\theta} \times \frac{d\sin 2\theta}{d2\theta} \times \frac{d2\theta}{d\theta}$, $\frac{dy}{d\theta} = \frac{d\sqrt{\cos 2\theta}}{d\cos 2\theta} \times \frac{d\cos 2\theta}{d2\theta} \times \frac{d2\theta}{d\theta}$

$\frac{dx}{dt} = \frac{1}{\cancel{2}\sqrt{\sin 2\theta}} \cdot \cos 2\theta \cdot \cancel{2}$, $\frac{dy}{d\theta} = \frac{1}{\cancel{2}\sqrt{\cos 2\theta}} \times -\sin 2\theta \times \cancel{2}$

$\frac{dx}{d\theta} = \frac{\cos 2\theta}{\sqrt{\sin 2\theta}}$, $\frac{dy}{d\theta} = -\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$

$\therefore \frac{dy}{dx} = \frac{\frac{-\sin 2\theta}{\sqrt{\cos 2\theta}}}{\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}} = -\frac{(\sin 2\theta).(\sin 2\theta)^{1/2}}{\cos 2\theta.(\cos 2\theta)^{1/2}}$

$= \cancel{\tan 2\theta}\ \frac{(\sin 2\theta)^{3/2}}{(\cos 2\theta)^{3/2}} = (\tan 2\theta)^{3/2}$

Solve it, find $\frac{dy}{dx}$

(1) $x = a\cos t$, $y = b\sin t$

(2) $x = a(1-\cos t)$, $y = a(t+\sin t)$

(3) $x = a\cos^3 t$, $y = a\sin^3 t$

(4) $x = \cos t + \cos 2t$, $y = \sin t + \sin 2t$

(5) If $x = a(t - \sin t)$, $y = a(1-\cos t)$

find $\frac{dy}{dx}$ at $t = \pi/2$

Differentiation Using Logarithm

(1) $\log_b y = x \Rightarrow b^x = y$

(2) $\log mn = \log m + \log n$

(3) $\log \frac{m}{n} = \log m - \log n$

(4) $\log m^n = n \log m$

Q. If $y = x^x$ find $\frac{dy}{dx}$

Sol^n. Given $y = x^x$

Taking log both sides

$\log y = \log x^x$

$\log y = x \log x$

Diff ______ to 'x'

$$\frac{d \log y}{dy} \times \frac{dy}{dx} = x \cdot \frac{d \log x}{dx} + \log x \cdot \frac{dx}{dx}$$

or, $\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1 = 1 + \log x$

$$\frac{dy}{dx} = y[1+\log x] = x^x[1+\log x]$$

Q. Differentiate $(\sin x)^x$ w.r.t 'x'

Soln.

Let $y = (\sin x)^x$

Taking log both sides

$\log y = \log (\sin x)^x$

$\log y = x \log(\sin x)$

Diff ___ to (x)

$$\frac{d \log y}{dy} \cdot \frac{dy}{dx} = x \cdot \frac{d \log \sin x}{d \sin x} \cdot \frac{d \sin x}{dx} + \log \sin x \cdot \frac{dx}{dx}$$

or, $\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x$

or, $\frac{dy}{dx} = y \left[x \cot x + \log \sin x \right]$

$\therefore \frac{dy}{dx} = (\sin x)^x \left[x \cot x + \log \sin x \right]$

Q. Differentiate $(\sin x)^{\log x}$ w.r.t x

Soln.

Let $y = (\sin x)^{\log x}$

$\log y = \log (\sin x)^{\log x}$

$\log y = \log x \cdot \log(\sin x)$

Diff ___ to (x)

$\frac{1}{y} \cdot \frac{dy}{dx} = \log x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot \frac{1}{x}$

$\frac{dy}{dx} = y \left[\log x \cdot \cot x + \log(\sin x) \cdot \frac{1}{x} \right]$

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Q. Differentiate $(x+1)^2 (x+2)^3 (x+3)^4$ w.r.t 'x'.

Sol^n. Let $y = (x+1)^2 (x+1)^3 (x+3)^4$

Taking log both sides

$$\log y = \log (x+1)^2 \cdot (x+2)^3 \cdot (x+3)^4$$

or, $$\log y = \log (x+1)^2 + \log (x+2)^3 + \log (x+3)^4$$

or, $$\log y = 2 \log (x+1) + 3 \log (x+2) + 4 \log (x+3)$$

Diff — to 'x'

$$\frac{d \log y}{dy} \times \frac{dy}{dx} = 2 \cdot \frac{d \log (x+1)}{d(x+1)} \times \frac{d(x+1)}{dx} + 3 \frac{d \log (x+2)}{d(x+2)} \times \frac{d(x+2)}{dx}$$
$$+ 4 \cdot \frac{d \log (x+3)}{d(x+3)} \times \frac{d(x+3)}{dx}$$

or, $$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x+1} \cdot 1 + 3 \cdot \frac{1}{x+2} \cdot 1 + 4 \cdot \frac{1}{x+3} \cdot 1$$

$$\frac{dy}{dx} = y \left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3} \right]$$

$$\therefore \frac{dy}{dx} = (x+1)^2 (x+2)^3 (x+3)^4 \left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3} \right]$$

Ans.

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Q. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find dy/dx

Sol^n. Let $y = u + v$

Diff — to 'x'

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{— (1)}$$

Let $u = x^{\cos x} \Rightarrow \log u = \log x^{\cos x}$

$\Rightarrow \log u = \cos x . \log x$

Diff — to (x)

$$\frac{d \log u}{du} \times \frac{du}{dx} = \cos x . \frac{d \log x}{dx} + \log x . \frac{d \cos x}{dx}$$

or, $\frac{1}{u} . \frac{du}{dx} = \cos x . \frac{1}{x} + \log x(-\sin x)$

or, $\frac{du}{dx} = u \left[\frac{\cos x}{x} - \sin x . \log x \right]$

or, $\frac{du}{dx} = x^{\cos x} \left[\frac{\cos x}{x} - \sin x . \log x \right]$

Let $v = (\cos x)^{\sin x} \Rightarrow \log v = \log(\cos x)^{\sin x}$

$\Rightarrow \log v = \sin x . \log \cos x$

or, Diff — to (x)

$$\frac{1}{v} . \frac{dv}{dx} = \log \cos x \frac{d \sin x}{dx} + \sin x . \frac{d \log \cos x}{d \cos x} \times \frac{d \cos x}{dx}$$

or, $\frac{dv}{dx} = v \left[\log \cos x . (\cos x) + \sin x . \frac{1}{\cos x} (-\sin x) \right]$

$\frac{dv}{dx} = (\cos x)^{\sin x} \left[\cos x \log \cos x - \sin x . \tan x \right]$

$\therefore$ From (1)

$$\frac{dy}{dx} = x^{\cos x} \left[\frac{\cos x}{x} - \sin x . \log x \right] + (\cos x)^{\sin x} \left[\cos x . \log \cos x - \sin x . \tan x \right]$$

Q. Differentiate $\sqrt{(x-1)(x-2)(x-4)}$ w.r.t (x)

Soln. Let $y = \sqrt{(x-1)(x-2)(x-4)}$

$\log y = \log\{\sqrt{(x-1)(x-2)(x-4)}\}$

or, $\log y = \log\{(x-1)(x-2)(x-4)\}^{1/2}$

or $\log y = \frac{1}{2} \log (x-1)(x-2)(x-4)$

or $\therefore \log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) + \log(x-4)\right]$

Diff — to (x)

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-4}\right]$$

$$\therefore \frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-4}\right]$$

$$\frac{dy}{dx} = \frac{\sqrt{(x-1)(x-2)(x-4)}}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-4}\right]$$

Solve it. (1) $x^{\cos x}$ (2) $(\cos x)^{\cot x}$

(3) $(\sqrt{x})^{x}$ (4) $x^{x} + x^{1/x}$

(5) If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

(6) If $y = x^{\sin x} + (\sin x)^{x}$. find $\frac{dy}{dx}$

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Derivative of one function with respect to another function :-

To obtain the derivative of $f(x)$ w.r.to $g(x)$.

Put $y = f(x)$ and $z = g(x)$.

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$$

Q. Differentiate $\sin x$ w.r.to e^x

Soln. Let $y = \sin x$, $z = e^x$

$$\frac{dy}{dx} = \frac{d\sin x}{dx} = \cos x \quad , \quad \frac{dz}{dx} = \frac{de^x}{dx} = e^x$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\cos x}{e^x}$$

Q. Differentiate $\log x$ w.r.to $\frac{1}{x}$

Soln. Let $y = \log x$; $z = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{x} \qquad \frac{dz}{dx} = \frac{-1}{x^2}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\frac{1}{x}}{\frac{-1}{x^2}} = -x$$

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Q. Differentiate e^x w.r.to $\sqrt{x}$

soln. Let $y = e^x$, $z = \sqrt{x}$

$$\frac{dy}{dx} = e^x \qquad \frac{dz}{dx} = \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^x}{\frac{1}{2\sqrt{x}}} = 2\sqrt{x}\cdot e^x$$

Solve it

1. Differentiate x^6 with respect to $\frac{1}{\sqrt{x}}$.
2. Differentiate $\log x$ with respect to $\cot x$.
3. Differentiate $e^{\sin x}$ with respect to $\cos x$.

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Derivative of inverse trigonometric Function :-

Important Result :-

1) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

2) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

3) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

4) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

5) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

6) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

Q. Differentiate. (a) $\sin^{-1} 2x$ (b) $\tan^{-1}\sqrt{x}$ w.r.t. x

Soln. (a) Let $y = \sin^{-1} 2x$

$$\frac{dy}{dx} = \frac{d\sin^{-1}2x}{d\,2x} \times \frac{d\,2x}{dx}$$

$$= \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

$$= \frac{1}{\sqrt{1-4x^2}} \cdot 2$$

(b) Let $y = \tan^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{d\tan^{-1}\sqrt{x}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{(1+x) \cdot 2\sqrt{x}}$$

Q. Differentiate the following w.r.t 'x'

(1) $\sec(\tan^{-1}x)$ (2) $\sin(\tan^{-1}x)$ (3) $\cot(\cos^{-1}x)$

Sol^n. (1) Let $y = \sec(\tan^{-1}x)$ ← (1)

Putting $\tan^{-1}x = t$

$x = \tan t$

$$\therefore \frac{dx}{dt} = \sec^2 t \Rightarrow \frac{dt}{dx} = \frac{1}{\sec^2 t}$$

from (1), $y = \sec t$

$$\therefore \frac{dy}{dx} = \frac{d\sec t}{dt} \times \frac{dt}{dx}$$

$$= \sec t \cdot \tan t \cdot \frac{1}{\sec^2 t} = \frac{\tan t}{\sec t}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{1+\tan^2 t}} = \frac{x}{\sqrt{1+x^2}}$$

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Q. Differentiate the following w.r.t x

(1) $\cos^{-1}\frac{2x}{1+x^2}$ (2.) $\sin^{-1}\frac{4x}{1+4x^2}$

Soln. (1) let $y = \cos^{-1}\frac{2x}{1+x^2}$ —(1)

put $x = \tan t \Rightarrow \tan^{-1}x = t$

$\frac{dx}{dt} = \sec^2 t$

From (1), $y = \cos^{-1}\frac{2\tan t}{1+\tan^2 t}$

$y = \cos^{-1}\frac{2\tan t}{\sec^2 t} = \cos^{-1}\frac{2\tan t}{1+\tan^2 t}$

$y = \cos^{-1}(\sin 2t)$

$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - 2t\right)\right]$

$y = \frac{\pi}{2} - 2t = \frac{\pi}{2} - 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = 0 - 2.\frac{1}{1+x^2}$

$\therefore \frac{dy}{dx} = \frac{-2}{1+x^2}$

9. Differentiate the following w.r.t 'x'

(1) $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ (2) $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$

Soln. (1)

Let $y = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ — (1)

Putting $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

From (i)

$$y = \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$$

$$y = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$y = \tan^{-1}(\tan\theta)$$

$$y = \theta$$

$$y = \cos^{-1}x$$

$$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

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(2) $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$

Sol^n. Let $y = \tan^{-1}\left(\sqrt{1+x^2}+x\right)$

putting $x = \tan t \Rightarrow \tan^{-1}x = t$

$$y = \tan^{-1}\left(\sqrt{1+\tan^2 t}+\tan t\right)$$

$$y = \tan^{-1}\left(\sqrt{\sec^2 t}+\tan t\right)$$

$$y = \tan^{-1}\left[\sec t+\tan t\right]$$

$$y = \tan^{-1}\left[\frac{1}{\cos t}+\frac{\sin t}{\cos t}\right]$$

$$y = \tan^{-1}\left[\frac{1+\sin t}{\cos t}\right]$$

$$y = \tan^{-1}\left[\frac{\left(\cos t/2+\sin t/2\right)^2}{\left(\cos t/2-\sin t/2\right)\left(\cos t/2+\sin t/2\right)}\right]$$

$$y = \tan^{-1}\left[\frac{\cos t/2+\sin t/2}{\cos t/2-\sin t/2}\right]$$

Divide by each $\cos t/2$

$$y = \tan^{-1}\left[\frac{1+\tan t/2}{1-\tan t/2}\right]$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+t/2\right)\right)$$

$$y = \frac{\pi}{4}+t/2$$

$$y = \frac{\pi}{4}+\frac{\tan^{-1}x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+x^2}$$

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Q. Differentiate w.r.t 'x'

1. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Sol^n. Let $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$

putting $x = \cos t$. then

$y = \tan^{-1}\sqrt{\frac{1-\cos t}{1+\cos t}} = \tan^{-1}\sqrt{\frac{\cancel{2}\sin^2 t/2}{\cancel{2}\cos^2 t/2}}$

$y = \tan^{-1}\sqrt{\tan^2 t/2}$

$y = \tan^{-1}(\tan t/2)$

$y = t/2$

$y = \frac{\cos^{-1}x}{2}$

$\frac{dy}{dx} = \frac{1}{2}\left[-\frac{1}{\sqrt{1-x^2}}\right]$

$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$

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Higher Order Derivatives

$$y = f(x)$$

Differentiating w.r.to 'x'. then

$$\frac{dy}{dx} = \frac{d f(x)}{dx}$$

$\Rightarrow$ $\frac{dy}{dx}$ or y_1 or $y' = f'(x)$

It is called first Order Derivatives

Again differentiate $\frac{dy}{dx}$ w.r.t 'x'

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}[f'(x)]$$

$\therefore$ $\frac{d^2y}{dx^2}$ or $y_2 = f''(x)$

It is called second Order Derivatives

Q. If $y = A\cos x + B\sin x$, show that

$$\frac{d^2y}{dx^2} + y = 0$$

Soln. Given, $y = A\cos x + B\sin x$

Differentiating w.r.to (x)

$$\frac{dy}{dx} = A(-\sin x) + B(\cos x)$$

or, $$\frac{dy}{dx} = -A\sin x + B\cos x$$

Again differentiate

$$\frac{d^2y}{dx^2} = -A\cos x + B(-\sin x)$$

or, $$\frac{d^2y}{dx^2} = -[A\cos x + B\sin x]$$

or, $$\frac{d^2y}{dx^2} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0$$

Q. If $y = 2\sin 2x - 5\cos 2x$, show that $\frac{d^2y}{dx^2} + 4y = 0$

Soln. $y = 2\sin 2x - 5\cos 2x$ —(i)

Diff — to (x)

$$\frac{dy}{dx} = 2\frac{d\sin 2x}{d2x} \times \frac{d2x}{dx} - 5\frac{d\cos 2x}{d2x} \times \frac{d2x}{dx}$$

or $$\frac{dy}{dx} = 2\cos 2x \cdot 2 - 5(-\sin 2x) \cdot 2$$

or $$\frac{dy}{dx} = 2[2\cos 2x + 5\sin 2x]$$

Again diff

$$\frac{d^2y}{dx^2} = 2[2(-\sin 2x \cdot 2) + 5\cos 2x \cdot 2]$$

$$\frac{d^2y}{dx^2} = 2\times 2[-2\sin 2x + 5\cos 2x]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -4[2\sin 2x - 5\cos 2x]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -4y \quad [\text{from } 1]$$

$$\therefore \frac{d^2y}{dx^2} + 4y = 0$$

Q. If $x = a\cos^3 t$, $y = a\sin^3 t$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

Soln. $x = a\cos^3 t$, $y = a\sin^3 t$

$$\frac{dx}{dt} = a \cdot 3\cos^2 t(-\sin t) \quad , \quad \frac{dy}{dt} = a \cdot 3\sin^2 t \cdot \cos t$$

$$\therefore \frac{dx}{dt} = -3a\cos^2 t \cdot \sin t \quad , \quad \frac{dy}{dt} = 3a \cdot \sin^2 t \cdot \cos t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t \cdot \cos t}{-3a\cos^2 t \cdot \sin t} = -\tan t$$

$$\therefore \frac{dy}{dx} = -\tan t$$

Again diff,

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt}(-\tan t) \cdot \frac{1}{-3a \cdot \cos^2 t \cdot \sin t}$$

$$= -\sec^2 t \cdot \frac{1}{-3a \cdot \cos^2 t \cdot \sin t}$$

$$= \frac{\sec^4 t \cdot \operatorname{cosec} t}{3a}$$

At $t = \pi/4$

$$\frac{d^2y}{dx^2} = \frac{1}{3a} \sec^4 \frac{\pi}{4} \cdot \operatorname{cosec} \frac{\pi}{4}$$

$$= \frac{1}{3a} (\sqrt{2})^4 \cdot (\sqrt{2}) = \frac{1}{3a} \times 4\sqrt{2}$$

Q. If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

Soln. $y = e^{m\sin^{-1}x}$

Diff — to (x)

$$\frac{dy}{dx} = \frac{de^{m\sin^{-1}x}}{d\,m\sin^{-1}x} \times \frac{d\,m\sin^{-1}x}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{m\sin^{-1}x} \cdot m\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{ym}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = m\cdot y$$

S.B.S. $(1-x^2)\cdot\left(\frac{dy}{dx}\right)^2 = m^2y^2$

Again diff

$$(1-x^2)\cdot 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = m^2\cdot 2y\cdot\frac{dy}{dx}$$

$$\Rightarrow \cancel{2\frac{dy}{dx}}\left[(1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx}\right] = m^2y\cdot\cancel{2\frac{dy}{dx}}$$

$$\therefore (1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

Q. If $y = \cos(m\sin^{-1}x)$ show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\cdot\frac{dy}{dx} + m^2y = 0$$

Soln. $y = \cos(m\sin^{-1}x)$ — (1)

Diff — to (x)

$$\frac{dy}{dx} = \frac{m\cdot -\sin(m\sin^{-1}x)}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = -m\sin(m\sin^{-1}x)$$

Squaring both sides

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2 \sin^2(m\sin^{-1}x)$$

or, $(1-x^2)\cdot\left(\frac{dy}{dx}\right)^2 = m^2\left[1-\cos^2(m\sin^{-1}x)\right]$

or, $(1-x^2)\cdot\left(\frac{dy}{dx}\right)^2 = m^2\left[1-y^2\right]$ [from (1)

or, $(1-x^2)\cdot\left(\frac{dy}{dx}\right)^2 = m^2 - m^2y^2$

Again diff —— to 'x'

$$(1-x^2)\cdot 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} = 0 - m^2\cdot 2y\cdot\frac{dy}{dx}$$
$$+\left(\frac{dy}{dx}\right)^2(-2x)$$

or, $\cancel{2\frac{dy}{dx}}\left[(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}\right] = -m^2y\cdot\cancel{2\frac{dy}{dx}}$

or, $(1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

Q. If $\frac{dy}{dx} = P$, show that $\frac{d^2y}{dx^2} = P\cdot\frac{dP}{dy}$

Sol^n. Given, $\frac{dy}{dx} = P$

Diff —— to 'x'

$$\frac{d^2y}{dx^2} = \frac{dP}{dx}$$

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$$\frac{d^2y}{dx^2} = \frac{dP}{dx} = \frac{dP}{dy} \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dP}{dy} \cdot P \quad . \text{ Prof} .$$

solve it (1) If $x^2 - 4y^2 = 4$

show that $\frac{d^2y}{dx^2} = -\frac{1}{4y^3}$

(2) find $\frac{d^2y}{dx^2}$ if

(a) $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$

(b) $x = 2\cos t - \cos 2t$, $y = 2\sin t - \sin 2t$ at $t = \frac{\pi}{2}$

(3) If $y = e^{\tan^{-1}x}$, prove that

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

(4) If $y = ae^{-3x} + be^{-4x}$

prove that $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$

(5) If $x = r\cos\theta$, $y = r\sin\theta$. find $\frac{dy}{dx}$

and $\frac{d^2y}{dx^2}$ at $\theta = \pi/4$

Application of Derivatives

1) Slope of tangent is

$$\frac{dy}{dx} = f'(x)$$

slope of tangent of $P(x_1, y_1)$ is

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = f'(x_1, y_1)$$

2) Tangent parallel to x-axis

$$\frac{dy}{dx} = f'(x) = 0$$

3) If two lines are parallel then their slopes then their slopes are equal

$$m_1 = m_2$$

If they are perpendicular. then

$$m_1 \times m_2 = -1$$

4) Slope of the Normal

$$= \frac{-1}{\text{slope of tangent}}$$

$$= \frac{-1}{\frac{dy}{dx}}$$

Q. find the slope of the curve $2x^2 - xy + 3y^2 = 18$ at the point (3, 1). Also find the slope of the normal at this point.

Soln. Eqn of curve is

$$2x^2 - xy + 3y^2 = 18$$

Diff —— to x

$$2 \cdot 2x - \left[x \cdot \frac{dy}{dx} + y \cdot 1\right] + 3 \cdot (2y) \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 4x - x\frac{dy}{dx} - y + 6y \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow (6y - x) \cdot \frac{dy}{dx} = y - 4x$$

$$\therefore \frac{dy}{dx} = \frac{y - 4x}{6y - x} \quad \text{—— (1)}$$

Slope of the curve at (3, 1) is

$$\frac{dy}{dx} = \frac{1 - 4 \times 3}{6 \times 1 - 3} = \frac{1 - 12}{6 - 3} = \frac{-11}{3}$$

Q. find the gradient of the curve $y = \sqrt{x^3}$ at $x = 4$

Soln. Eq^n of curve is

$$y = \sqrt{x^3}$$

Diff ——— to 'x' is

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^3}} \cdot 3x^2$$

$$= \frac{3x^2}{2\sqrt{x^3}}$$

Gradient of tangent at $x=4$ is

$$\frac{dy}{dx} = \frac{3\cdot(4)^2}{2\cdot\sqrt{4^3}} = \frac{3\times 16}{2\times 8} = 3$$

Q. Find the slope of tangent if $x = a\cos^3\theta$, $y = b\sin^3\theta$, at the point $\theta = \frac{\pi}{4}$

Soln. Given

$$x = a\cos^3\theta \quad , y = b\sin^3\theta$$

$$\frac{dx}{d\theta} = a\cdot 3\cos^2\theta\cdot(-\sin\theta) , \frac{dy}{d\theta} = b\cdot 3\sin^2\theta\cdot\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cancel{3}b\sin^2\theta\cdot\cos\theta}{-\cancel{3}a\cos^2\theta\cdot\sin\theta} = \frac{b}{a}\tan\theta \quad \text{—(1)}$$

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At $\theta = \frac{\pi}{4}$

$\therefore \frac{dy}{dx} = \frac{-6}{9} \cdot \tan \pi/4 = \frac{-6}{9} \cdot 1 = \frac{-6}{9}$

Q. At what points does the curve $y = x^3 - 24x + 2$ have slope equal to 3?

Soln. eqn of curve is

$$y = x^3 - 24x + 2 \quad \text{---(i)}$$

Diff —— to x

$$\frac{dy}{dx} = 3x^2 - 24 \quad \text{---(ii)}$$

from (1) and (2)

$3x^2 - 24 = 3$

$3x^2 = 27$

$x^2 = 9$

$x = 3, -3$

from (1), $x = 3$

$$y = 3^3 - 24(3) + 2 = 27 - 72 + 2 = -43$$

Point $(3, -43)$

when $x = -3$ from (1)

$\therefore y = (-3)^3 - 24(-3) + 2 = -27 + 72 + 2 = 47$

Teacher's Signature :

Point $(-3, 47)$

Q. find the point on the curve $y = x^2 - 6x + 8$, where the tangent is parallel to x-axis.

soln. eqn of curve

$$y = x^2 - 6x + 8 \quad —(1)$$

Diff ——— w.r.t. x

$$\frac{dy}{dx} = 2x - 6 \quad —(2)$$

∵ tangent is parallel to x-axis. then slope is zero. and slopes of parallel lines are equal.

$$2x - 6 = 0 \Rightarrow x = 3$$

putting $x = 3$ in eqn (1)

$$y = 3^2 - 6 \times 3 + 8 = 9 - 18 + 8 = -1$$

point is $(3, -1)$

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Q. Find the point on the curve $y = 7x - 3x^2$ where the inclination of the tangent is $45°$.

Soln. slope of tangent $= \tan\theta$

$= \tan 45°$

$= 1$ —— (1)

Eqn of curve is

$y = 7x - 3x^2$ —— (2)

$\therefore \frac{dy}{dx} = 7 - 6x$

$\therefore$ slope of tangent $\left(\frac{dy}{dx}\right) = 7 - 6x$

$1 = 7 - 6x$

$\Rightarrow 6x = 6$

$x = 1$

from (2), putting $x = 1$, then

$y = 7(1) - 3(1)^2 = 7 - 3 = 4$

$\therefore$ point is $(1, 4)$

Q. The slope of the curve $2y^3 = ax^2 + b$ at $(1, -1)$ is same as the slope of $x + y = 0$. Find a, b.

Soln. Eqn of curve

$2y^3 = ax^2 + b$

at $(1,-1)$.

$2(-1)^3 = a(1)^2 + b$

$\Rightarrow a+b = -2$

Diff ——— (1)

$6y^2 \cdot \dfrac{dy}{dx} = 2ax$

$\dfrac{dy}{dx} = \dfrac{2ax}{6y^2} = \dfrac{ax}{3y^2}$

$\therefore$ Slope of curve = slope of tangent $= \dfrac{ax}{3y^2}$

slope of curve at $(1,-1) = \dfrac{a(1)}{3(-1)^2} = \dfrac{a}{3}$

slope of $x+y=0$ is $\dfrac{-1}{1} = -1$

$\therefore \dfrac{a}{3} = -1 \Rightarrow a = -3$

put $a = -3$ in eq^n $a+b = -2$

$-3+b = -2$

$b = 1$

Solve it

1. find the slopes of the tangent and normal to the curve
$$y = \sqrt{x} \quad at \quad x = 9$$

2. find the slope of the tangent to the curve $x^2+y^2 = 25$ at $(-3, 4)$.

3. At what point on the curve $y = 3x - x^2$, the slope is -5?

4. find the slope of tangent if $y = a\cos^3\theta$, $x = a\sin^3\theta$, at $\theta = \pi/4$

5. find the points on the curve $y = x^3 - 3x + 1$ at which the tangent is parallel to x-axis

6. find the points on the curve $y = 2 - x^3$ at which tangents are parallel to $12x + y = 2$

Q. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$

Soln. Eqn of curve -

$$y = 3x^4 - 4x$$

$$\Rightarrow \frac{dy}{dx} = \frac{d\,3x^4}{dx} - \frac{d\,4x}{dx}$$

$$\frac{dy}{dx} = 3 \cdot \frac{d\,x^4}{dx} - 4\,\frac{dx}{dx}$$

$$\frac{dy}{dx} = 3 \cdot 4x^{4-1} - 4 \cdot 1$$

$$\frac{dy}{dx} = 12x^3 - 4 \quad \text{——(1)}$$

At $x = 4$, From (1)

$$\left(\frac{dy}{dx}\right)_{x=4} = 12(4)^3 - 4$$

$$= 12 \times 64 - 4$$

$$= 768 - 4$$

$$= 764 \text{ Ans.}$$

Q. Find the slope of tangent to the curve $y = \dfrac{x-1}{x-2}$ at $x = 10$

Soln. eqn of curve

$$y = \frac{x-1}{x-2}$$

Differentiating by $\frac{dy}{dx}$ both sides with respect to (x)

$$\frac{dy}{dx} = \frac{(x-2)\frac{d(x-1)}{dx} - (x-1)\frac{d(x-2)}{dx}}{(x-2)^2}$$

$$= \frac{(x-2)(1) - (x-1)(1)}{(x-2)^2}$$

$$= \frac{(x-2) - (x-1)}{(x-2)^2}$$

$$= \frac{x-2-x+1}{(x-2)^2}$$

$$= \frac{-1}{(x-2)^2}$$

At $x = 10$

$$\frac{dy}{dx} = \frac{-1}{(10-2)^2} = \frac{-1}{(8)^2} = \frac{-1}{64}$$

Q. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

Ans. Eqn of curve

$$y = x^3 - x + 1$$

$$\frac{dy}{dx} = \frac{dx^3}{dx} - \frac{dx^1}{dx} + \frac{d \cdot 1}{dx}$$

$$\frac{dy}{dx} = 3x^{3-1} - 1 + 0$$

$$\frac{dy}{dx} = 3x^2 - 1 \quad \text{—} (1)$$

$\because$ x-coordinate is 2

$$x = 2$$

$$\frac{dy}{dx} = 3(2)^2 - 1$$

$$= 3 \cdot 4 - 1$$

$$= 12 - 1$$

$$= 11$$

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Q. find the slope of the Normal to the curve $x = a\cos\theta$, $y = b\sin\theta$ at $\theta = \pi/4$

Soln. $x = a\cos\theta$, $y = b\sin\theta$

$$\frac{dx}{d\theta} = a\,\frac{d\cos\theta}{d\theta}, \quad \frac{dy}{d\theta} = b\cdot\frac{d\sin\theta}{d\theta}$$

$$\frac{dx}{d\theta} = a(-\sin\theta), \quad \frac{dy}{d\theta} = b\cos\theta$$

$$\frac{dx}{d\theta} = -a\sin\theta, \quad \frac{dy}{d\theta} = b\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{1}{\frac{dx}{d\theta}}$$

$$\frac{dy}{dx} = b\cos\theta \times \frac{1}{-a\sin\theta} = \frac{-b}{a}\cot\theta$$

$$\text{slope of Normal} = \frac{-1}{\frac{dy}{dx}}$$

$$= \frac{+1}{\frac{b}{a}\cot\theta} = \frac{a}{b\cot\theta}$$

At $\pi/4$,

$$\text{slope of normal} = \frac{a}{b\cot\pi/4} = \frac{a}{b\cdot 1} = \frac{a}{b}$$

Q. Find the slope of the normal to the curve $x = 1 - a\sin\theta$, $y = b\cos^2\theta$, at $\theta = \pi/2$

Soln.

$x = 1 - a\sin\theta$ $\qquad$ $y = b\cos^2\theta = b(\cos\theta)^2$

$\frac{dx}{d\theta} = \frac{d\,1}{d\theta} - \frac{a\,d\sin\theta}{d\theta}$ $\qquad$ $\frac{dy}{d\theta} = \frac{b\,d(\cos\theta)^2}{d\cos\theta} \times \frac{d\cos\theta}{d\theta}$

$= 0 - a\cos\theta$ $\qquad$ $= b\cdot 2(\cos\theta)^{2-1}(-\sin\theta)$

$= -a\cos\theta$ $\qquad$ $= -2b\sin\theta\cdot\cos\theta$

$\therefore$ slope of normal $= -\frac{1}{\frac{dy}{dx}}$

$= \frac{-1}{\frac{dy}{d\theta} \times \frac{1}{\frac{dx}{d\theta}}}$

$= \frac{-1}{-2b\sin\theta\cdot\cos\theta \times \frac{1}{-a\cos\theta}}$

$= \frac{-a}{2b\sin\theta}$ ——— (1)

At $\theta = \pi/2$, from (1)

$\cancel{\frac{dy}{dx}}$ Slope of normal

$= \frac{-a}{2b\sin\pi/2} = \frac{-a}{2b\cdot 1} = \frac{-a}{2b}$

Q. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the joining the points $(2, 0)$ and $(4, 4)$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x_1, y_1 \quad\quad\quad x_2, y_2$

Soln. eq^n of curve

$$y = (x-2)^2$$

$$\frac{dy}{dx} = \frac{d(x-2)^2}{d(x-2)} \times \frac{d(x-2)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2(x-2)^{2-1} \cdot 1$$

$$\Rightarrow \frac{dy}{dx} = 2(x-2)$$

Slope of tangent $= 2(x-2) = \frac{dy}{dx}$

Slope of joining two points $= \frac{y_2 - y_1}{x_2 - x_1}$
$(2, 0)$ and $(4, 4)$
(x_1, y_1) (x_2, y_2)

$$= \frac{4-0}{4-2}$$

$$= \frac{4}{2} = 2$$

$\therefore$ slope of tangent = slope of joining two points

$$\cancel{2}(x-2) = \cancel{2}$$

$$x - 2 = 1 \Rightarrow x = 3$$

If $x = 3$, then From (1),

$y = (3-2)^2 = 1$ $\quad\therefore$ points are $(3, 1)$

Equation of tangent at (x_1, y_1)

$$y - y_1 = m(x - x_1), \quad m = \text{slope of tangent}$$

Equation of Normal at (x_1, y_1)

$$y - y_1 = -\frac{1}{m}(x - x_1), \quad -\frac{1}{m} = \text{slope of Normal}$$

Q. Find the equation of tangent and normal to the curves at the given points

$$y = x^2 \text{ at } (-1, 1)$$

Soln. Given curve

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

At $(-1, 1)$, $\frac{dy}{dx} = m = 2(-1) = -2$

Eqn of tangent at $\overset{x_1, y_1}{(-1, 1)}$

$$y - 1 = -2(x - (-1))$$

$\Rightarrow y - 1 = -2(x+1) \Rightarrow y - 1 = -2x - 2$

$\Rightarrow 2x + y + 1 = 0$

Eqn of Normal at $(-1, 1)$

$$y - 1 = \frac{+1}{+2}(x - (-1)) \Rightarrow y - 1 = \frac{1}{2}(x+1)$$

$\Rightarrow 2y - 2 = x + 1$

Teacher's Signature :

$\Rightarrow x - 2y + 3 = 0$

7. Find the equations of the tangent and Normal to the curve

$$y = x^4 - 6x^3 + 13x^2 - 10x + 5$$ at the point (1,3)

Soln. Given $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$\therefore \quad \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

$\left(\frac{dy}{dx}\right)_{(1,3)} = 4 \times 1^3 - 18 \times 1^2 + 26 \times 1 - 10$

$= 4 - 18 + 26 - 10 = 2$

$\therefore$ Eqn of tangent at (1,3)

$y - 3 = 2(x-1)$

$\Rightarrow \quad y - 3 = 2x - 2$

$\Rightarrow \quad 2x - y + 1 = 0$

Eqn of Normal at (1,3)

$y - 3 = \frac{-1}{2}(x-1)$

$\Rightarrow \quad 2y - 6 = -x + 1$

$\Rightarrow \quad x + 2y - 7 = 0$

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Q. find the equation of the tangent to the curve $y = \sqrt{5x-3} - 2$ which is parallel to the line $4x - 2y + 3 = 0$

Sol/n. Given line, $4x - 2y + 3 = 0$

$\therefore$ Slope of line $= -\dfrac{\text{coefficient of } x}{\text{coefficient of } y}$

$$= -\frac{4}{-2}$$

Slope of line $= 2$

$\therefore$ Slope of tangent $= 2$

Let the point of contact be (x_1, y_1)

Now, $y = \sqrt{5x-3} - 2 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{5x-3}} \cdot 5$

$\left(\dfrac{dy}{dx}\right)_{(x_1, y_1)} = \dfrac{5}{2\sqrt{5x_1-3}}$

$\therefore \dfrac{5}{2\sqrt{5x_1-3}} = 2$

S.B.S, $\dfrac{25}{4(5x_1-3)} = 4$

$\Rightarrow 25 = 16(5x_1 - 3)$

$\Rightarrow 25 = 80x_1 - 48$

$\Rightarrow 73 = 80x_1$

$\Rightarrow x_1 = \dfrac{73}{81}$

$\therefore y_1 = \sqrt{5x_1 - 3} - 2$

$= \sqrt{5 \times \frac{73}{80} - 3} - 2 = \dfrac{-3}{4}$

$\therefore$ Point is $\left(\dfrac{73}{81}, \dfrac{-3}{4}\right)$

Eq^n of tangent is

$y - \left(\dfrac{-3}{4}\right) = \left(x - \dfrac{73}{81}\right)2$

$\Rightarrow 80x - 40y - 103 = 0$

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Q. At what points will be tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to x-axis? Also, find the equations of tangents to the curve at these points.

Soln. Given $y = 2x^3 - 15x^2 + 36x - 21$

$$\frac{dy}{dx} = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$$

At (x_1, y_1), $\frac{dy}{dx} = 6(x_1^2 - 5x_1 + 6)$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 0$$

$\Rightarrow 6(x_1^2 - 5x_1 + 6) = 0$

$\Rightarrow x_1^2 - 5x_1 + 6 = 0$

$\Rightarrow x_1^2 - 3x_1 - 2x_1 + 6 = 0$

$\Rightarrow x_1(x_1 - 3) - 2(x_1 - 3) = 0$

$\Rightarrow (x_1 - 2)(x_1 - 3) = 0$

$\Rightarrow x_1 = 2, 3$

when $x_1 = 2$ then $y_1 = 2.2^3 - 15.2^2 + 36.2 - 21 = 7$

$x_1 = 3$, then $y_1 = 2.3^3 - 15.3^2 + 36.3 - 21 = 6$

$\therefore$ Points are $(2,7)$ and $(3,6)$

Q. Determine the points on the curve $2y = 3 - x^2$ at which the tangent is parallel to the line $x+y=0$.

Soln. Let the point be (x_1, y_1), then

Slope of tangent = slope of line

Given line, $x + y = 0$

$y = -x$

$\Rightarrow \frac{dy}{dx} = -1 \quad -(i)$

Now, $2y = 3 - x^2 \Rightarrow 2\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = -x$

$\therefore \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -x_1 \quad -(ii)$

$\therefore$ from (i) and (ii)

$-x_1 = -1 \Rightarrow x_1 = 1$

$\because (x_1, y_1)$ lies on the curve $2y = 3 - x^2$

$\therefore 2y_1 = 3 - x_1^2$

when $x_1 = 1$, $2y_1 = 3 - 1^2$

$\Rightarrow \cancel{2}y_1 = \cancel{2}$

$y_1 = 1$

$\therefore$ Point is $(x_1, y_1) = (1, 1)$

Q. Find the points on the curve $4x^2+9y^2=1$, where the tangents are perpendicular to the line $x+2y=0$

Soln. Eqn of line $x+2y=0 \Rightarrow y = -\frac{x}{2}$

$m_1 =$ slope of line $= -\frac{\text{coeff. of } x}{\text{coeff of } y} = \frac{-1}{2}$ —(i)

Let the point be (x_1, y_1)

Eqn of curve is

$4x^2+9y^2=1$

$\Rightarrow 8x + 18y \cdot \frac{dy}{dx} = 0$

$\Rightarrow 18y \frac{dy}{dx} = -8x$

$\Rightarrow \frac{dy}{dx} = \frac{-8x}{18y} = \frac{-4x}{9y}$

At (x_1, y_1)

$m_2 = \frac{dy}{dx} = -\frac{4x_1}{9y_1}$ —(ii)

From (i) and (ii)

as perpendicular, $m_1 \times m_2 = -1$

$-\frac{4x_1}{9y_1} \times \left(\frac{-1}{2}\right) = -1$

$\Rightarrow y_1 = \frac{-2}{9} x_1$ —(iii)

$\because (x_1, y_1)$ lies on $4x^2+9y^2=1$

$\Rightarrow 4x_1^2+9y_1^2=1$

$4x_1^2 + 9 \cdot \frac{4}{81} x_1^2 = 1$

$\Rightarrow x_1^2 = \frac{9}{40}$

$\Rightarrow x_1 = \pm\frac{3}{2\sqrt{10}}$

$\therefore y_1 = \frac{-2}{9} x_1$

$= \frac{-2}{9} \times \frac{3}{2\sqrt{10}}$

Points are

$\left(\frac{3}{2\sqrt{10}}, \frac{1}{3\sqrt{10}}\right)$

and $\left(\frac{-3}{2\sqrt{10}}, \frac{-1}{3\sqrt{10}}\right)$

Solve it

(1) find the equation of tangent and Normal

i) $y = x^3 - 2x + 7$ at $(1, 6)$

ii) $y = x^3$ at $P(1,1)$

iii) $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

iv) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1$

(2) . find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line

$y - 4x + 5 = 0$

(3) find the equation of the tangent to the curve $x^2 + 2y = 8$, which is perpendicular to the line $x - 2y + 1 = 0$

Q. Find the maximum and minimum values

(i) $y = x^3 + 6x^2 - 15x + 5$

(ii) $y = x^3 - 7x^2 + 11x + 5$

(iii) $y = x^4 - 4x$

(iv) $y = 4 - x - x^2$

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Q. find the local maxima or local minima if any of

(i) $f(x) = \frac{1}{x^2+2}$ (ii) $f(x) = x^3 - 3x$

Also find local maximum and local minimum values.

soln. i) $f(x) = \frac{1}{x^2+2}$

$$f'(x) = \frac{(x^2+2)\cdot\frac{d.1}{dx} - 1.\frac{d(x^2+2)}{dx}}{(x^2+2)^2}$$

$$= \frac{(x^2+2)\cdot 0 - 2x}{(x^2+2)^2} = \frac{-2x}{(x^2+2)^2}$$

for local maxima and local minima

$$f'(x) = 0 \Rightarrow \frac{-2x}{(x^2+2)^2} = 0$$

$$\Rightarrow -2x = 0 \Rightarrow x = 0$$

$x=0$ is a point of local maximum

$$f(x) = \frac{1}{x^2+2} \Rightarrow f(0) = \frac{1}{0+2} = \frac{1}{2}$$

(ii). $f(x) = x^3 - 3x \Rightarrow f'(x) = 3x^2 - 3$

for local maxima and local minima

$$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0$$

$$\Rightarrow 3(x^2-1) = 0$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow (x+1)(x-1) = 0 \Rightarrow x = 1, -1$$

$x=1$ local minimum value, $f(1) = 1^3 - 3\times 1 = -2$

$x=-1$ local maximum, $f(-1) = (-1)^3 - 3\times(-1) = 2$

Q. Find all the points of local maxima and local minima and also find maximum and minimum value.

Soln. $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

$\therefore f'(x) = -\frac{3}{4} \times 4x^3 - 8 \times 3x^2 - \frac{45}{2} \times 2x + 0$

$= -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

Now, $f'(x) = 0 \Rightarrow -3x(x^2+8x+15) = 0$

$\Rightarrow -3x[x^2+5x+3x+15] = 0$

$\Rightarrow -3x[x(x+5)+3(x+5)] = 0$

$\Rightarrow -3x(x+5)(x+3) = 0$

$\Rightarrow x = 0, -5, -3$

$f''(x) = -9x^2 - 48x - 45$

when $x = 0 \Rightarrow f''(0) = -45 < 0$

$\therefore x = 0$ is a point of local maximum

$f(0) = 105$

when $x = -5 \Rightarrow f''(-5) = -30 < 0$

point of local maximum

$f(-5) = \frac{295}{4}$

when $x = -3 \Rightarrow f''(-3) = 18 > 0$

Point of local minimum

$f(-3) = \frac{231}{4}$

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Q. find the maximum and minimum values of $8x^4 - 8x^3 + 12x^2 - 48x + 25$ on $[0, 3]$

Soln. let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$

$f'(x) = 12x^3 - 24x^2 + 24x - 48$

$f''(x) = 36x^2 - 48x + 24$

for local maxima and minima

$f'(x) = 0 \Rightarrow 12x^3 - 24x^2 + 24x - 48 = 0$

$\Rightarrow 12(x^3 - 2x^2 + 2x - 4) = 0$

$\Rightarrow 12(x-2)(x^2+2) = 0$

$x = 2$

$f''(2) = 72 > 0$

$\therefore$ $x = 2$ is a point local minima

$f(2) = -39$, $f(0) = 25$, $f(3) = 16$

minimum values is -39 at $x = 2$

maximum values is 25 at $x = 0$

Q. find two positive numbers x and y such that $(x+y)=60$ and xy^3 is maximum.

soln. let $x+y=60$ and let $P=xy^3$

Now,
$$P = xy^3$$
$$= (60-y).y^3 \quad [\because x=60-y]$$

$$\therefore \frac{dP}{dy} = (60-y).3y^2+y^3(-1)$$
$$= (60-y).3y^2-y^3$$
$$= 180y^2-4y^3$$
$$= 4y^2(45-y)$$

$$\frac{d^2P}{dy^2} = 360y-12y^2 = 12y(30-y)$$

Now,
$$\frac{dP}{dy}=0 \Rightarrow 4y^2(45-y)=0$$
$$\Rightarrow y=0, 45$$

$$\frac{d^2P}{dy^2} = 12\times45(30-45) = -8100<0$$

$\therefore$ $y=45$ is a point of maximum

$\therefore$ Numbers are 45 and 15

Mean Deviation about the mean:- Individual (x) are given

① Mean ($\bar{x}$) : If $x_1, x_2, x_3 \dots x_n$ are given. then

$$\bar{x} = \frac{\text{Sum of observation}}{\text{Number of observation}}$$

i) $MD(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{N}$

ii) $MD(M) = \frac{\sum |x_i - M|}{N}$

Q. Find the mean deviation about the mean for the following data 15, 17, 10, 13, 7, 18, 9, 6, 14, 11

Sol^n.

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$
15	3	3
17	5	5
10	-2	2
13	1	1
7	-5	5
18	6	6
9	-3	3
6	-6	6
14	2	2
11	1	1
		= 34

$$\bar{x} = \frac{15+17+10+13+7+18+9+6+14+11}{10}$$

$$= \frac{120}{10} = 12$$

$$MD(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{N}$$

$$= \frac{34}{10}$$

$$= 3.4$$

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Q. Find the MD about the median

11, 3, 8, 7, 5, 14, 10, 2, 9

Soln. Arrange this

2, 3, 5, 7, 8, 9, 10, 11, 14

$n = 9$ (odd)

$$\text{Median } (M) = \frac{n+1}{2} \text{ th observation}$$

$$= \frac{9+1}{2} \text{ th } \text{"}$$

$$= 5 \text{ th } \text{"}$$

$$= 8$$

x_i	$x_i - M$	$\lvert x_i - M \rvert$
2	-6	6
3	-5	5
5	-3	3
7	-1	1
8	0	0
9	1	1
10	2	2
11	3	3
14	6	6

$\Sigma \lvert x_i - M \rvert = 27$

$$MD(M) = \frac{\Sigma \lvert x_i - M \rvert}{N}$$

$$= \frac{27}{9}$$

$$= 3$$

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Discrete Frequency Distribution (x_i and f_i)

(1) MD about Mean:– $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

(2) MD about the Median:–

$$MD(M) = \frac{\sum f_i |x_i - M|}{N}$$

Q. Find the MD about the mean

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

Soln.

x_i	f_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$	$f_i x_i$
3	6	-5	5	30	18
5	8	-3	3	24	40
7	15	-1	1	15	105
9	25	1	1	25	225
11	8	3	3	24	88
13	4	5	5	20	52
	66			138	528

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{528}{66}$

$\bar{x} = 8$

$$MD(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{138}{66}$$

$= 2.09$

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Variance:- Mean of squares of the deviation from the mean is called the variance. It is denoted by σ^2.

Standard Deviation:- The positive square root of variance is called the standard deviation and it is denoted by σ.

$$\sigma = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{N}}$$

(1). Find mean, variance and S.D for 5, 9, 8, 12, 6, 10, 6, 8

Sol:

x_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
5	-3	9
9	1	1
8	0	0
12	4	16
6	-2	4
10	2	4
6	-2	4
8	0	0
		$\Sigma(x_i-\bar{x})^2 = 2$

$\bar{x} = \frac{\text{Sum of total obs}}{\text{No. of obs}}$

$\bar{x} = \frac{5+9+8+12+6+10+6+8}{8}$

$\bar{x} = \frac{64}{8} = 8$

Variance $= \frac{\Sigma(x_i-\bar{x})}{N}$

$= \frac{38}{8} = 4.75$

$S.D = \sqrt{4.75} = 2.17$

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(2) Find the variance and S.D

x_i	10	15	18	20	25
f_i	3	2	5	8	2

Sol^n.

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i-\bar{x})^2$
10	3	30	-8	64	192
15	2	30	-3	9	18
18	5	90	0	0	0
20	8	160	2	4	32
25	2	50	7	49	98
	20	$\Sigma f_i x_i = 360$			340

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{20} = 18.$$

$$\text{variance} = \frac{340}{20} = 17$$

$$S.D = \sqrt{17} = 4.12$$

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Graphical Representation

Histogram :- A histogram is a bar graph that shows ~~data~~ data in intervals.

or

Histogram is the pictorial representation of a grouped frequency distribution by means of adjacent rectangles whose areas are proportional to the frequencies.

In this class boundaries are plotted along the x-axis and the frequencies along the y-axis. The height is equal to the corresponding class frequency.

When all the classes are not of equal widths, the height of a rectangle is proportional to the frequency density of the corresponding class-interval

$$\therefore \text{frequency density} = \frac{\text{class Frequency}}{\text{Width of the class interval}}$$

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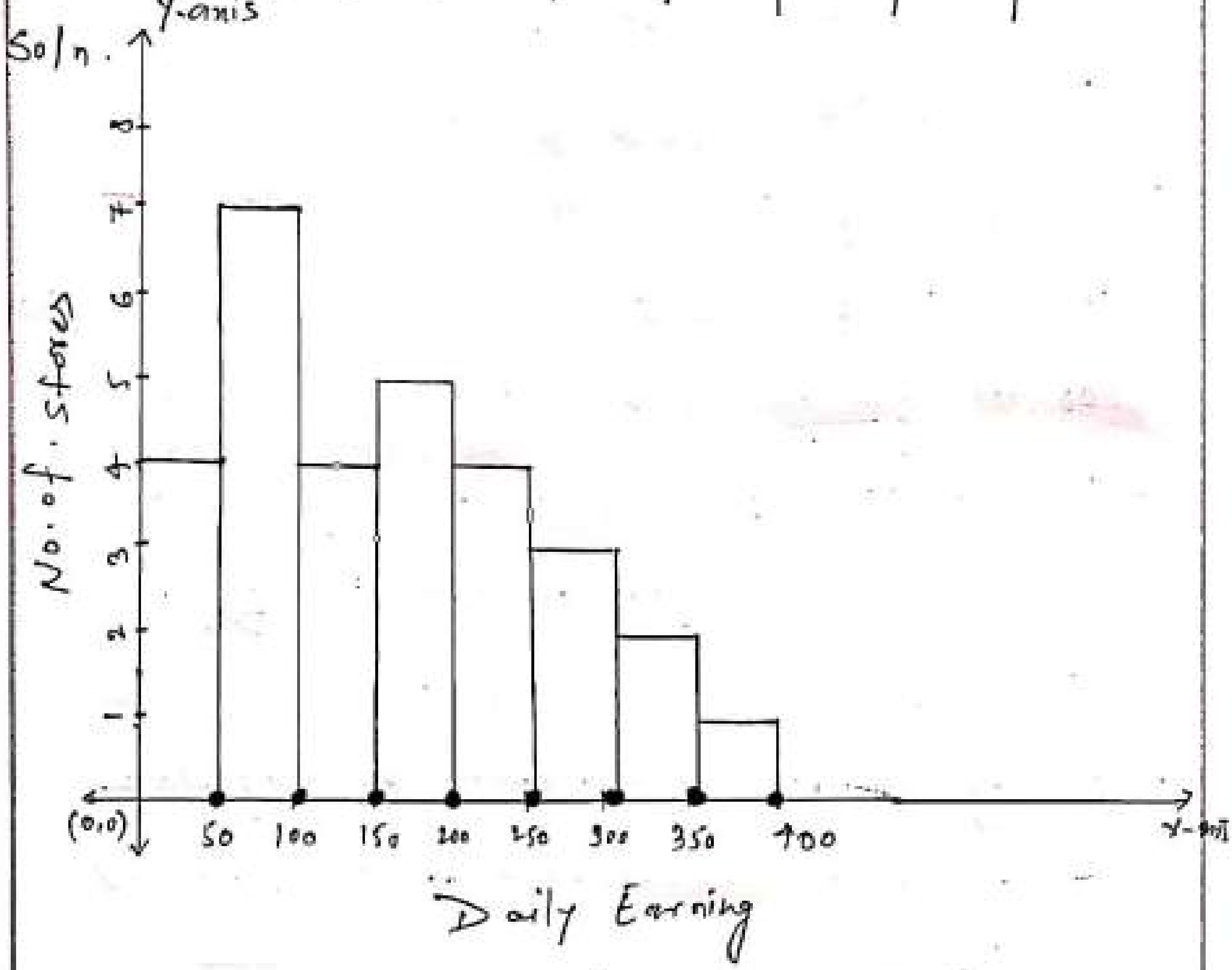
Q. Draw a Histogram to represent the following data. The daily earning of 30 drug stores are given below.

Daily earning	0-50	50-100	100-150	150-200	200-250	250-300	300-350	350-400
No of stores	4	7	4	5	4	3	2	1

Soln.



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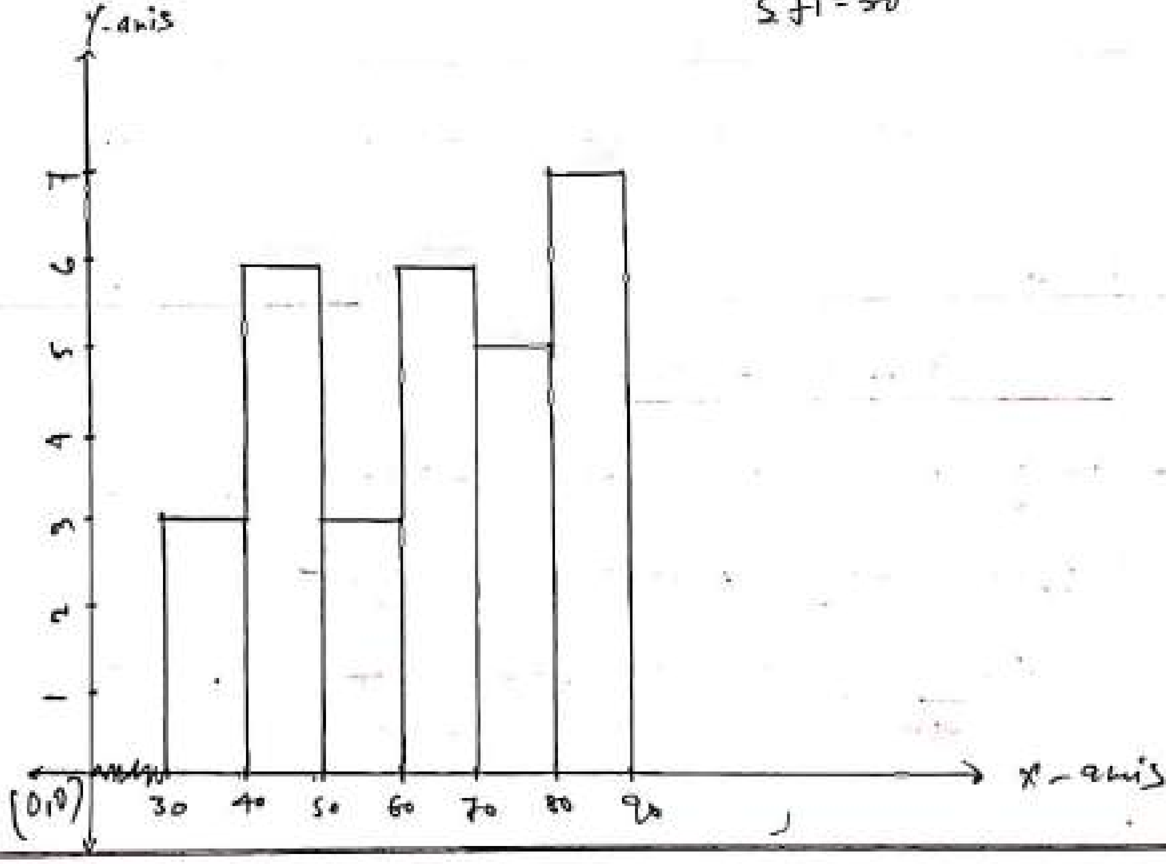
Q. Construct a frequency distribution table for the following data of weights (in gms) of 30 bolts using equal class intervals, one of them being 50-60 (60 not included). Hence draw a Histogram.

45, 53, 30, 85, 75, 85, 40, 60, 65, 40
60 75 70 60 70 85 85 80 35 45
40 50 60 60 55 45 30 80 85 75

Sol. Frequency distribution are

Class-I	Tally Marks	Frequency
30-40	III	3
40-50	卌 I	6
50-60	III	3
60-70	卌 I	6
70-80	卌	5
80-90	卌 II	7
		$\Sigma f_i = 30$



solve it

1) Draw a histogram for the following data

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	5	11	19	21	16	10	8	6

2) Draw a histogram for the following data

Weight in gm	10-15	15-20	20-25	25-35	35-55	55-70	70-75
No. of items	4	8	7	10	12	15	4

Construction of Histogram when class intervals are unequal :- If the class intervals are unequal

$$\text{Height} = \frac{\text{class frequency}}{\text{Width of the class intervals}}$$

If the width of a class is doubled than that of a normal class. then the height must be halved. Then the frequency must be divided by 2

Q. Draw histogram for the following freq. distribution

C.I	16-19	21-24	26-29	31-34	36-44	46-64
Freq	18	30	12	15	18	24

Soln.

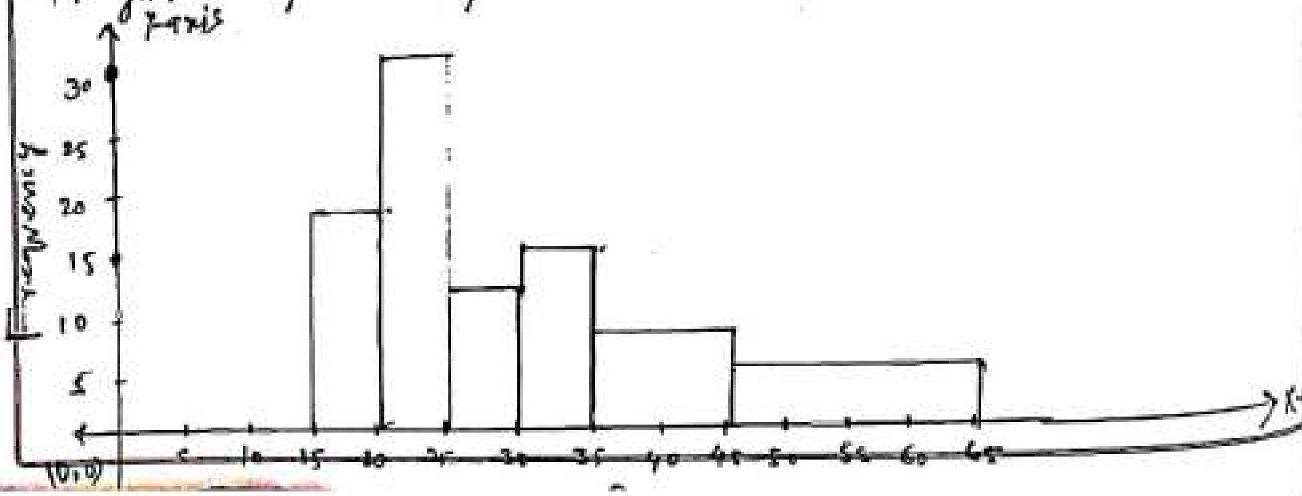
$$\text{Correction factor} = \frac{21-19}{2} = \frac{2}{2} = 1$$

First four classes have equal width 5 units. Fifth class has width twice that of the normal width and sixth class width four times the normal width.

$\therefore$ Height or freq. density of fifth class (35-45) = $\frac{\text{class frequency}}{\text{Width of class}}$

$$= \frac{18}{2} = 9$$

Height or fre. density of class (45-65) is = $\frac{24}{4} = 6$



Graphical determination of the mode :-

To determine, first we draw the histogram.

∴ Mode is observation to the maximum frequency.

It lies in the rectangle of maximum height.

Q. The crushing strength of 45 cement concrete blocks are given below

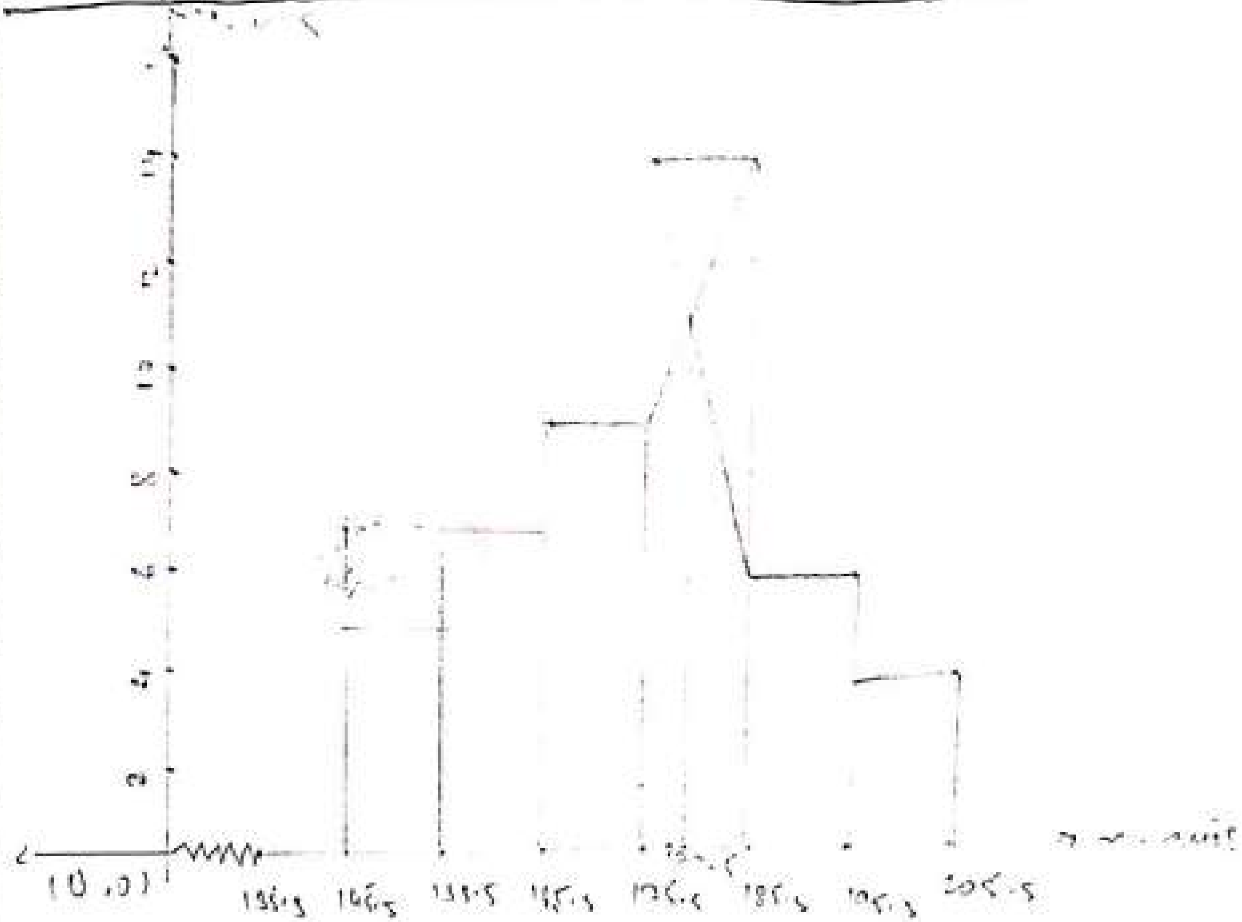
Crushing strength	146-155	156-165	166-175	176-185	186-195	196-205
No. of blocks	5	7	9	14	6	4

a) Represent the data by histogram

b) find the mode graphically and check it analytically

Soln.

C.I	145.5 – 155.5	155.5 – 165.5	165.5 – 175.5	175.5 – 185.5	185.5 – 195.5	195.5 – 205.5
f_i	5	7	9	14	6	4



Maximum frequency is 14 and its class 175.5 – 185.5

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$= 175.5 + \frac{14-9}{2\times 14 - 9 - 6} \times 10$$

$$= 175.5 + \frac{50}{13} = 175.50 + 3.85$$

$$= 179.35$$

Cumulative Frequency curve or ogive (S type curve).

Less than type curve :-

Q. Draw an ogive for the following frequency distribution by less than method.

Marks	0 - 10	10-20	20-30	30-40	40-50	50-60
No. of students	7	10	23	51	6	3

Soln.

Marks	Number of students	Upper Limit	Cumulative Frequency
0 - 10	7	10	7
10 - 20	10	20	17
20 - 30	23	30	40
30 - 40	51	40	91
40 - 50	6	50	97
50 - 60	3	60	100

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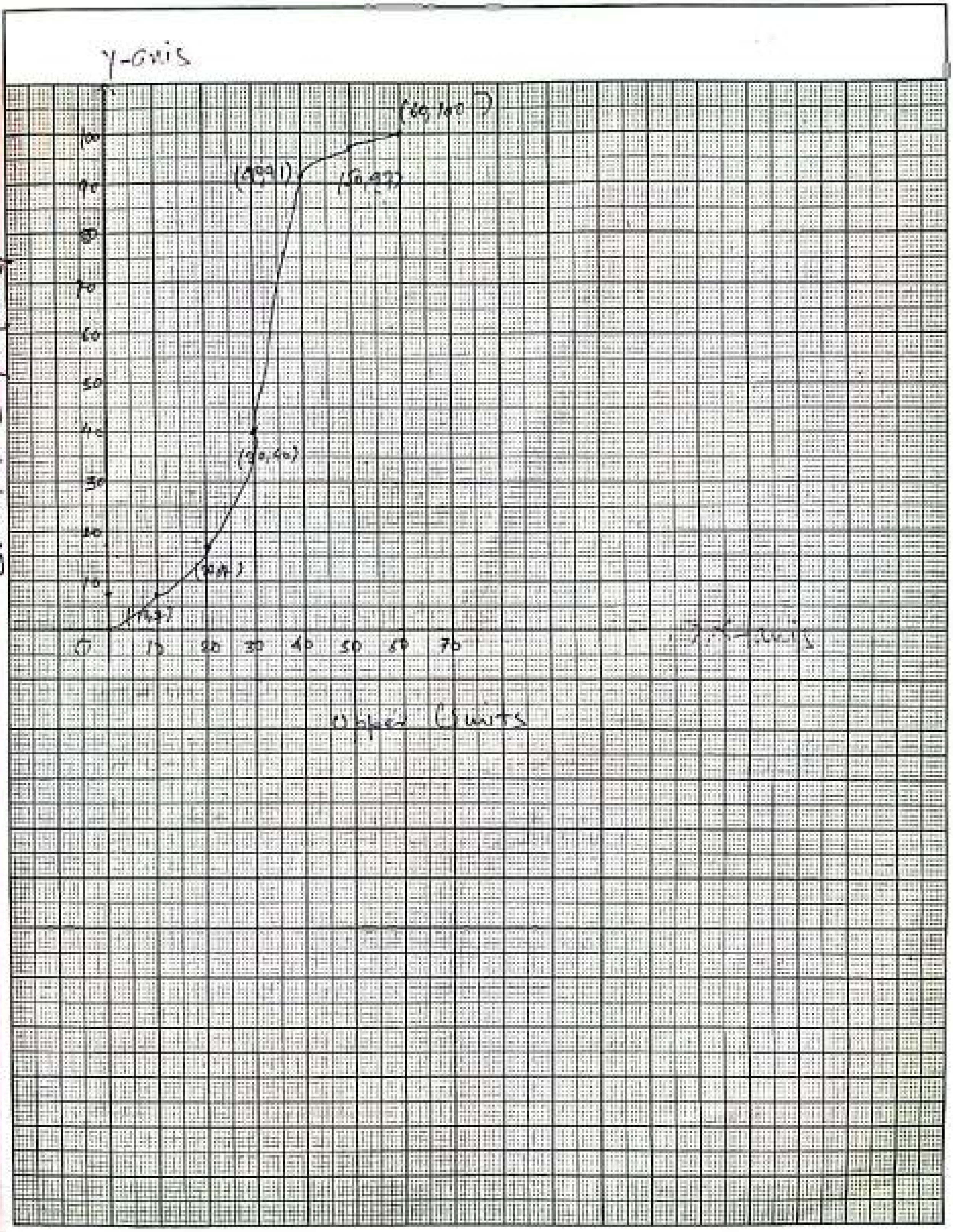

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Cumulative frequency curve or ogive (S type curve)

More than type:-

Q. For the following frequency distribution, draw cumulative frequency curve of 'more than' type and obtain the median value.

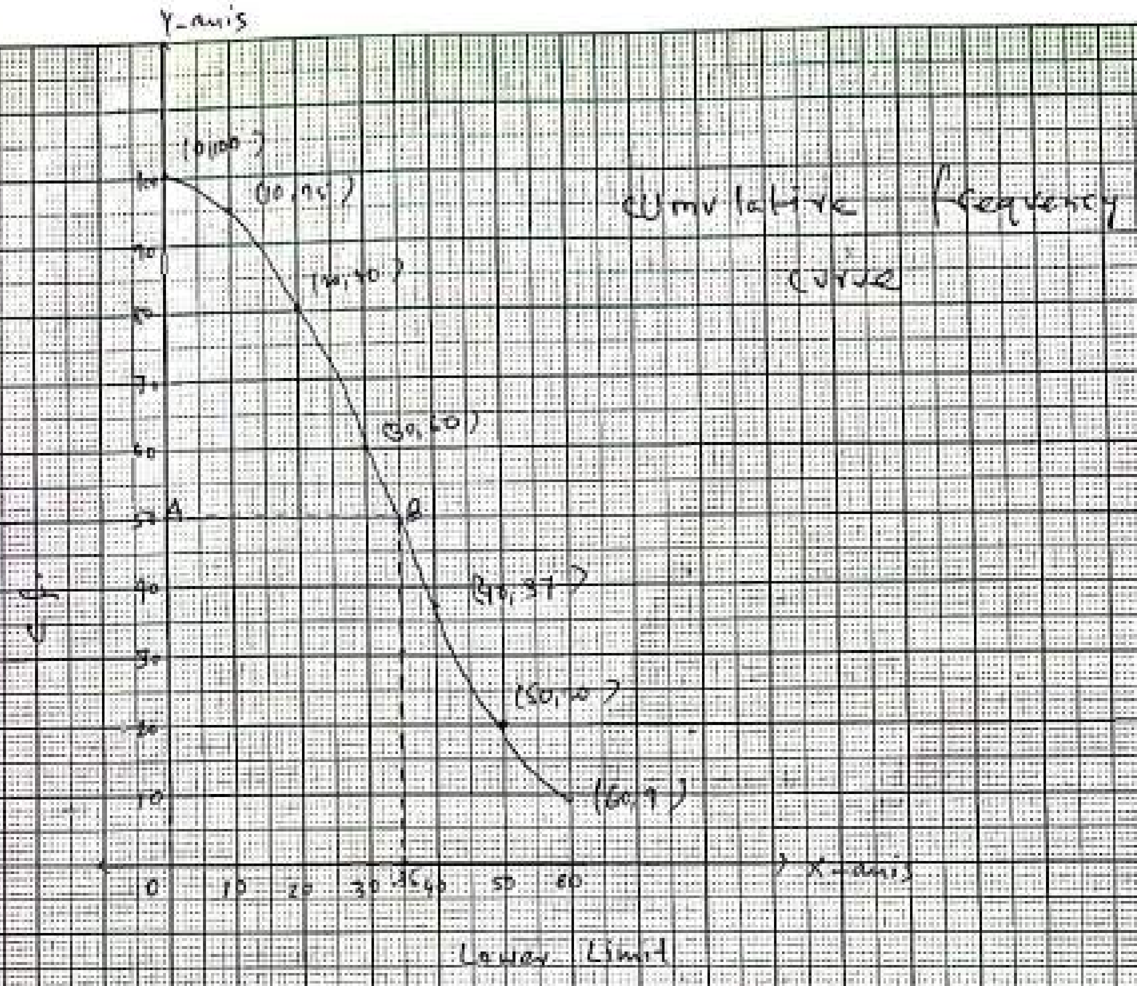
Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Soln.

Class Interval	Frequency	Lower Limit	cf.
0-10	5	0	100
10-20	15	10	95
20-30	20	20	80
30-40	23	30	60
40-50	17	40	37
50-60	11	50	20
60-70	9	60	9
	N=100		

$\frac{N}{2} = \frac{100}{2} = 50$

$\Rightarrow \frac{N}{2} = 50$



Median = 35

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(1) The following table shows the grouped frequency distribution of marks of 60 students. Draw the ogives.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of students	4	8	12	15	12	6	3

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(2) Find graphically the median from ogive of the following distribution. Also find it analytically.

Class Intervals	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65
Frequency	25	35	50	90	75	60	35	25	15

Solve it

1. find the median graphically of the following data

Class interval	0-10	10-20	20-30	30-40	40-50
No. of persons	14	23	27	21	15

2. Draw a histogram and find mode graphically

class interval	145-155	156-165	166-175	176-185	186-195	196-205
frequency	5	7	9	14	6	4

3. Draw the cumulative frequency diagram and hence determine the median marks of students

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	5	12	16	4	3

Mean

Direct Method :

$$\bar{X} = \frac{x_1 + x_2 + \cdots + x_n}{N} = \frac{\text{Sum of observation}}{\text{Number of observation}}$$

Assumed mean Method :

$$\bar{X} = A + \frac{\Sigma d_i}{N}$$

Q. find the mean of the following observation
a) by direct method b) by assumed mean method
480, 492, 495, 500, 505, 515, 485

Soln. Direct method

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{N}$$

$$= \frac{480 + 492 + 495 + 500 + 505 + 515 + 485}{7}$$

$$= \frac{3472}{7} = 496$$

Assumed mean method : $A = 495$

x_i	$d_i = x_i - A$
480	−15
485	−10
492	−3
495	0
500	5
505	10
515	20

$\Sigma d_i = 7$

$$\bar{x} = A + \frac{\Sigma d_i}{N}$$

$$= 495 + \frac{7}{7}$$

$$= 495 + 1$$

$$= 496$$

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Q. find K, if mean of the following observation is 2.5.

1, 2, 3, 2, 3, K, 1, 3, 1, 2, 3, 5

Soln. Direct method

$$\bar{x}\text{ (mean)} = \frac{1+2+3+2+3+K+1+3+1+2+3+5}{12}$$

$$2.5 = \frac{26+K}{12} \Rightarrow 26+K = 30$$

$$K = 4$$

Mean for frequency distribution (Ungrouped data) (x, f)

Direct Method

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1+f_2+\cdots+f_n} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

Assumed mean method:

$$\bar{x} = A + \frac{\Sigma f_i d_i}{N}$$

Q. find the mean of the following data:

a) Direct method b) Assumed mean method

A = 34

x_i	f_i	f_ix_i	$d_i = x_i - A$	f_id_i
32	4	128	−2	−8
33	10	330	−1	−10
34	13	442	0	0
35	8	280	1	8
36	5	180	2	10
	= 40	1360		0

a) Direct Method

$$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{1360}{40} = 34$$

b) Assumed mean method

$$\bar{x} = A + \frac{\Sigma f_id_i}{N}$$

$$= 34 + \frac{0}{40}$$

$$= 34 + 0$$

$$= 34$$

Q. find the mean

Ans.

C.I	x_i	f_i	f_ix_i
15-20	17.5	4	70
20-25	22.5	5	112.5
25-30	27.5	11	302.5
30-35	32.5	6	192.
35-40	37.5	5	187.5
40-45	42.5	8	340
45-50	47.5	9	427.5
50-55	52.5	6	315
55-60	57.5	4	230
60-65	62.5	2	125
		60	2305

$\Sigma f_i = 60$, $\Sigma f_ix_i = 2305$

$$\text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

$$= \frac{2305}{60}$$

$$= 38.42$$

Q. find the mean by step deviation method

Ans.

C.I	x_i	f_i	$u_i = \frac{x_i - a}{c}$ $a=31, c=3$	f_iu_i
36-38	37	2	2	4
33-35	34	5	1	5
30-32	31	17	0	0
27-29	28	10	-1	-10
24-26	25	8	-2	-16
21-33	22	1	-3	-3
		43		-20

$$\bar{U} = \frac{\Sigma f_iU_i}{\Sigma f_i}$$

$$= \frac{-20}{43}$$

$\therefore$ Mean $\bar{x}$

$$= a + c \cdot \bar{U}$$

$$= 31 + 3\left(-\frac{20}{43}\right)$$

$$= 31 - \frac{60}{43}$$

$$= 29.60$$

Solve it

(1) find the mean by assumed mean method
295, 345, 320, 315, 280

(2) Calculate the mean

x_i	4	7	10	13	16	19
f_i	7	10	25	20	25	30

(3) find the mean
a) By direct method b) by step deviation method

Marks	No. of students
0 - 10	3
10 - 20	9
20 - 30	15
30 - 40	8
40 - 50	5

Median

Arranging the data in ascending order

Case I: When the number is odd (3, 5, 7, ...)

$$M = \left(\frac{n+1}{2}\right)\text{th observation}$$

Case II: When the number is even

$$M = \frac{\frac{n}{2}\text{th obs} + \left(\frac{n}{2}+1\right)\text{th obs}}{2}$$

Q. Find the median for the data
34, 32, 48, 38, 24, 30, 27, 21, 35

Soln. Arranging the data in ascending order
21, 24, 27, 30, 32, 34, 35, 38, 48

$n = 9$ (odd)

$$\text{Median} = \left(\frac{n+1}{2}\right)\text{th observation}$$
$$= \left(\frac{9+1}{2}\right)\text{th observation}$$
$$= 5\text{th obs} = 32$$

Q. Find the median of daily wages of 10 workers
50, 70, 40, 75, 30, 60, 80, 90, 100, 55

Soln. Arranging it, 30, 40, 50, 55, 60, 70, 75, 80, 90, 100

$n = 10$ (even)

$$\text{Median} = \frac{\frac{n}{2}\text{th obs} + \left(\frac{n}{2}+1\right)\text{th obs}}{2}$$
$$= \frac{\frac{10}{2}\text{th obs} + \left(\frac{10}{2}+1\right)\text{th obs}}{2}$$
$$= \frac{5\text{th obs} + 6\text{th obs}}{2} = \frac{60+70}{2} = 65$$

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Median for ungrouped Frequency Distribution:

Q. Calculate median for the following data:

x_i	4	7	10	13	16	19
f_i	7	10	15	20	25	30

soln.

x_i	f_i	c.f	
4	7	7	
7	10	17	
10	15	32	
13	20	52	
16	25	77	M. class
19	30	107	
	N=107		

N = 107 which is odd

Median = $\left(\frac{n+1}{2}\right)$th place observation

$= \left(\frac{107+1}{2}\right)$th " "

$= 54$th place "

$= 16$

Median for Grouped frequency distribution

$$\text{Median} = L + \frac{\frac{N}{2} - cf}{f} \times i$$

L = lower class boundry of the median class

N = Total frequency

cf = Less than c.f of a class interval previous to the median class

f = frequency of the median class

i = Width of the class interval

Q. Calculate the median of the frequency distribution.

Wages in Rs Per day	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Workers	16	21	20	28	10	3	1	1

S/N. C.I	f_i	C.F
10-20	16	16
20-30	21	37
30-40	20	57
40-50	28	85
50-60	10	95
60-70	3	98
70-80	1	99
80-90	1	100
	100 = N	

$N = 100$

$\frac{N}{2} = \frac{100}{2} = 50$

which lies in the c.I 30-40

$\therefore$ Median class is 30-40

$l = 30$, $cf = 37$, $f = 20$, $c = 10$

$$\text{Median} = l + \frac{N/2 - cf}{f} \times i$$

$$= 30 + \frac{50-37}{2} \times 10 = 30 + 6.5 = 36.5$$

Solve it.

(1) The score of cricket player in test series 3, 24, 0, 44, 44, 0, 90, 9, 87, 11. find median

(2) ~~below~~ weight of 60 boys are. find median

W. in kg	35	40	45	50	55
No. of boys	8	14	19	12	7

(3) Calculate median

Wages	1-3	3-5	5-7	7-9	9-11	11-13
No. of workers	2	6	10	12	9	5

Q. Calculate the mode

Wages	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Workers	16	21	20	28	10	3	1	1

Soln. Maximum frequency = 28

∴ Modal class is 40-50

C.I	Frequency	
10-20	16	
20-30	21	
30-40	20	f_1
40-50	28	f_m
50-60	10	f_2
60-70	3	
70-80	1	
80-90	1	

$f_m = 28$

$f_1 = 20$

$f_2 = 10$

$i = 10$

$L = 40$

$$\text{Mode} = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i$$

$$= 40 + \frac{28-20}{2(28)-20-10} \times 10$$

$$= 40 + \frac{8 \times 10}{26} = 40 + 3.077$$

$$= 43.077$$

Q. The crushing strength of 45 cement concrete blocks are

Crushing strength	146-155	156-165	166-175	176-185	186-195	196-205
No. of blocks	5	7	9	14	6	4

Soln.

C.I	Frequency	
145.5-155.5	5	
155.5-165.5	7	
165.5-175.5	9	f_1
175.5-185.5	14	f_m
185.5-195.5	6	f_2
195.5-205.5	9	

$f_m = 14$, $f_1 = 9$, $f_2 = 6$, $i = 10$, $L = 175.5$

$$\text{Mode} = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i$$

$$= 175.5 + \frac{14-9}{2(14)-9-6} \times 10$$

$$= 175.5 + \frac{5}{13} \times 10$$

$$= 175.5 + 3.846$$

$$= 179.346$$

Mode

Mode of data is that value which occurs with the maximum frequency or maximum number of times.

Q. The daily earnings of 12 workers in a workplace Rs. 16, 19, 12, 14, 13, 17, 16, 19, 20, 15, 16, 13 find the mode.

Soln. Arrange in increasing order

12, 13, 13, 14, 15, 16, 16, 16, 17, 19, 19, 20

Mode = Maximum times
= 16

Mode for Ungrooped Frequency Distribution
Mode can be found by maximum frequency

Q. Find the mode of the frequency distribution

Age in years	13	14	15	16	17	18
No. of students	10	12	20	14	9	3

Soln. In this inspection the maximum frequency is 20.

Mode = 15

Mode for Grouped frequency distribution

$$M = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times i$$

L = lower limit
f_1 = Preceding the modal class
f_2 = frequency of " " "
f_m = maximum frequency , i = Difference of Limits

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i = Imaginary Number $= \sqrt{-1}$

Power of i

1) $i^0 = 1$

2) $i^1 = i = \sqrt{-1}$

3) $i^2 = -1$

4) $i^3 = -i$

5) $i^4 = 1$

$$4 \overline{)\,19\,}(4 \quad \frac{16}{3}$$

Ex. Evaluate i^{19}

Soln. $i^{19} = i^4 \times i^4 \times i^4 \times i^4 \times i^3$

$= 1 \times 1 \times 1 \times 1 \times -i$

$= -i$

$i^{19} = (i^4)^4 \times i^3$

$= 1 \times -i$

$= -i$

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Q. Evaluate $i^{38} + i^{26}$

Sol^n: $i^{38} = (i^4)^9 \times i^2$

$= 1^9 \times -1$

$= 1 \times -1$

$= -1$

$i^{26} = (i^4)^6 \times i^2$

$= 1 \times -1$

$= -1$

$\therefore i^{38} + i^{26} = -1 - 1 = -2$

Q. Evaluate $i^9 + \frac{1}{i^9}$

Sol^n: $\frac{i^9}{1} + \frac{1}{i^9}$

$$= \frac{i^{18} + 1}{i^9}$$

$$= \frac{(i^4)^4 \times i^2 + 1}{(i^4)^2 \times i^1}$$

$$= \frac{1 \times -1 + 1}{1^2 \times i}$$

$$= \frac{-1 + 1}{i}$$

$$= \frac{0}{i}$$

$= 0$

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Conjugate of C.N

$$z = a+ib \qquad \frac{1}{a+ib}$$

Conjugate of z,

$$\bar{z} = \overline{(a+ib)}$$

$$= a - ib$$

Modulus of C.N

$$z = a+ib$$

$$|z| = |a+ib|$$

$$|z| = \sqrt{a^2+b^2}$$

Ex modulus of $3+4i$

Sol: $|3+4i| = \sqrt{3^2+4^2}$

$$= \sqrt{25}$$

$$= 5$$

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(2) Convert the complex Number $1+i\sqrt{3}$ in polar form

Soln.

$$z = 1 + i\sqrt{3} = x + iy$$

$$x = 1, \quad y = \sqrt{3}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3} = \sqrt{4} = 2$$

$$r = 2$$

$$\theta = \alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$$

$$\theta = \alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Polar form

$$z = r\left[\cos\theta + i\sin\theta\right]$$

$$= 2\left\{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right\}$$

Polar form (r, θ)

$z = x + iy$ (Cartesian form

Polar form $= r\cos\theta + ir\sin\theta$

$$\boxed{z = r[\cos\theta + i\sin\theta]}$$

$r = \sqrt{x^2 + y^2}$

$\left[\alpha = \tan^{-1}\left|\frac{y}{x}\right|\right]$

arg z

i) z lies I

$\theta = \alpha$

ii) z lies II

$\theta = \pi - \alpha$

iii) z lies in III

$\theta = -(\pi - \alpha)$

iv) z lies in 4 th

$\theta = 2\pi - \alpha$

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Q. Convert $(1+i)$ in polar form

Soln.

Let $Z = 1 + i.1 = x + iy$

R.P → Imag Part

$x = 1$

$y = 1$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{1}{1}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

45
$= 45 \times \frac{\pi}{180}$
$\frac{\pi}{4}$

Polar form

$z = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

Teacher's Signature : ______

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(3) Find the modulus and argument $-\sqrt{3}+i$

soln,

$$z = -\sqrt{3} + i \cdot 1 = x + i \cdot y$$

$$x = -\sqrt{3}, \quad y = 1 \quad (-, +)$$

modulus

$$z = |x+iy| = \sqrt{x^2+y^2}$$

$$\text{Modulus} = \sqrt{(-\sqrt{3})^2 + 1^2}$$

$$= \sqrt{4}$$

$$|-\sqrt{3}+i| = 2$$

$$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$$

$$= \tan^{-1}\left|\frac{1}{-\sqrt{3}}\right|$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Teacher's Signature :

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$$\operatorname{Arg} z = \theta = \pi - \alpha$$
$$= \pi - \frac{\pi}{6}$$
$$= \frac{5\pi}{6}$$

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A number in the form of a+ib is called complex Number

$$z = a+ib$$

z = complex Number
a = Real Number
b = Imaginary Number

Ex

$$z = 3+2i$$

Real NO = 3
Im NO. = 2

Imaginary Number = Negative square root

$$i = \sqrt{-1}$$

$$= \sqrt{-1},\ \sqrt{-4} \ldots$$

$$\sqrt{+16} = 4$$

$$\sqrt{-16} = \sqrt{-1 \times 16} = \underline{\sqrt{-1}} \times \underline{\sqrt{16}}$$

$$= i \times 4$$

$$= 4i$$

$$\sqrt{-81} = \sqrt{-1} \times \sqrt{81}$$

$$= i \times 9 = 9i$$

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Q. find the conjugate of $-3+4i$

Soln. $\overline{-3+4i} = -3-4i$

Q. find the conjugate of $\frac{1}{3-2i}$

Soln. $\frac{1}{3-2i} \times \frac{3+2i}{3+2i}$

$= \frac{3+2i}{3^2-(2i)^2}$

$= \frac{3+2i}{9-4 \cdot i^2}$

$= \frac{3+2i}{9-4(-1)}$

$= \frac{3+2i}{13} = \frac{3}{13} + \frac{2i}{13}$

$\overline{\left(\frac{3}{13} + \frac{2i}{13}\right)} = \frac{3}{13} - \frac{2i}{13}$

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Multiplicative inverse

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

Q. Find the multiplicative inverse of $4 - 3i$

Sol^n. Let $z = 4 - 3i$

$$\bar{z} = \overline{4 - 3i}$$

$$\bar{z} = 4 + 3i$$

$$|z| = \sqrt{(4)^2 + (-3)^2}$$

$$= 5$$

$$|z|^2 = 25$$

$$\therefore z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25}$$

$$= \frac{4}{25} + \frac{3i}{25}$$

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